

PROBLEM 1.3

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the magnitude of the force \mathbf{P} for which the tensile stress in rod AB is twice the magnitude of the compressive stress in rod BC .

SOLUTION

$$A_{AB} = \frac{\pi}{4} (50)^2 = 1963.5 \text{ mm}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{1963.5}$$

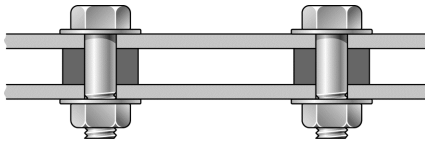
$$A_{BC} = \frac{\pi}{4} (75)^2 = 4417.9 \text{ mm}^2$$

$$\begin{aligned} \sigma_{BC} &= \frac{2(120) - P}{A_{BC}} \\ &= \frac{240 - P}{4417.9} \end{aligned}$$

Equating σ_{AB} to $2\sigma_{BC}$

$$\frac{P}{1963.5} = \frac{2(240 - P)}{4417.9}$$

$$P = 112.9 \text{ kN} \blacktriangleleft$$



PROBLEM 1.5

Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

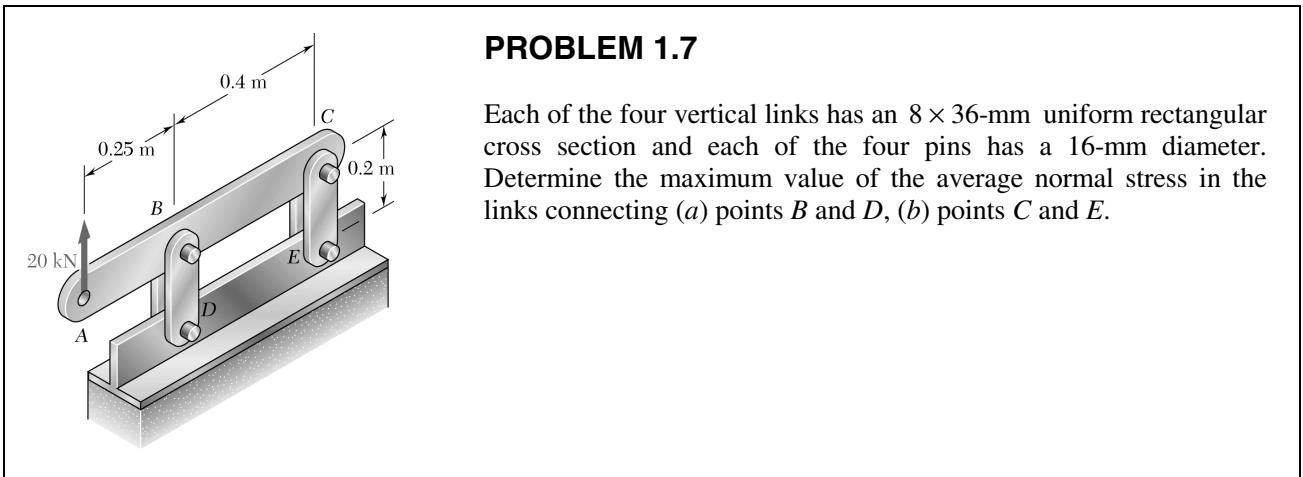
For the bolt,
$$\sigma_b = \frac{F_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad \text{or} \quad P_b = \frac{\pi}{4} \sigma_b d_b^2$$

For the spacer,
$$\sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad \text{or} \quad P_s = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

Equating P_b and P_s ,

$$\frac{\pi}{4} \sigma_b d_b^2 = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)} d_b = \sqrt{\left(1 + \frac{200}{130}\right)} (16) \quad d_s = 25.2 \text{ mm} \quad \blacktriangleleft$$

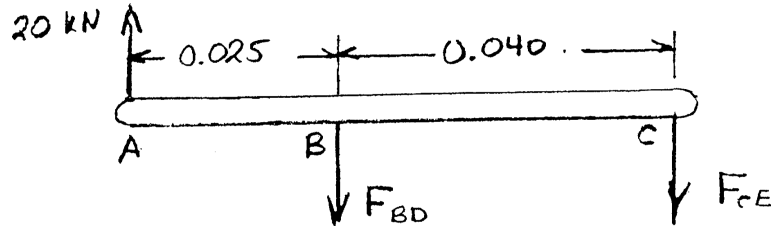


PROBLEM 1.7

Each of the four vertical links has an 8×36 -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D , (b) points C and E .

SOLUTION

Use bar ABC as a free body.



$$\sum M_C = 0 : (0.040) F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \quad \text{Link } BD \text{ is in tension.}$$

$$\sum M_B = 0 : -(0.040) F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \quad \text{Link } CE \text{ is in compression.}$$

$$\text{Net area of one link for tension} = (0.008)(0.036 - 0.016) = 160 \times 10^{-6} \text{ m}^2.$$

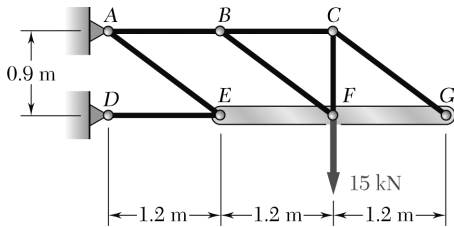
$$\text{For two parallel links, } A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$$

$$(a) \quad \sigma_{BD} = \frac{F_{BD}}{A_{\text{net}}} = \frac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6 \quad \sigma_{BD} = 101.6 \text{ MPa} \quad \blacktriangleleft$$

$$\text{Area for one link in compression} = (0.008)(0.036) = 288 \times 10^{-6} \text{ m}^2.$$

$$\text{For two parallel links, } A = 576 \times 10^{-6} \text{ m}^2$$

$$(b) \quad \sigma_{CE} = \frac{F_{CE}}{A} = \frac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^6 \quad \sigma_{CE} = -21.7 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 1.11

The rigid bar EFG is supported by the truss system shown. Knowing that the member CG is a solid circular rod of 18 mm diameter, determine the normal stress in CG .

SOLUTION

Using portion $EF GCB$ as a free body

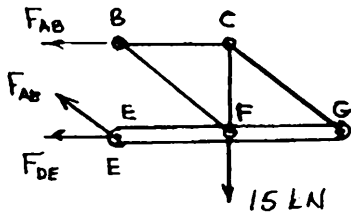
$$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AB} - 15 = 0$$

$$F_{AE} = 25 \text{ kN}$$

Using beam $EF G$ as a free body

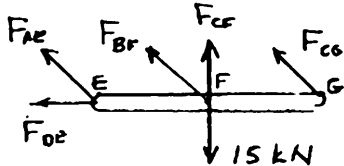
$$+\curvearrowright M_F = 0: -(1.2) \frac{0.9}{1.2} F_{AE} + (1.2) \left(\frac{0.9}{1.2} F_{CG} \right) = 0$$

$$F_{CG} = F_{AE} = 25 \text{ kN}$$



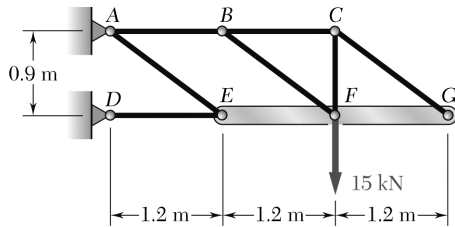
Cross sectional area of member CG

$$A_{CG} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.018)^2 = 254.4 \times 10^{-6} \text{ m}^2$$



Normal stress in CG .

$$\sigma_{CG} = \frac{F_{CG}}{A_{CG}} = \frac{25}{254.4 \times 10^{-6}} = 98.3 \text{ MPa}$$



PROBLEM 1.12

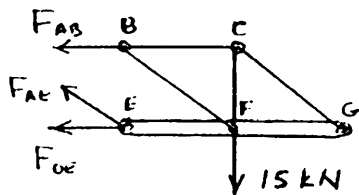
The rigid bar EFG is supported by the truss system shown. Determine the cross-sectional area of member AE for which the normal stress in the member is 105 MPa.

SOLUTION

Using portion $EFGCB$ as a free body

$$+\uparrow \Sigma F_y = 0: \frac{0.9}{1.5} F_{AE} - 15 = 0$$

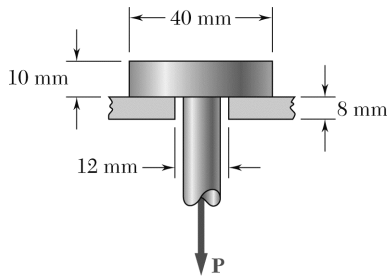
$$F_{AE} = 25 \text{ kN}$$



Stress in member AE $\sigma_{AE} = 105 \text{ MPa}$

$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}}$$

$$A_{AE} = \frac{F_{AE}}{\sigma_{AE}} = \frac{25 \times 10^3}{105 \times 10^6} = 238.1 \times 10^{-6} \text{ m}^2$$



PROBLEM 1.17

A load P is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load P that can be applied to the rod.

SOLUTION

For the steel rod,

$$A_1 = \pi d_1 t_1 = (\pi)(0.012)(0.010)$$

$$= 376.99 \times 10^{-6} \text{ m}^2$$

$$\tau_1 = \frac{P}{A_1} \rightarrow P_1 = \tau_1 A_1$$

$$P_1 = (180 \times 10^6)(376.99 \times 10^{-6}) = 67.86 \times 10^3 \text{ N}$$

For the aluminum plate,

$$A_2 = \pi d_2 t_2 = (\pi)(0.040)(0.008) = 1.00531 \times 10^{-3} \text{ m}^2$$

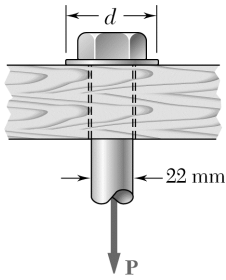
$$\tau_2 = \frac{P_2}{A_2} \rightarrow P_2 = \tau_2 A_2$$

$$P_2 = (70 \times 10^6)(1.0053 \times 10^{-6}) = 70.372 \times 10^3 \text{ N}$$

The limiting value for the load P is the smaller of P_1 and P_2 .

$$P = 67.86 \times 10^3 \text{ N}$$

$$P = 67.9 \text{ kN} \blacktriangleleft$$



PROBLEM 1.19

The load \mathbf{P} applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer, knowing that the axial normal stress in the steel rod is 35 MPa and that the average bearing stress between the washer and the timber must not exceed 5 MPa.

SOLUTION

$$\text{Steel rod: } A = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\sigma = 35 \times 10^6 \text{ Pa}$$

$$\begin{aligned} P &= \sigma A = (35 \times 10^6)(380.13 \times 10^{-6}) \\ &= 13.305 \times 10^3 \text{ N} \end{aligned}$$

$$\text{Washer: } \sigma_b = 5 \times 10^6 \text{ Pa}$$

Required bearing area:

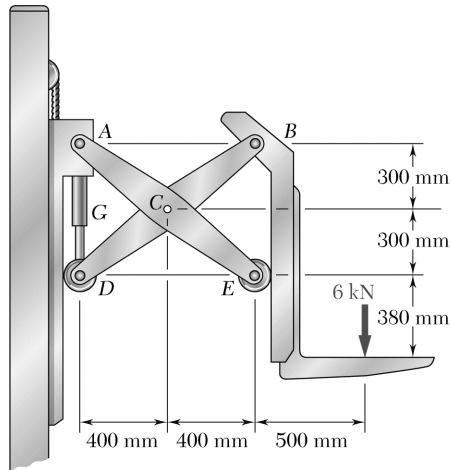
$$A_b = \frac{P}{\sigma_b} = \frac{13.305 \times 10^3}{5 \times 10^6} = 2.6609 \times 10^{-3} \text{ m}^2$$

$$\text{But, } A_b = \frac{\pi}{4}(d^2 - d_i^2)$$

$$\begin{aligned} d^2 &= d_i^2 + \frac{4A_b}{\pi} \\ &= (0.025)^2 + \frac{(4)(2.6609 \times 10^{-3})}{\pi} \\ &= 4.013 \times 10^{-3} \text{ m}^2 \\ d &= 63.3 \times 10^{-3} \text{ m} \end{aligned}$$

$$d = 63.3 \text{ mm} \blacktriangleleft$$

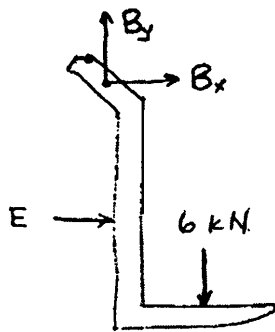
PROBLEM 1.23



Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 6 kN. Knowing that the thickness of member BD is 16 mm, determine (a) the average shearing stress in the 12-mm-diameter pin at B , (b) the bearing stress at B in member BD .

SOLUTION

Use one fork as a free body.



$$+\curvearrowright \Sigma M_B = 0: 0.6E - (0.5)(6) = 0$$

$$E = 5 \text{ kN} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: E + B_x = 0 \quad B_x = -E$$

$$B_x = 5 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: B_y - 6 = 0 \quad B_y = 6 \text{ kN}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{5^2 + 6^2} = 7.81 \text{ kN}$$

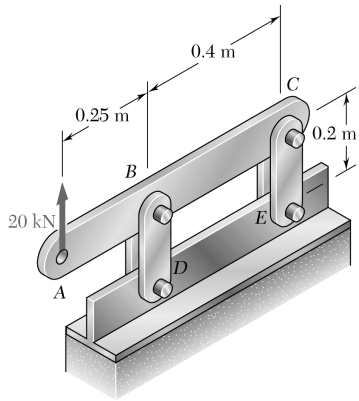
(a) Shearing stress in pin at B .

$$A_{\text{pin}} = \frac{\pi}{4} d_{\text{pin}}^2 = \frac{\pi}{4} (0.012)^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$\tau = \frac{B}{A_{\text{pin}}} = \frac{7.81 \times 10^3}{113.1 \times 10^{-6}} = 69 \text{ MPa} \quad \blacktriangleleft$$

(b) Bearing stress at B .

$$\sigma = \frac{B}{dt} = \frac{7.8 \times 10^3}{(0.012)(0.016)} = 40.6 \text{ MPa} \quad \blacktriangleleft$$



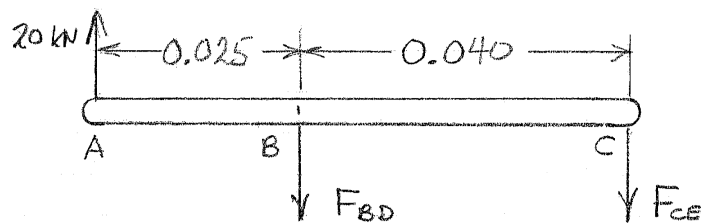
PROBLEM 1.27

For the assembly and loading of Problem 1.7, determine (a) the average shearing stress in the pin at B, (b) the average bearing stress at B in member BD, (c) the average bearing stress at B in member ABC, knowing that this member has a 10×50 -mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8×36 -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.

SOLUTION

Use bar ABC as a free body.



$$+\circlearrowleft \sum M_C = 0: (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B $\tau = \frac{F_{BD}}{2A}$ for double shear,

where $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \quad \tau = 80.8 \text{ MPa} \quad \blacktriangleleft$$

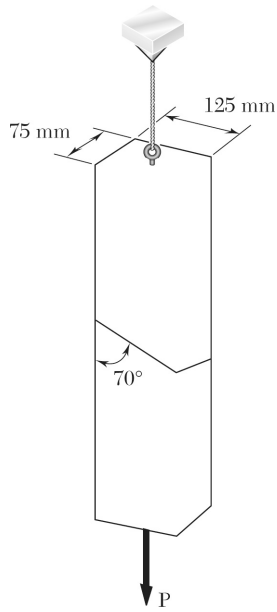
(b) Bearing: link BD $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \quad \sigma_b = 127.0 \text{ MPa} \quad \blacktriangleleft$$

(c) Bearing in ABC at B $A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \quad \sigma_b = 203 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 1.30



Two wooden members of 75×125 mm uniform rectangular cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (a) the largest load P which can be safely supported, (b) the corresponding shearing stress in the splice.

SOLUTION

$$A_o = (0.075)(0.125) = 9.375 \times 10^{-3} \text{ m}^2$$

$$\theta = 90^\circ - 60^\circ = 30^\circ \quad \sigma = 500 \times 10^3 \text{ Pa}$$

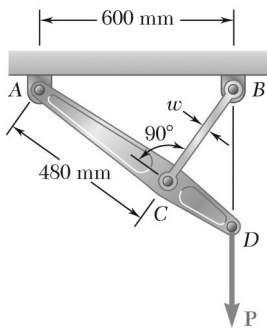
$$\sigma = \frac{P}{A_o} \cos^2 \theta$$

$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(9.375 \times 10^{-3})(500 \times 10^3)}{\cos^2 30^\circ} = 6.25 \times 10^3 \text{ N}$$

(a) $P = 6.25 \text{ kN}$ ◀

$$\tau = \frac{P \sin 2\theta}{2 A_\theta} = \frac{(6.25 \times 10^3) \sin 60^\circ}{(2)(9.375 \times 10^{-3})} = 288.68 \times 10^3$$

(b) $\tau = 289 \text{ kPa}$ ◀

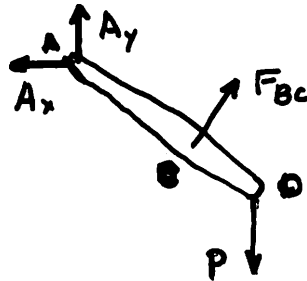


PROBLEM 1.37

Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load P ?

SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$$\Sigma M_A = 0:$$

$$(480)F_{BC} - (600)P = 0$$

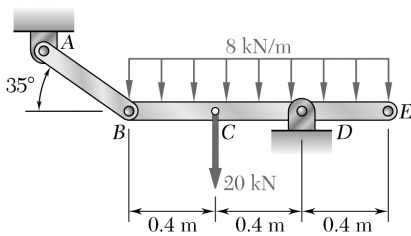
$$F_{BC} = \frac{600}{480}P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{ N}$$

Ultimate load for member BC : $F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{ N}$$

Factor of safety: $F.S. = \frac{F_U}{F_{BC}} = \frac{72 \times 10^3}{20 \times 10^3}$

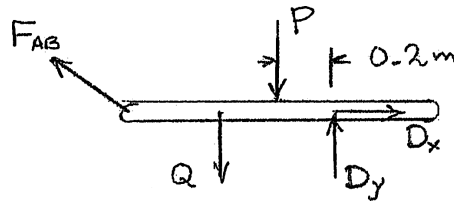
$$F.S. = 3.60 \blacktriangleleft$$



PROBLEM 1.41

Link AB is to be made of a steel for which the ultimate normal stress is 450 MPa. Determine the cross-sectional area for AB for which the factor of safety will be 3.50. Assume that the link will be adequately reinforced around the pins at A and B .

SOLUTION



$$P = (1.2)(8) = 9.6 \text{ kN}$$

$$+\circlearrowleft \sum M_D = 0 : \quad -(0.8)(F_{AB} \sin 35^\circ) \\ + (0.2)(9.6) + (0.4)(20) = 0$$

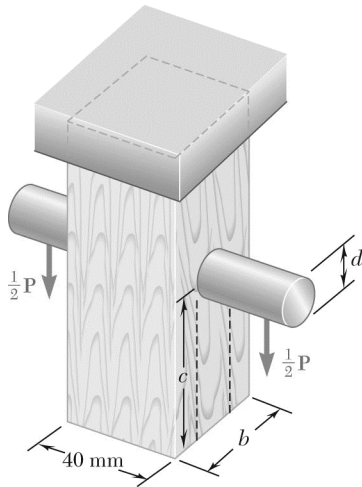
$$F_{AB} = 21.619 \text{ kN} = 21.619 \times 10^3 \text{ N}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{\sigma_{\text{ult}}}{\text{F.S.}}$$

$$A_{AB} = \frac{(\text{F.S.})F_{AB}}{\sigma_{\text{ult}}} = \frac{(3.50)(21.619 \times 10^3)}{450 \times 10^6} \\ = 168.1 \times 10^{-6} \text{ m}^2$$

$$A_{AB} = 168.1 \text{ mm}^2 \quad \blacktriangleleft$$

PROBLEM 1.45



A load P is supported as shown by a steel pin that has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 145 MPa in shear. Knowing that $b = 40$ mm, $c = 55$ mm, and $d = 12$ mm, determine the load P if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin:

$$\begin{aligned}P_U &= 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U \\ &= \frac{\pi}{4}(2)(0.012)^2(145 \times 10^6) = 32.80 \times 10^3 \text{ N}\end{aligned}$$

Based on tension in wood:

$$\begin{aligned}P_U &= A\sigma_U = w(b-d)\sigma_U \\ &= (0.040)(0.040 - 0.012)(60 \times 10^6) \\ &= 67.2 \times 10^3 \text{ N}\end{aligned}$$

Based on double shear in the wood:

$$\begin{aligned}P_U &= 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5 \times 10^6) \\ &= 33.0 \times 10^3 \text{ N}\end{aligned}$$

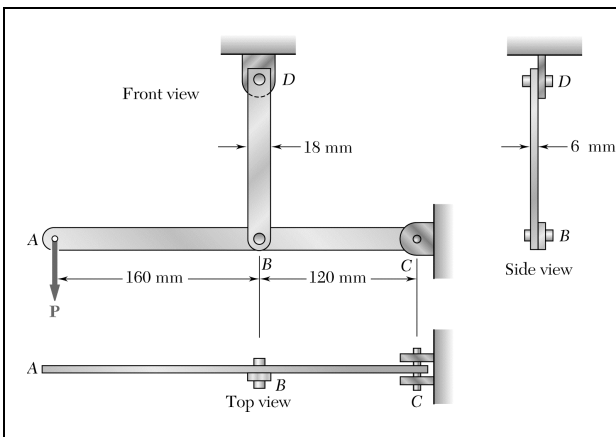
Use smallest

$$P_U = 32.8 \times 10^3 \text{ N}$$

Allowable:

$$P = \frac{P_U}{\text{F.S.}} = \frac{32.8 \times 10^3}{3.2} = 10.25 \times 10^3 \text{ N}$$

10.25 kN ◀

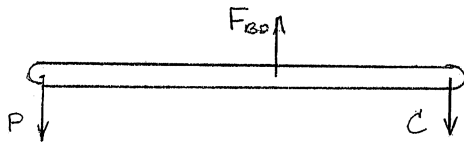


PROBLEM 1.51

In the steel structure shown, a 6-mm-diameter pin is used at C and 10-mm-diameter pins are used at B and D . The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link BD . Knowing that a factor of safety of 3.0 is desired, determine the largest load P that can be applied at A . Note that link BD is not reinforced around the pin holes.

SOLUTION

Use free body ABC .



$$+\circlearrowleft \Sigma M_C = 0 : 0.280P - 0.120 F_{BD} = 0$$

$$P = \frac{3}{7} F_{BD} \quad (1)$$

$$+\circlearrowleft \Sigma M_B = 0 : 0.160P - 0.120C = 0$$

$$P = \frac{3}{4} C \quad (2)$$

Tension on net section of link BD .

$$F_{BD} = \sigma A_{\text{net}} = \frac{\sigma_U}{\text{F.S.}} A_{\text{net}} = \left(\frac{400 \times 10^6}{3} \right) (6 \times 10^{-3})(18 - 10)(10^{-3}) = 6.40 \times 10^3 \text{ N}$$

Shear in pins at B and D .

$$F_{BD} = \tau A_{\text{pin}} = \frac{\tau_U}{\text{F.S.}} \frac{\pi}{4} d^2 = \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (10 \times 10^{-3})^2 = 3.9270 \times 10^3 \text{ N}$$

Smaller value of F_{BD} is $3.9270 \times 10^3 \text{ N}$.

$$\text{From (1)} \quad P = \left(\frac{3}{7} \right) (3.9270 \times 10^3) = 1.683 \times 10^3 \text{ N}$$

$$\text{Shear in pin at } C. \quad C = 2\tau A_{\text{pin}} = 2 \frac{\tau_U}{\text{F.S.}} \frac{\pi}{4} d^2 = (2) \left(\frac{150 \times 10^6}{3} \right) \left(\frac{\pi}{4} \right) (6 \times 10^{-3})^2 = 2.8274 \times 10^3 \text{ N}$$

$$\text{From (2)} \quad P = \left(\frac{3}{4} \right) (2.8274 \times 10^3) = 2.12 \times 10^3 \text{ N}$$

Smaller value of P is allowable value. $P = 1.683 \times 10^3 \text{ N}$

$P = 1.683 \text{ kN} \blacktriangleleft$