Problem Set 1

1.1) Known

$F = 30\,lbf$

$W = 10\,lbf$

$F - W = ma = (30-10)\,lbf = \left(\frac{10\,lbf}{32.2\,ft/s^2}\right) a$ $\boxed{a = 64.4\ ft/s^2}$

1.2) Known

$F = 30\,lbf$

$g = 5.31\,ft/s^2$

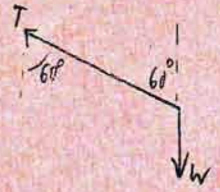
$F - W = ma$ $(30 - 1.65)\,lbf = \left(\frac{1.65\,lbf}{5.31\,ft/s^2}\right) a$ $\boxed{a = 91.23\ ft/s^2}$

1.3) Known

$L = 1m$

$m = 1kg$

$\theta = 60°$



$W = mg = 9.81\,m/s$ $\Sigma F_y = 0$ $T\cos 60 - W = 0$

$\boxed{T = 19.62N}$

$a = \frac{T}{m} = \frac{19.62N}{1kg} = 19.62\,m/s^2$ $r = L\sin 60 = 0.866m$

$a = \omega^2 r$ $\omega^2 = \frac{a}{r} = \frac{19.62\sin 60}{0.866}$ $\omega = 4.43\,rad/s \cdot (rev/2\pi\,rad) \cdot (60s/min) = \boxed{42.3\,RPM}$

1.4) F, F, F, T 1.5) d

1.6)

psia	psig	psfa	psfg	kPa(a)	kPa(g)
14.696	0	2116.2	0	101.33	0
39.696	25	5716.2	3600	273.70	172.37
6.840	-7.856	985	-1131.2	47.16	-54.17
29.39	14.694	4232.2	2116	202.64	101.31
14.695	0.001	2116.1	0.1	101.32	0
14502.4	145037.7	20887551.4	20885435.2	100101.33	10^5

1.7) Known

$m = 3000kg$

$d = 0.25m$

$r = 0.125m$

$F = mg = (3000\,kg)(9.81\,m/s) = 29.43\,kN$ $A = \pi r^2 = 0.0491\,m^2$

$P = \frac{F}{A} = \frac{29.43\,kN}{0.0491\,m^2} = \boxed{600\,kPa}$

1.8) Known

$P_{atm} = 19.399\ kPa(a) = 2.814\ psia$

$P_{O_2} = 100\ kPa(g)$ $\quad P_A = 8\,psig + 2.814\,psia = 10.814\,psia = \boxed{74.6\ kPa(a)}$

$h = 12000\,m$ $\quad P_{O_2} = 100\,kPa(g) + 74.6\,kPa(a) = \boxed{174.6\,kPa(a)}$

1.9) Known

$PV = mRT \quad m_1 R_1 T_1 = m_2 R_2 T_2$

$P_1 = 20\,psig = 34.7\,psia \quad P_1 V_1 = P_2 V_2$

$V_2 = \frac{1}{3} V_1$

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{(34.7\,psia)(3)}{(1)} = 104.1\,psia = 89.4\,psig$$

1.10) Known

$h_1 = 266\,m$

$h_2 = 915\,m$

$T = -20°C = 253°K$

$P = 100\,kPa$

$R = 0.287\,kJ/kgK$

$B = 0.00650\,K/m$

a) $$\rho = \frac{P}{RT} = \frac{100\,kPa}{(0.287\,kJ/kgK)(253\,K)} = 1.377\,\frac{kN\cdot kg/m^3}{kN\cdot m} = \boxed{1.377\,kg/m^3}$$

b) $$-\rho g = \frac{dP}{dy}$$

$$\int dP = -\int \rho g\, dy$$

$$\Delta P = (-1.377\,kg/m^3)(9.81\,m/s^2)(915\,m - 266\,m)$$

$$= -8767\,kg/m\cdot s^2 = \boxed{-8767\,Pa}$$

---

c) $$-\rho g = \frac{dP}{dy} \rightarrow \frac{-Pg}{RT} = \frac{dP}{dy} \rightarrow \int_{P_1}^{P_2} \frac{dP}{P} = \int_{h_1}^{h_2} \frac{-g}{RT}\,dy \rightarrow \ln\left(\frac{P_2}{P_1}\right) = \frac{-g}{RT}(h_2 - h_1)$$

$$\rightarrow \ln(P_2) = \frac{-g}{RT}(h_2 - h_1) + \ln(P_1) = \frac{-(9.81\,m/s^2)(915-266)m}{(287\,J/kgK)(253\,K)} + \ln(100000)$$

$\ln(P_2) = 11.42524$

$P_2 = e^{11.425245}$

$= 91605\,Pa$

$$\Delta P = P_2 - P_1 = (91605 - 100000)\,Pa = \boxed{-8395\,Pa}$$

---

d) $$T = T_0 - B(y - y_0) = 253 - 0.00650\,K/\cancel{m}\,(915 - 266)\cancel{m} = 248.78\,K$$

$$P_2 = P_0\left[\frac{T_0 - B(z - z_0)}{T_0}\right]^{\frac{g}{BR}} = (100000\,Pa)\left(\frac{248.78\,\cancel{K}}{253\,\cancel{K}}\right)^{\frac{9.81\,m/s^2}{(0.00650\,K/m)(287\,J/kgK)}} = 91524.8\,Pa$$

$\Delta P = P_2 - P_1 = (91524.8 - 100000)\,Pa$

$\boxed{= -8475\,Pa}$

$$\rho = \frac{P}{RT} = \frac{91524.8\,Pa}{(287\,J/kgK)(248.78\,K)} = \boxed{1.282\,kg/m^3}$$

1.11) Known

$h_2 = 10 km$

$P_2 = 100 MPa$

$P_1 = 0.1 MPa$

$k = 2100 MPa$

$SG = 1.03$

$h_1 = 0 km$

$\rho_1 = SG(1000 kg/m^3)$
$= 1030 kg/m^3$

$m = \rho V$

$$k = -V\left(\frac{\partial P}{\partial V}\right) = \frac{-\Delta P/\Delta V}{V} = \frac{-\Delta P V}{\Delta V} = \frac{-\Delta P(m/\rho_1)}{(m/\rho_2)-(m/\rho_1)}$$

$$\rightarrow = \frac{-\Delta P(m/\rho_1)}{(m\rho_1 - m\rho_2)/\rho_2\rho_1} = \frac{\cancel{m}(-P_\Delta \rho_1 \rho_2)}{\cancel{m}\rho_1(\rho_1-\rho_2)} = \frac{-\Delta P \rho_2}{\rho_1 - \rho_2}$$

$$\rightarrow \rho_1 - \rho_2 = \frac{-\Delta P \rho_2}{k} \longrightarrow \rho_2\left(\frac{\Delta P}{k} - 1\right) = -\rho_1$$

$$\rightarrow \rho_2 = \frac{-\rho_1}{(\Delta P/k - 1)} = \frac{-1030 kg/m^3}{\left(\frac{(100-0.1)MPa}{2100 MPa} - 1\right)} = \boxed{1081 kg/m^3}$$

1.12) Known

$\mu = 2\times10^{-5} Ns/m^2$

$l = 0.05m$

$r_p = 0.025m$

$r_c = 0.02505m$

$v = 12 m/s$

$A_p = 7.8540\times10^{-3} m^2$

$A_c = 7.8697\times10^{-3} m^2$

$$\tau_p = \mu \frac{dv}{dy} A_p = \frac{(2\times10^{-3} Ns/m^2)(12 m/s)(7.8540\times10^{-3} m^2)}{(0.02505-0.025)m}$$

$$= \boxed{3.77 N}$$

1.13) Known

$R = 0.06m$

$r = 0.03m$

$L = 0.15m$

$t = 0.001m$

$m = 0.03kg$

$\omega = 30 RPM = \pi\ rad/s$

$W = mg = 0.2943N$

a)

$$M_{pulley} = M_p = rW = 0.03m(0.2943N) = 0.008829 Nm$$

$$\tau = \mu dV/dy \quad v = R\omega \quad A_{cylinder} = A_c = RL$$

$$M_L = \tau R A_a = \mu R dV A_c/dy = 2\mu R^2\omega\pi RL/t = 2\mu R^3 L\pi\omega/t$$

$$M_L - M_p = 0 \quad M_L = M_p \quad 2\mu PL\pi\omega/t = M_p$$

$$\mu = \frac{M_p t}{2R^3 L\pi\omega} = \frac{(0.008829 N\cdot m)(0.001 m)}{2(0.06)^3 m^3(0.15m)(\pi)(\pi\ rad/s)} = \boxed{1.38\times10^{-2} Ns/m^2}$$

1.13...) b) $M_{\text{bottom}} = M_b = \int r\,dF = \int r\mu(r\omega/t)(2\pi r)\,dr = \int_0^R (2\mu\omega\pi r^3/t)\,dr = (2\pi\mu\omega/t)\int_0^R r^3\,dr$

$= \mu\omega\pi R^4/2t$

$M_p - M_L - M_b = 0 \qquad M_p = M_L + M_b$

$0.008829\ N{\cdot}m = 2\mu R^3 L\pi\omega/t + \mu\omega\pi R^4/2t$

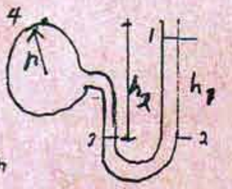
$\longrightarrow \mu = \dfrac{(0.008829\ N{\cdot}m)}{\omega\pi\left(2R^3L/t + R^4/2t\right)} = \dfrac{(0.008829\ N{\cdot}m)}{(\pi)(\pi\ rad/s)\left[2(0.06m)^3(0.15m)/0.001m + (0.06m)^4/2(0.001m)\right]}$

$= \dfrac{0.008829\ Nm}{0.7035\ m^3/s} = \boxed{1.26\times10^{-2}\ Ns/m^2}$

Problem Set 2

2.1) Known

$h_1 = 0.15m$
$h_2 = 0.05m$
$S.G. = 13.6m$
$P_1 = 0 \text{ kPa(g)}$
$r = 0.03m$



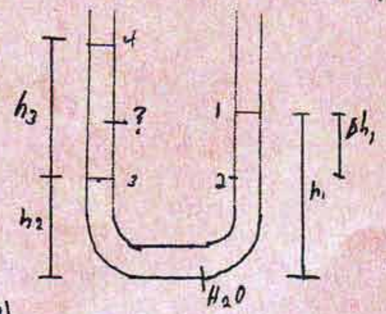
$P_2 = P_1 + \rho_{Hg} g h_1$
$= 1000 kg/m^3 (13.6)(9.81 m/s^2)(0.15 m)$
$= 20012.4 \text{ kg/ms}^2$
$= 20.0 \text{ KPa}$

Note: $kg/ms^2 = Pa$

$P_2 = P_3 \quad P_4 = P_3 - \rho_{H_2O} g h_2$
$= 20.0 kPa - 1000 kg/m^3 (9.81 m/s^2)(0.08 m)$
$= 20.0 kPa - 784.8 \text{ kg/ms}^2$
$= 20.0 kP - 0.78 kPa$
$\boxed{= 19.2 \text{ KPa}}$

2.2) Known

$h_1 = 0.20m$
$h_2 = 0.10m$
$h_3 = 0.15m$
$\Delta h_1 = 0.10m$
$\Delta h_2 = 0.15m$
$P_1 = 0 \text{ kpa(g)}$
$\rho_2 = ?$



$P_2 = P_1 + \rho_{H_2O} g \Delta h_1$
$= 0 + 1000 kg/m^3 (9.81 m/s^2)(0.10 m)$
$= 981 \text{ kg/ms}^2$
$= 0.981 \text{ KPa} \qquad P_2 = P_3$

$P_3 = P_4 + \rho_\alpha g \Delta h_3$

$\rho_\alpha = \dfrac{P_3 - P_4}{g \Delta h_3} = \dfrac{981 \text{ Pa} - 0}{(9.81 m/s^2)(0.15 m)} = 666.7 \text{ kg/m}^3$

$SG = \rho / 1000 kg/m^3 = 666.7 kg/m^3 \boxed{= 0.667}$

2.3) Known

$h_1 = 12m$
$h_2 = 6m$
$h_3 = 4m$
$h_4 = 0.9m$
$\rho_{oil} = 2950 kg/m^3$
$\rho_{ALC} = 630 kg/m^3$
$P_1 = 259.8 kPa$
$\rho_{H_2O} = 998 kg/m^3$

$P_2 = P_1 - \rho_{ALC} g \Delta h_1$
$= 259800 Pa - 630 kg/m^3 (9.81 m/s^2)(6-4) m$
$= 247439 \text{ Pa}$

$P_3 = P_2 + \rho_{oil} g h_4$
$= 247439 Pa + 2950 kg/m^3 (9.81 m/s^2)(0.90 m)$
$= 273484.6 \text{ Pa}$

$P_4 = P_3 + \rho_{H_2O} g \Delta h_2$
$= 273484.6 Pa + 998 kg/m^3 (9.81 m/s^2)(12-4-0.9) m$
$= 342996 \text{ Pa}$

$\boxed{P_4 = 343 \text{ kPa}}$

2.4) Known

$P_1 = 50\ psig = 7200\ psfg$
$h_1 = 20ft$
$h_2 = 20\ in = 1.67ft$
$h_3 = 30\ in = 2.5\ ft$
$h_4 = 5ft$
$SG_{Hg} = 13.6$
$SG_{oil} = 0.8$

$P_2 = P_1 - \rho_{oil}\, g\, \Delta h_1$
$= 7200\ psfg - 0.8(1.9379\ slugs/ft^3)(32.2ft/s^2)(20-1.67)ft$
$= 7200psfg - 915.0\ slugs/fts^2$
$= 6285\ psfg$

Note: $slugs/fts^2 = psf$

$P_3 = P_2 + \rho_{Hg}\, g\, h_3$
$= 6285 + 13.6(1.9379\ slug/ft^3)(32.2ft/s^2)(2.5ft)$
$= 6285\ psfg + 2122\ slug/fts^2$
$= 8407\ psfg$

$P_4 = P_3 - \rho_{H_2O}\, g\, \Delta h_2$
$= 8407 - 1.9379\ slug/ft^3(32.2ft/s^2)(2.5+1.67+5)ft$
$= 8407 - 572\ slug/fts^2$
$= 7835\ psfg$

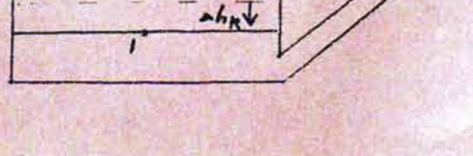
$P_A = 7835\ psfg / 144\ ft^2/in^2$
$= 54.4\ psig + 14.80\ psia$
$= 69.2\ psia$

$\boxed{P_A = 69.2\ psia}$

2.5) Known

$SG_{Hg} = 13.6$
$A_R = 25A_T$
$\Delta h_T = 0.10m$

$\Delta V_R = \Delta V_T$
$A_R \Delta h_R = A_T \Delta h_T$
$25\cancel{A_T}\Delta h_R = \cancel{A_T}\Delta h_T$



$\Delta h_R = \dfrac{\Delta h_T}{25} = \dfrac{0.10}{25} = 4.0\times10^{-3}$

$\Delta h = \Delta h_R + \Delta h_T \sin\theta$
$= 4.0\times10^{-3} + 0.1\sin 15° = 0.0299\ m$

$\Delta_1 P_2 = \rho_{Hg}\, g\, \Delta h = 13.6(1000\ kg/m^3)(9.81m/s^2)(0.0299m) = 3989\ Pa$

$\Delta_2 P_3 = \Delta_1 P_2 - \rho_{H_2O}\, g\, \Delta h = 3989\ Pa - 1000\ kg/m^3(9.81m/s^2)(0.0299m)$
$= 3989\ Pa - 293$
$= 3696\ Pa$

$\boxed{\Delta P = 3.70\ kPa}$

2.6) Known

$\Delta h = 500m$
$S.G._{salt} = 1.015$
$S.G._{H_2O} = 1.0$
$T = 4°C = 277K$
$R = 287$

Salt water @ 500m

$P_2 = \cancel{P_1} + \rho_{salt} g \Delta h$
$= 1.015 (1000\ kg/m^3)(9.81 m/s^2)(500m)$
$= 4978575\ Pa$

Pump @ surface

$P_1 = P_2 - \rho_{H_2O} g \Delta h$
$= 4978575\ Pa - 1000\ kg/m^3 (9.81 m/s^2)(500m)$
$= 73575\ Pa$

$\boxed{P_{pump} = 73.6\ KPa\ (g)}$

Air compressor @ surface

$\frac{P_0}{P} = e^{-(z-z_0)\ g/RT}$ $\quad p = P_0 e^{(z-z_0) g/RT} = 4978575\ e^{-500(9.81/287 \cdot 277)}$
$= 4978575 e^{-0.061699}$
$= 4680690\ Pa$

$\boxed{P_{Air} = 4681\ kPa}$

2.7) Known

$\Delta h = 0.15m$
$l = 0.25m$

$\Delta P_h = \rho g h$
$\Delta P_l = \rho a l$

$\Delta P_h = \Delta P_l$
$\cancel{\rho} gh = \cancel{\rho} a l$
$a = gh/l$
$= (9.81 m/s^2)(0.15m)/0.25m$
$= 5.9 m/s$

$\boxed{a = 5.9 m/s\ \text{Left}}$

2.8) Known

$\omega = 600\ Rev = 20\pi\ rad/s$
$\rho = 1.2\ kg/m^3$
$P_{axis} = 99125\ Pa$
$a_n = \omega^2 r$
$\rho = 1.2\ kg/m^3$
$r = 1.0m$

a) $\Delta P = \int_a^r \rho a_n dr$
$= \int_a^r \rho \omega^2 r\, dr$
$= \frac{1}{2} \rho \omega^2 r^2$

$\boxed{P_r = P_{axis} + \frac{1}{2} \rho \omega^2 r^2}$

---

b) $P_r = P_{axis} + \frac{1}{2} \rho \omega^2 r^2$
$= 99125 + \frac{1}{2}(1.2 kg/m^3)(20\pi)^2 (1.0m)^2$
$= 101494\ Pa$

$\boxed{P_r = 101494\ Pa}$

2.9) $P = P_{atm} + \rho g z$ ①

$\frac{dP}{dr} = \rho a r = -\rho r \omega^2$

$P - P_0 = \int_0^r \rho r \omega^2 dr$

$P - P_0 = \frac{1}{2} \rho \omega^2 r^2$

$P - P_{atm} = \rho g z_0 + \frac{1}{2} \omega^2 r^2$

$P = P_{atm} + \rho g z_0 + \frac{1}{2} \rho \omega^2 r^2$ ②

① = ② $\cancel{\rho} g z = \cancel{\rho} g z_0 + \frac{1}{2} \cancel{\rho} \omega^2 r^2$

$\boxed{z = z_0 + \omega^2 r^2 / 2g}$

b) $z = \frac{\omega^2 r^2}{2g}$

① $\frac{dz}{dr} = r\omega^2/g$ ② $\frac{dz}{dr} = \tan 45° = 1$

$r\omega^2/g = 1 \quad \omega = \sqrt{g/r} = \sqrt{9.8/0.05} = 14.0 \text{ rad/s} \boxed{= 134 \text{ RPM}}$

Problem Set 3

3.1) Known

$R_1 = 0.075m$

$R_2 = 0.125m$

$\bar{v}_i = 10 m/s$

a) $Q = 0 = \sum \bar{v}_i A_i = \bar{v}_i A_i - \bar{v}_e A_e$

$$0 = (10 m/s)\left[\pi(0.075m)^2\right] - \bar{v}_e\left[\pi(0.125m)^2\right]$$

$$\boxed{\bar{v}_e = 3.6 m/s}$$

---

b) $Q = 0 = \int_0^r \bar{v}_i\,dA = \bar{v}_i A_i - \int_0^{R_2} \bar{v}_e\,dA$

$$0 = \bar{v}_i A_i - \int_0^{R_2} \bar{v}_{CL}\left[1 - (r^2/R^2)\right] 2\pi r\,dr$$

$$\bar{v}_i A_i = \int_0^{R_2} \bar{v}_{CL} 2\pi r\,dr - \int_0^{R_2} \bar{v}_{CL}(r^3/R^2)\,dr = \bar{v}_{CL}\left[\pi R_2^2 - \pi R_2^{\cancel{4}\,2}/(2R_2^2)\right]$$

$$\bar{v}_{CL} = \frac{2\bar{v}_i A_i}{\pi R_2^2} = \frac{2\pi(10 m/s)(0.075m)^2}{\pi(0.125m)^2} = 7.2 m/s$$

$$\boxed{\bar{v}_{CL} = 7.2 m/s}$$

3.2) Known

$Q_{in} = 500$ Usgal/min $= 1.114\ ft^3/s$ $\qquad \bar{v}_e = \sqrt{2gh}$

$R_1 = 5ft$

$R_2 = 1.25in = 0.1042\ ft$

a) $Q = 0 = Q_{in} - \bar{v}_e A_e \rightarrow (\sqrt{2gh})(\pi R_2^2) = Q_{in}$

$$\rightarrow h = \frac{(Q_{in}/\pi R_2^2)^2}{2g} = \frac{\left[(1.114\ ft^3/s)/\pi(0.1042ft)^2\right]^2}{2(32.2 ft/s^2)} = 16.6\ ft$$

$$\boxed{h = 16.6 ft}$$

---

b) $h = 5ft \qquad Q = \frac{dV}{dt} = \frac{\pi r^2 dh}{dt} = Q_{in} - \bar{v}_e A_e$

$$\frac{dh}{dt} = \left[Q_{in} - (\pi R_2^2)\sqrt{2gh}\right]/\pi R_1^2$$

$$= \left[1.114 ft^3/s - (\pi[0.1042ft]^2)\sqrt{2(32.2ft/s^2)(5ft)}\right]/\pi(5ft)^2$$

$$= (0.5019\ ft^3/s)/25\pi\,\cancel{ft^2}$$

$$= 0.00639\ ft/s = 0.383\ ft/min$$

$$\boxed{dh/dt = 0.383\ ft/min}$$

3.3) <u>Known</u>

$R_1 = 25mm = 0.025m$
$R_2 = 300mm = 0.30m$
$h = 10mm = 0.01m$
$\bar{V}_i = 10m/s$

$Q = 0 = \bar{V}_i A_i - \bar{V}_e A_e$

$0 = (10m/s)\left[\pi(0.025m)^2\right] - \bar{V}_e\left[2\pi(0.30m)(0.01m)\right]$

$\bar{V}_e = \dfrac{0.0196\ m^3/s}{0.0188\ m^2} = 1.04\ m/s$

$\boxed{\bar{V}_e = 1.04 m/s}$

3.4) <u>Known</u>

$\bar{V}_1 = 100 ft/s$
$\bar{V}_3 = 19 ft/s$
$\rho = 64.4\ lbm/ft^3$
$= 2\ slugs/ft^3$
$A_1 = 0.1 ft^2$
$A_3 = 1.0 ft^2$
$A_2 = A_3 - A_1 = 0.9 ft^2$

a) $Q = 0 = \bar{V}_1 A_1 + \bar{V}_2 A_2 - \bar{V}_3 A_3$

$\bar{V}_2 = \dfrac{\bar{V}_3 A_3 - \bar{V}_1 A_1}{A_2} = \dfrac{(19 ft/s)(1.0 ft^2) - (100 ft/s)(0.1 ft^2)}{(0.9 ft^2)} = 10\ ft/s$

$\boxed{\bar{V}_2 = 10 ft/s}$

---

b) $\dot{m}_2 = \rho \bar{V}_2 A_2 = (2\ slug/ft^3)(10 ft/s)(0.9 ft^2) = 18\ slug/s$

$\boxed{\dot{m}_2 = 18\ slug/s}$

3.5) <u>Known</u>

$r_1 = 0.15m$
$r_2 = 0.08m$
$\bar{V}_1 = 2m/s$
$P_1 = 100 kPa(g)$
$P_{atm} = 101\ kPa(a)$

$F_x = -(\dot{m}\bar{V})_{out} + \cancel{(\dot{m}\bar{V})_{in}}$

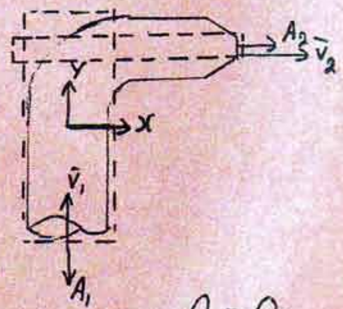
$= -\rho \bar{V}_2 A_2 \bar{V}_2 = -\rho A_2 \bar{V}_2^2$

$\Sigma F_x = F_x + \cancel{P_2 A_2} - \cancel{P_{atm} A_2} \qquad P_2 = P_{atm}$

$F_x = -(1000 kg/m^3)\left[\pi(0.08m)^2\right](7.03 m/s^2)^2$

$= -994\ kg\,m/s^2$

$\boxed{F_x = -944 N}$



$Q_1 = Q_2$

$\bar{V}_1 A_1 = \bar{V}_2 A_2$

$\bar{V}_2 = \bar{V}_1 A_1 / A_2$

$= \dfrac{\cancel{\pi}(0.15m)^2(2m/s)}{\cancel{\pi}(0.08m)^2}$

$= 7.03 m/s$

$\Sigma F_y = F_y - P_1 A_1 + P_{atm} A_1$

$= \cancel{(\dot{m}\bar{V})_{out}} - (\dot{m}\bar{V})_{in} + A_1(P_{atm} - P_1)$

$= -\rho A_1 \bar{V}_1^2 + A_1(P_{atm} - P_1)$

$= -(1000 kg/m^3)\left[\pi(0.15m)^2\right](2m/s)^2 + \left[\pi(0.15m)^2\right](101000 Pa - 201000 Pa)$

$= -282.7\ kg\,m/s - 7068.6\ kg\,m/s$

$= -7351\ N$

$\boxed{F_y = -7351\ N}$

3.6) Known

$P_1 = 100$ Psia

$T_1 = 80°F = 540°R$

$\bar{v}_1 = 50$ ft/s

$r_1 = 3$ in

$P_2 = 200$ Psig $= 214.7$ Psia

$\bar{v}_2 = 100$ ft/s

$\dot{m}_2 = 0.181$ lbm/s

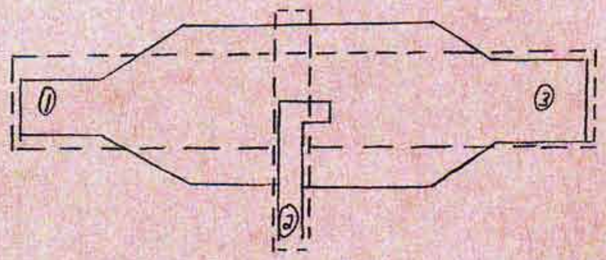
$r_2 = 0.5$ in

$P_3 = 70$ Psia

$T_3 = 1500°F = 1960°R$

$r_3 = 6$ in

$R = 53.3$ ft lbf/lbm°R



$\rho = P/RT \quad \dot{m} = \rho \bar{v} A = P\bar{v}A/RT$

a) $\dot{m}_1 = \dfrac{P_1 \bar{v}_1 A_1}{RT_1} = \dfrac{(100\ lbf/in^2)(50\ ft/s)[\pi(3in)^2]}{(53.3\ ft\ lbf/lbm°R)(540°R)} = 4.91$ lbm/s

$m_3 = m_1 + m_2$

$= 4.91$ lbm/s $+ 0.181$ lbm/s

$= 5.09$ lbm/s

$\boxed{m_3 = 5.09\ lbm/s}$

$\dot{m}_3 = \dfrac{P_3 \bar{v}_3 A_3}{RT_3} \quad \bar{v}_3 = \dfrac{\dot{m}_3 R T_3}{P_3 A_3} = \dfrac{(5.09\ lbm/s)(53.3\ ft\ lbf/lbm\ °R)(1960°R)}{(70\ lbf/in^2)[\pi(6in)^2]} = 67.2$ ft/s

$\boxed{\bar{V}_3 = 67.2\ ft/s}$

---

b) $\Sigma F_y = -P_2 A_2 + P_{atm} A_2 = -(P_2 - P_{atm})A_2 = -(214.7 - 14.7)\ lbf/in^2 \cdot \pi(0.5in)^2$

$= -157$ lbf

$\boxed{F_y = 157\ lbf}$

$\Sigma F_x = -P_1 A_1 - P_{atm}(A_3 - A_1) + P_3 A_3$

$= -(100\ lbf/in^2)[\pi(3in^2)] - (14.7\ lbf/in^2)[\pi(6in)^2 - \pi(3in)^2] + (70\ lbf/in^2)[\pi(6in^2)]$

$= 3842$ lbf

$\boxed{F_x = 3842\ lbf}$

3.7) Known

$Q_1 = 25\ m^3/s$

$r_1 = 0.375$ m

$\rho = 1.3\ kg/m^3$

$P_{atm} = 101$ kPa(a)

$r_2 = 0.25$ m

$P_2 = 5$ kPa(g)

$\bar{v}_1 = 0$

$\dot{m}_1 = \rho Q = (1.3\ kg/m^3)(25\ m^3/s)$

$= 32.5$ kg/s

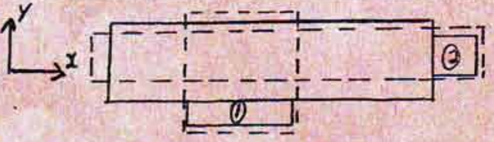
$\dot{m}_1 = \dot{m}_2$

$\dot{m}_2 = \rho\ \bar{V}_2 A_2$

$\bar{V}_2 = \dot{m}_2 / \rho A_2$

$= (32.5\ kg/s)/(1.3\ kg/m^3)[\pi(0.25m)^2]$

$= 127.3$ m/s



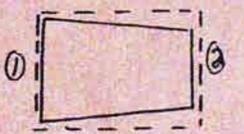
$3.7\ldots)\ \sum F_y = F_y + \cancel{P_{atm}A_1} - \cancel{P_{atm}A_1} = \cancel{(\dot m v)_{out}} - \cancel{(\dot m v)_{in}} = 0$ $\boxed{F_y = 0}$

Note: use of (g) instead of both sides in absolute

$$\sum F_x = (\dot m v)_{out} - \cancel{(\dot m v)_{in}} + P_2 A_2$$
$$= (32.5\,kg/s)(127.3\,m/s) + (5000\,Pa\,(g))[\pi(0.25\,m)^2]$$
$$= 5119\ N$$

$\boxed{F_x = 5119\ N}$

3.8) Known



$r_1 = 0.15\,m$
$r_2 = 0.10\,m$
$\bar v_1 = 1000\,km/h = 277.8\,m/s$
$\bar v_2 = 800\ m/s$
$P_2 = 15\ kPa(g)$
$\dot m_1 = 20\,kg/s$
$\dot m_2 = (20\,kg/s)(1/50) = 0.4\,kg/s$
$\dot m_3 = \dot m_1 + \dot m_2 = 20.4\,kg/s$

a) $F_x = (\dot m v)_{out} - (\dot m v)_{in}$
$= (20.4\,kg/s)(800\,m/s) - (20\,kg/s)(277.8\,m/s)$
$= 10764\ kg\,m/s^2$

$\boxed{F_x = 10764\ N}$ × $F_x = 10476$

b) $\sum F_x = F_x + P_2 A_2$
$= 10764\ N + (15000\,Pa)[\pi(0.10\,m)^2]$
$= 11235\ N$

$\boxed{F_x = 11235\,N}$

3.9) Known

$\bar v_1 = 100\,ft/s$
$\rho = 64.4\ lbm/ft^3$
$A_1 = 0.1\,ft$
$A_2 = 0.9\,ft$
$A_3 = 1.0\,ft$

$P_1 = P_2$
$\dot m = \rho \bar v A$

$F_x = (\dot m v)_{out} - (\dot m v)_{in}$
$= \dot m_3 \bar v_3 - \dot m_1 \bar v_1 - \dot m_2 \bar v_2$
$= \rho(\bar v_3^2 A_3 - \bar v_1^2 A_1 - \bar v_2^2 A_2)$
$= (64.4\,lbm/ft^3)[(19\,ft/s)^2(1.0\,ft^2) - (100\,ft/s)^2(0.1\,ft^2) - (10\,ft/s)^2(0.9\,ft^2)]$
$= -46947.6\ \cancel{lbm}\,\cancel{ft}/s^2 \cdot (lbf\,\cancel{s^2}/32.2\,\cancel{lbm\,ft})$
$= -1458\ lbf$

$\sum F_x = F_x + P_3 A_3 - P_2 A_2 - P_1 A_1 = F_x + P_3 A_3 - P_2(A_2 + A_1)$
$0 = -1458\,lbf + P_3(1.0\,ft^2) - P_2(0.9\,ft^2 + 0.1\,ft^2)$
$P_3 - P_2 = 1458\ lbf/1.0\,ft^2$
$= 1458\ lbf/\cancel{ft^2} \cdot (\cancel{ft^2}/144\,in^2)$
$= 10.1\ lbf/in^2$

$\boxed{P_3 - P_2 = 10.1\ psi}$

Problem Set 4

4.1) Known

$P_1 = P_{atm} = 101\,200\ Pa(a)$

$T_{atm} = 18°C = 291\,k$

$\Delta h = 0.15\,m$

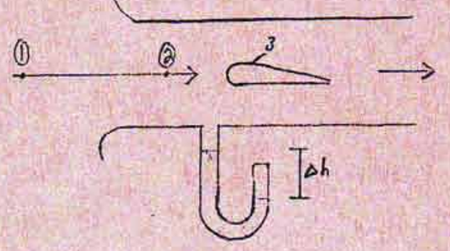
$P_f = 95000\ Pa(a)$

Assume: steady state

streamline

incompressible

$\bar{V}_1 = 0$



a) $P_1 + \cancel{\rho g z_1} + \cancel{\tfrac{1}{2}\rho \bar{V}_1^2} = P_2 + \rho g z_2 + \tfrac{1}{2}\rho \bar{V}_2^2$

$P_T = P_1 = P_2 + \tfrac{1}{2}\rho \bar{V}_2^2 = \boxed{101\,200\ Pa}$

b) $P_2 = P_1 - \rho_w g \Delta h$ $\quad P_T = P_2 + \tfrac{1}{2}\rho_a \bar{V}_2^2$ $\quad \rho_a = P/RT$

$P_2 = P_T - \rho_w g \Delta h$ $\quad P_T - P_2 = \tfrac{1}{2}\rho_a \bar{V}_2^2$

① $P_T - P_2 = \rho_w g \Delta h$ $\quad$ ② $P_T - P_2 = \tfrac{1}{2} P \bar{V}_2^2 / RT$

①=② $\quad \rho_w g \Delta h = \tfrac{1}{2} P \bar{V}_2^2 / RT \rightarrow \bar{V}_2 = \sqrt{\dfrac{2\rho_w g \Delta h R T}{P}}$

$$\bar{V}_2 = \sqrt{\frac{2(1000\ kg/m^3)(9.81\,m/s^2)(0.15\,m)(287\ J/kg\,k)(291\,k)}{101\,200\ Pa} \cdot \left(\frac{Pa\ m^2}{N}\right) \cdot \left(\frac{Nm}{J}\right)}$$

$= \sqrt{2428.76\ m^2/s^2} = 49.3\ m/s \cdot (3.6\ km\,s/m\,h)$

$= 177.4\ km/h$ $\qquad \boxed{\bar{V}_2 = 177.4\ km/h}$

c) $\underbrace{P_2 + \tfrac{1}{2}\rho_a \bar{V}_2^2}_{P_{atm}} + \cancel{\rho_a g z_2} = P_f + \tfrac{1}{2}\rho_a \bar{V}_f^2 + \cancel{\rho_a g z_f}$

$\rightarrow \bar{V}_f = \sqrt{\dfrac{2(P_{atm} - P_f)}{\rho_a}} = \sqrt{\dfrac{2(101200 - 95000)\ N/m^2}{(1.212\ kg/m^3)} \cdot \left(\dfrac{kg\ m}{N s^2}\right)} = \sqrt{10231.0\ m^2/s^2} = \boxed{101.1\ m/s}$

d) Altitude = 5000 m

Table $\rho_a = 0.73643\ kg/m^3$

info. $P_{atm} = 54048\ Pa$

i) $P_{stag} = P_{atm} + \tfrac{1}{2}\rho_a \bar{V}_2^2$

$= 54048\ Pa + \tfrac{1}{2}(0.73643\ kg/m^3)(49.3\ m/s)^2$

$\boxed{= 54942\ Pa}$

ii) $P_{stag} = P_a - \tfrac{1}{2}\rho_a \bar{V}_f^2 \rightarrow P_f = P_{stag} - \tfrac{1}{2}\rho_a \bar{V}_f^2$

$\rightarrow P_f = 54942 - \tfrac{1}{2}(0.73643\ kg/m^3)(101.1\ m/s)^2$

$\boxed{= 51178\ Pa}$

Problem Set 5

5.1) Known

$P_{atm} = 14.7$ Psia

$T_{atm} = 60°F = 520°R$

$\rho_a = 0.076\ lbm/ft^3$

$r = 3.5ft$

$\dot{m}_1 = 1200\ lbm/s$

$\dot{m}_c = 0.83(1200) = 996\ lbm/s$

$\dot{m}_{H_1} = 0.17(1200) = 204\ lbm/s$

$\bar{V}_c = 900\ ft/s$

$\bar{V}_H = 1450\ ft/s$

$\dot{m}_f = 22000\ lbm/h = 6.1\ lbm/s$

Assume

$V_0 = 0$ negligible

$V_F = 0$ negligible

Pressure equal on all sides

$\dot{m}_{H_1} + \dot{m}_F = \dot{m}_H = 210.1\ lbm/s$

a) $\Sigma F_x = (mv)_{out} - \cancel{(mv)_{in}}$

$= \dot{m}_c \bar{V}_c + \dot{m}_H \bar{V}_H$

$= (996\ lbm/s)(900 ft/s) + (210.1\ lbm/s)(1450\ ft/s)$

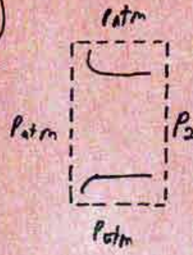
$= 1201045\ \cancel{lbm\ ft/s^2} \cdot (lbf \cdot \cancel{s^2}/32.2\ \cancel{lbm\ ft})$

$= 37300\ lbf$

$\boxed{F_{stand} = 37300\ lbf}$

---

b)



Known

$r = 3.5ft$

$\dot{m}_2 = \rho_a \bar{V}_2 A_2 \quad \bar{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{1200\ \cancel{lbm}/s}{(0.076\ \cancel{lbm}/ft^3)(\pi(3.5\cancel{ft})^2)} = 410.3 ft/s$

$\Sigma F_x = (mv)_{out} - \cancel{(mv)_{in}}$

$= (1200\ \cancel{lbm/s})(410.3\ \cancel{ft/s}) \cdot (lbf\ \cancel{s^2}/32.2\ \cancel{lbm\ ft})$

$= 15290.7\ lbf$

$P_1 + \cancel{\rho_a g z_1} + \cancel{\tfrac{1}{2}\rho_a \bar{V}_1^2} = P_2 + \cancel{\rho_a g z_2} + \tfrac{1}{2}\rho_a \bar{V}_2^2$

$P_2 = P_1 - \tfrac{1}{2}\rho_a \bar{V}_2^2$

$= 14.7\ lbf/in^2 - \tfrac{1}{2}(0.076\ \cancel{lbm/ft^3})(410.3\ \cancel{ft/s})^2 \cdot (lbf \cdot \cancel{s^2}/32.2\ \cancel{lbm\ ft})(1\cancel{ft^2}/144 in^2)$

$= 14.7\ lbf/in^2 - 1.4\ lbf/in^2 = 13.3\ lbf/in^2$

$P_{2g} = -1.4$ psig

b) $\Sigma F_x = F_x - P_a A$

$F_x = \Sigma F_x / 2bolts$

$= 15290\ lbf / 2bolts \quad \boxed{= 7645\ lbf/bolt}$

5.2) Known

$h_1 = 20m$

$h_2 = 3m$

$w = 10m$

$Q = 554 m^3/s$

$F_x = \int P dA$

$= \int \rho g h dA$

$= \int \rho g w h dh$

a) $F_{x_a} = \int_0^{20m} \rho g w h dh = \tfrac{1}{2}\rho g w h^2 \Big|_0^{20}$

$= \tfrac{1}{2}(1000 kg/\cancel{m^3})(9.81 m/s^2)(10m)(20\cancel{m} - 0)^2$

$= 1.962 \times 10^7\ kg\,m/s^2 \quad \boxed{F_{x_a} = 1.96 \times 10^7\ N}$

b) $F_{x_b} = \int_3^{20} \rho g w h dh = \tfrac{1}{2}(1000 kg/\cancel{m^3})(9.81 \cancel{m}/s^2)(10m)[(20\cancel{m})^2 - (3\cancel{m})^2]$

$= 19178550\ N$

$\dot{m} = \rho Q$

$\bar{V} = Q/A$

5.2b...) $\Sigma F_x = F_{x_b} - \left[(\dot{m}v)_{out} - (\dot{m}v)_{in}\right]$

$= F_{x_b} - \left[\rho Q^2/A_2 - \rho Q^2/A_1\right]$

$= F_{x_b} - \rho Q^2\left[\frac{1}{A_2} - \frac{1}{A_1}\right]$

$= 19178550\ N - (1000\ kg/m^3)(554\ m^3/s)^2\left[1/(3m)(10m) - 1/(20m)(10m)\right]$

$= 19178550 N - 8695953 N$

$= 10482597\ N$

$\boxed{\Sigma F_x = 1.05 \times 10^7 N}$

5.3) known

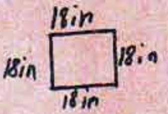
$Q_1 = 12000\ cfm = 200\ ft^3/s$

$T_1 = 70°F = 530°R$

$\Delta h_{H_2O} = 20\ in_{H_2O}\ (g) = 1.67$

$r_1 = 8in$

$A_2 = (18\ in)\times(18\ in) = 324\ in^2 = 2.25 ft^2$



$\bar{V}_1 = 0$

$\therefore F_y = \bar{V}A = 0$

$h = 5\ ft$

$\dot{m} = \rho Q$

$\bar{V} = Q/A$

$F_x = (\dot{m}\bar{v})_{out} - \cancel{(\dot{m}v)_{in}}$

$= \rho Q\bar{V}_2 = \rho Q^2/A_2$

$= \frac{(0.075\ lbm/ft^3)(200\ ft^3/s)^2}{(2.25 ft^2)} \cdot \left(\frac{lbf\ s^2}{32.2\ lbm\ ft}\right)$

$= 41.67\ lbf$

$P_{2g} = \rho g h$

$= (62.2\ lbm/ft^3)(32.2\ ft/s^2)(1.67 ft) \cdot (lbf\ s^2/32.2\ lbm\ ft)$

$= 103.9\ lbf/ft^2 (g)$

Assume: air density to be $0.075\ lbm/ft^3$

Note: If using absolute

↓

$\Sigma F_x = P_{atm}A_2 + P_2A_2 + F_x$

$\Sigma F_x = F_x + P_{2g}A_2$

$= 41.67\ lbf + (103.9\ lbf/ft^2(g))(2.25 ft^2)$

$= 275\ lbf$

$\boxed{\Sigma F_x = 275\ lbf}$

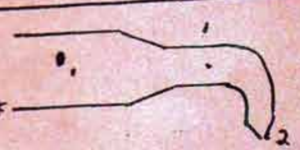
$\Sigma M_{y_{Floor}} = F_x h = (275\ lbf)(5f) = 1375\ lbf\text{-}ft$

5.4) known

$\bar{V} = \dot{m}/\rho A$

$P_1 + \frac{1}{2}\rho\bar{V}_1^2 + \cancel{\rho g z_1} = P_2 + \frac{1}{2}\rho\bar{V}_2^2 + \cancel{\rho g z_2}$

nozzel lies in horizontal



$SG = 0.86$

$P_1 = 200\ kPa(g)$

$\dot{m} = 20\ kg/s$

$r_0 = 0.075m$

$r_1 = 0.05m$

$r_2 = 0.025m$

$l_1 = 0.30m$

$l_2 = 0.25m$

a) $P_1 = P_2 + \frac{1}{2}\rho(\bar{V}_2^2 - \bar{V}_1^2)$

$= P_2 + \frac{1}{2}\rho\left[\dot{m}^2/\rho^2(1/A_2^2 - 1/A_1^2)\right]$

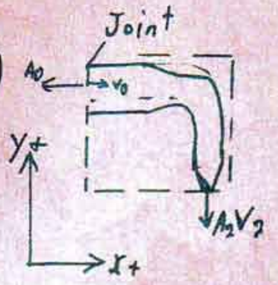
$= 200000\ Pa + \frac{(20 kg/s)^2}{2(0.86)(1000 kg/m^3)} \cdot \left[\left(\frac{1}{\pi(0.025m)^2}\right)^2 - \left(\frac{1}{\pi(0.05m)^2}\right)^2\right]$

$= 200000 Pa + 60291\ Kg/ms^2$

$= 260291\ Pa(g) + 101300\ Pa(a)$

$= 361591\ Pa(a)$

$\boxed{P_1 = 361.6\ kPa(a)}$

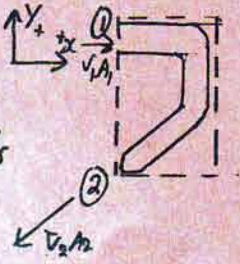
b) 

$$F_x = (\cancel{mv})_{out} - (mv)_{in} - \Delta PA$$
$$= -\dot{m}^2/\rho A_0 - (260\ 291 - 200000)P_{a(g)} \cdot [\pi(0.075m)^2]$$
$$= -(20 kg/s)^2/(0.86)(1000 kg/m^3)[\pi(0.075m)^2] - 1065.4 N$$
$$= -26.3 N - 1065.4 N$$
$$= -1091.7 \qquad \boxed{F_x = -1091.7 N}$$

$$-F_y = (mv)_{out} - (\cancel{mv})_{in} \qquad \Delta P \text{ not included in a nozzel}$$
$$= \dot{m}^2/\rho A_2$$
$$= (20 kg/s)^2/(0.86)(1000 kg/m^3)[\pi(0.025)^2]$$
$$= 236.9 N \qquad \boxed{F_y = 237 N}$$

$$\circlearrowleft + \Sigma M_{z\,flange} = -F_y \cdot l_1 + F_x \cdot 0 = (-236.9 N)(0.3 m) = -71 N\cdot m \qquad \boxed{M_z = -71 N\cdot m}$$

5.5) Known



$P_1 = P_F = 300$ kpa g

$Q = 250$ l/min $= 4.17\times10^{-3} m^3/s$

$r_1 = 0.015 m$

$r^2 = 0.0075 m$

$l = 2m$

$$\dot{m} = \rho Q = (1000 kg/m^3)(4.17\times10^{-3} m^3/s)$$
$$= 4.17 kg/s$$
$$\bar{v}_1 = Q/A_1 = Q/[\pi r_1^2]$$
$$= (4.17\times10^{-3} m^3/s)/[\pi(0.015m)^2]$$
$$= 5.9 m/s$$
$$\bar{v}_2 = Q/A_2 = (4.17\times10^{-3} m^3/s)/[\pi(0.0075m)^2]$$
$$= 23.6 m/s$$

note: dir^n of flow on diagram

$$\Sigma F_x = (mv)_{out} + (mv)_{in} = F_x - P_1 A_1$$
$$F_x = \dot{m}_2 \bar{v}_2 \sin 45 + \dot{m}_1 \bar{v}_1 + P_1 A_1$$
$$= (4.17 kg/s)[(23.6 m/s)\sin 45 + 5.9 m/s] + (300\,000 Pa)[\pi(0.015m)^2]$$
$$= 306.2 N \qquad \boxed{F_x = 306 N}$$

$$\Sigma F_y = (mv)_{out} = (4.17 kg/s)[(23.6 m/s)\sin 45]$$
$$= 69.6 N \qquad \boxed{F_y = 69.6 N}$$

$$\circlearrowleft + \Sigma M_z = -F_y \cdot l = -(69.6N)(2m) = -138.4 N\cdot m \qquad \boxed{M_z = 138.4 N\cdot m}$$

5.6) Known

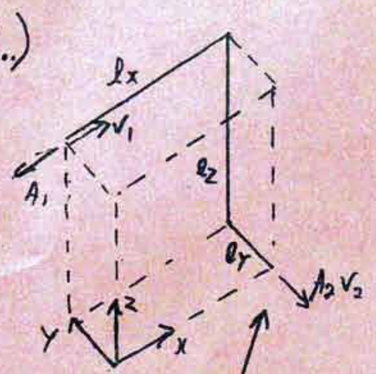
$\dot{m} = 5 kg/s$ $\qquad l_x = 2m$

$\bar{v}_1 = 100 m/s$ $\qquad P_1 = 200 kPa(g) = 301 kPa(a)$ $\qquad l_z = 5m$

$\bar{v}_2 = 300 m/s$ $\qquad P_2 = P_{atm} = 101 kPa(a)$ $\qquad l_y = 1m$

$r = 0.075 m$

5.6...)



Note: negative dir^{n}

$\Sigma F_x = \cancel{(\dot{m}v)_{out}} - (\dot{m}v)_{in} = F_x + P_1 A_1$

$F_x = -\dot{m}_1 \bar{v}_1 - P_1 A_1$

$= (5\,kg/s)(100\,m/s) - (200000\,Pa(g))[\pi(0.075m)^2]$

$= -4034\,N$

$F_x = -4034\,N$

$\Sigma F_y = (\dot{m}v)_{out} - \cancel{(\dot{m}v)_{in}}$ Note: atmospheric on both sides

$= \dot{m}_2 V_2 = -(5\,kg/s)(300\,m/s)$

$= -1500\,N$ $\boxed{F_y = -1500\,N}$

$\Sigma F_z = 0$ By inspection - atmospheric both sides
- no flow enter/exit

$\Sigma M_y = F_x \cdot 0 = 0$

$\Sigma M_z = F_y \cdot l_x = (-1500N)(2m) = -3000\,Nm$

$\Sigma M_x = F_y \cdot l_z = (-1500N)(5m) = -7500\,N\cdot m$

$\boxed{M_y = 0 \quad M_z = -3000\,Nm \quad M_x = -7500\,Nm}$

5.7) Known

$r_1 = 0.3m$ (in)

$r_2 = 0.4m$ (out)

$Q = 1000\,m^3/min$
$= 16.7\,m^3/s$

$\Delta h_1 = 0.1m$ .. $H_2O(g)$

$\dot{m}_s = 6000\,kg/h$
$= 1.67\,kg/s$

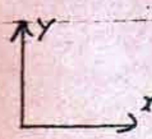
$T_a = 20°C = 293K$

$r_{cyc} = 1.5m$

$P_1 = \rho g \Delta h = (1000\,kg/m^3)(9.81\,m/s^2)(0.1m)$
$= 981\,Pa$

$\dot{m}_a = \rho Q = (1.225\,kg/m^3)(16.7\,m^3/s)$
$= 20.46\,kg/s$ ← From Tables

$\dot{m}_1 = \dot{m}_a + \dot{m}_s = (20.46 + 1.67)\,kg/s = 22.13\,kg/s$



$\bar{V}_1 = Q/A_1 = (16.7\,m^3/s)/[\pi(0.3m)^2] = -59.06\,m/s$

$\bar{V}_2 = Q/A_2 = (16.7\,m^3/s)/[\pi(0.4\,m)^2] = 33.22\,m/s$

$\Sigma F_x = \cancel{(\dot{m}v)_{out}} - (\dot{m}v)_{in} = F_x - P_1 A_1$

$F_x = -\dot{m}_1 v_1 + P_1 A_1 = -(22.13\,kg/s)(-59.06\,m/s) + (981\,Pa)[\pi(0.3m)^2]$

$\boxed{F_x = 1584\,N}$

$\Sigma F_y = (\dot{m}v)_{out} - \cancel{(\dot{m}v)_{in}}$ Note: atmospheric on both sides

$= \dot{m}_a V_2 = (20.46\,kg/s)(33.22\,m/s)$

$\boxed{F_y = 679.7\,N}$

$\Sigma M_z = F_x \cdot (r_{cyc} - r_1) = (1584N)[(1.5m) - (0.3m)] = 1900.8\,N\cdot m$ $\boxed{M_z = 1900.8\,N\cdot m}$

5.8) Known

$Q = 1\,l/s = 0.001\,m^3/s$

$A = 30\,mm^2 = 3\times10^{-5}\,m^2$

$r = 0.2\,m$

$\dot{m} = \rho Q = (1000\,kg/m^3)(0.001\,m^3/s) = 1\,kg/s$

$\bar{v} = Q/2A = (0.001\,m^3/s)/2(3\times10^{-5}\,m/s) = 16.67\,m/s$

$\bar{V}_{nozzel} = \bar{v}\cos 30 = 14.4\,m/s \qquad \dot{m}_{nozzel} = \dot{m}/2 = 0.5\,kg/s$

$$\Sigma F_y = (mv)_{out} - \cancel{(mv)_{in}} = 2\dot{m}_{nozzel}\bar{V}_{nozzel}$$
$$= 2(1\,kg/s)(14.4\,m/s) = 2(0.5\,kg/s)(14.4\,m/s) = 14.4\,N$$

a) $\Sigma M_z = F_y \cdot r = (14.4\,N)(0.2\,m) = 2.88\,N\cdot m \quad \boxed{M_z = 2.88\,N\cdot m}$

b) $\omega = 500\,RPm = 52.36\,rad/s$

$\bar{v} = r\omega = 0.2m(52.36\,rad/s)$
$= 10.47\,m/s$

$\Sigma M_z = \dot{m}\bar{v}r = (1kg/s)(14.4 - 10.47)m/s\,(0.2m) = 0.786\,Nm$

$\boxed{M_z = 0.786\,Nm}$

c) $\omega = \bar{v}/r = (14.4\,m/s)/(0.2\,m) = 72.2\,rad/s \cdot (60\,min/s/2\pi) = 689\,RPM \quad \boxed{\omega = 689\,RPM}$

5.9) Known

$r_h = 1.3\,ft$

$r_t = 1.7\,ft$

$r_m = 1.5\,ft$

$Q = 250\,ft^3/s$

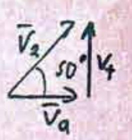
$\omega = 1000\,RPM = 104.72\,rad/s$

$\rho = 0.077\,lbm/ft^3$

$\dot{m} = \rho Q = (0.077\,lbm/ft^3)(250\,ft^3/s) = 19.25\,lbm/s$

$\bar{V}_1 = Q/A_1 = (250\,ft^3/s)/\pi(1.7^2 - 1.3^2)ft^2 = 66.3\,ft/s$

$\bar{V}_a = V_1$



$\bar{V}_t = \bar{V}_a \tan 50$
$= 66.3 \tan 50 = 79.0\,ft/s$

a) $\Sigma F_t = \dot{m}\bar{V}_t = (19.25\,\cancel{lbm/s})(79.0\,ft/s) \cdot (lbf\,s^2/32.2\,\cancel{lbm\,ft})$
$= 47.22\,lbf$

$\Sigma M = r_m F_t = (1.5\,ft)(47.22\,lbf) = 70.8\,ft\cdot lbf$

$W = M\omega = (70.8\,\cancel{ft\cdot lbf})(104.72\,rad/s) \cdot (hp\,s/550\,\cancel{ft\,lbf})$
$= 13.5\,hp$

$\boxed{W = 13.6\,hp}$

b) If the swirl doesn't change $\boxed{M = 70.8\,ft\cdot lbf}$

Problem Set 6

6.1) Known

a) $v_2 = 150\ ft/s$

$\eta = 0.70$

$\Delta z = 25ft$

$d_2 = 1in \quad r_2 = 0.5in$

$A_2 = \pi r^2 = \pi(0.5/12)^2 = 0.005454ft^2$

$P_1 = P_2$

Assume: $\Delta u = 0$ negligible, adiabatic $Q_H = 0$

$$\cancel{Q_H} - W_p = \Sigma\left(\cancel{\tfrac{P}{\rho}} + \tfrac{v^2}{2} + gz + \cancel{u}\right)\rho v A = \left(\tfrac{v_2^2}{2} - \cancel{\tfrac{v_1^2}{2}} + g\Delta z\right)\rho v_2 A_2$$

$$-W_p = \left[(150ft/s)^2/2 + (32.2ft/s)(25ft)\right](1.94\,slug/ft^3)(150ft/s)(0.00545ft^2)$$

$$= (12055\ ft^2/s^2)(1.58595\ slug/s)\cdot\frac{lbf\,s^2}{slug\,ft}\cdot\frac{1hp\,s}{550\,lbf\,ft}$$

$$W_p = -34.76\ hp \rightarrow W_p/0.7 \rightarrow \boxed{W_{p_{new}} = -50hp}$$

-ive means input

b) $v_3^2 = v_2^2 + 2gh \rightarrow h = \frac{v_2^2}{2g} = \frac{(150ft/s)^2}{2(32.2ft/s^2)} = 350ft \longrightarrow \boxed{h = 350ft}$

$v_3 = 0$

6.2) Known

a) $d_1 = 6in \rightarrow r_1 = 0.25ft$

$d_2 = 4in \rightarrow r_2 = 0.1\bar{6}ft$

$Q = 8ft^3/s$

$\eta = 0.70$

$\Delta z = 0$

$A_1 = \pi r_1^2 = 0.196ft^2$

$A_2 = 0.0873ft^2$

$v_1 = Q/A_1 = (8ft^3/s)/(0.196ft^2) = 40.82ft/s$

$v_2 = 91.6\ ft/s$

Assume: $\Delta u = 0$ negligible, adiabatic $Q_H = 0$

$P_1 = -10\,psig = -1440\,psfg$

$P_2 = 25\,psig = 3600\,psfg$

$$\cancel{Q_H} - W_p = \Sigma\left(P/\rho + \tfrac{v^2}{2} + \cancel{gz} + \cancel{u}\right)\rho Q \qquad -W_p = \left(\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2}\right)\rho Q$$

$$-W_p = \left[(3600+1440)\tfrac{lbf}{ft^2}/(1.94\,slug/ft^3) + \left[(91.6ft/s)^2 - (40.82ft/s)^2\right]/2\right](1.94\,slug/ft^3)\cdot(8ft^3/s)$$

$$= \left[2597.9\ lbf\,ft/slug\cdot\frac{slug\,ft}{lbf\,s^2} + 3362.1\ ft^2/s^2\right](15.52\ slug/s)$$

$$= 92499.2\ slug\,ft^2/s^3\cdot\frac{lbf\,s^2}{slug\,ft}\cdot\frac{1hp\,s}{550\,lbf\,ft} = 168.2hp \longrightarrow \boxed{W_p = 168.2hp}$$

b) $W_p/0.7 \longrightarrow \boxed{W_{p_{new}} = 240.3hp}$

6.3) Known

$h_1 = 100m$ $\quad A_2 = \pi r^2 = 38.48m^2$ $\quad$ Take water level as datum

$h_2 = -5m$ $\quad$ Assume: $\Delta u = 0$ negligible

$Q = 200 m^3/s$ $\quad v_2 = Q/A_2 = (200m^3/s)/(38.48m^2) = 5.20 m/s$ $\quad P_1 = P_{atm}$

$d_2 = 7m$ $r_2 = 3.5m$ $\quad$ Adiabatic $Q_H = 0$

$\eta = 0.94$ $\quad v_1 = 0$

$$\cancel{Q_H} - W_t = \Sigma\left(\tfrac{P}{\rho} + \tfrac{v^2}{2} + gz + \cancel{u}\right)\rho v A \longrightarrow -W_t = \left(\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - \cancel{v_1^2}}{2} + g(z_2 - z_1)\right)\rho Q$$

$$W_t = -\rho Q\left(\frac{P_{atm} + \rho g(-h_2) - P_{atm}}{\rho} + \frac{v_2^2}{2} + g(z_2 - z_1)\right)$$

$$= -(1000 kg/m^3)(200 m^3/s)\left[\frac{\cancel{\rho}(9.81m/s^2)(5m)}{\cancel{\rho}} + \frac{(5.20m/s)^2}{2} + (9.81m/s^2)((-5m) - 100m)\right]$$

$$= 193.5 \ kg\,m^2/s^3 \cdot \frac{Ns^2}{kg\,m} \cdot \frac{J}{Nm} \cdot \frac{Ws}{J} \longrightarrow W_t = 193.5 W$$

$$W_t \cdot 0.94 \longrightarrow \boxed{W_{new} = 182 W}$$

6.4) Known

$Q = 12000 ft^3/min = 200 ft^3/s$ $\quad$ Assume: $Q_H = 0$ Adiabatic $\quad \rho = P/RT$

$T_1 = 70°F = 530°R$ $\quad \Delta z = 0$ negligible

$T_2 = 315°F = 775°R$ $\quad v_1 = v_2 = 0$

$P_3 = P_1 = 14.7 psia = 2116.8 psfa$ $\quad R = 53.3 \frac{ft\,lbf}{lbm°R} \cdot 32.2 \frac{lbm\,ft}{lbf\,s^2} = 1716.3 ft^2/°Rs^2$

$P_2 = 30 psig = 6436.8 psfa$ $\quad c_v = 0.17 \frac{Btu}{lbm°R} \cdot 778 \frac{ft\,lbf}{Btu} \cdot 32.2 \frac{lbm\,ft}{lbf\,s^2} = 4258.8 ft^2/°Rs$

$V_3 = 1600 ft/s$

$$\dot m_1 = \dot m_2 = \dot m_3 = \rho_1 Q = \frac{P_1 Q}{RT_1} = \frac{(2116.8 psfa)(200 ft^3/s)}{(1716.3 ft^2/°Rs^2)(530°R)} \cdot \frac{1 slug\, ft}{lbf\, s^2} = 0.465 slug/s$$

a) $$\cancel{Q_H} - W = \Sigma\left[P/\rho + \cancel{v^2/2} + \cancel{gz} + u\right]\dot m \longrightarrow -W = \left[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} + u_2 - u_1\right]\dot m$$

$$W = -\dot m\left[\frac{P_2}{P_2/RT_2} - \frac{P_1}{P_1/RT_1} + c_p(T_2 - T_1)\right] = -\dot m\left[R(T_2 - T_1) + c_p(T_2 - T_1)\right] = -\dot m(R + c_p)(T_2 - T_1)$$

$$W = -(0.465 slug/s)(1716.3 + 4258.8) ft^2/°Rs^2 (775 - 530)°R \cdot \frac{lbf\,s^2}{slug\,ft} \cdot \frac{hp\,s}{550 lbf\,ft}$$

$$\boxed{W = -1238 hp}$$

6.4b) similar to part a but no work and add V_2

$$0 = -\dot{m}\left[(R+c_v)(T_3-T_2)+\frac{V_2^2}{2}\right] \longrightarrow V_2^2 = 2(R+c_v)(T_3-T_2)$$

$$T_3 = \frac{-V_2^2}{2(R+c_v)} + T_2 = \frac{-(1600\ ft/s)^2}{2(1716+4258.8)ft^2/°R s^2} + 775°R = 561°R = 101°F \longrightarrow \boxed{T_3 = 101°F}$$

6.5) Known

$W = -100\,W$
$Q_H = 1000\,W$
$P_1 = P_2 = 101325\,Pa$
$T_1 = 25°C = 298\,K$
$Q = 60\,m^3/h$

$d_1 = 0.04\,m$ $r_1 = 0.02\,m$
$d_2 = 0.03$ $r_2 = 0.015\,m$
$A_1 = \pi r^2 = 1.26\times10^{-3}\,m^2$
$A_2 = 7.07\times10^{-4}$

$c_p = 1\,kJ/kg\,K$ $\rho = P/RT$
$R = 287\,J/kg\,K$

$$\dot{m} = \rho Q = \frac{P_1 Q}{RT_1} = \frac{(101325\,Pa)(60\,m^3/h)}{(287\,J/kg\,K)(298\,K)} = 0.0197\,kg/s$$

$V_2 = Q/A = 60\,m^3/h / 7.07\times10^{-4}\,m^2$
$= 23.6\,m/s$

Assume: Δz negligible, $V_1 = 0$, $\rho_1 = \rho_2$, $P_1 = P_2$

$$Q_H - W = \Sigma\left(\cancel{P/\rho} + \frac{V^2}{2} + \cancel{gz} + u\right)\dot{m} = \dot{m}\left(\frac{V_2^2 - \cancel{V_1^2}}{2} + c_p(T_2-T_1)\right) \longrightarrow Q_H - W = \dot{m}\left(\frac{V_2^2}{2} + c_p(T_2-T_1)\right)$$

$$\longrightarrow T_2 = \left[\frac{(Q_H - W)/\dot{m} - V_2^2/2}{c_p}\right] + T_1 = \left[\frac{(1000+100)\frac{Nm}{s}/0.0197\,kg/s - (23\,m/s)^2/2}{(1000\,J/kg\,K)}\right] + 298$$

$\boxed{T_2 = 353.6\,K = 80.6°C}$ Iterations not necessary

6.6) Known

$h = 450\,ft$
$Q = 10000\,ft^3/s$
$\eta = 0.94$
$H_L = 0.016\ H_{DYN}\ L/D$
$L = 585\,ft$
$D = 26\,ft$
$P_1 = P_2 = P_{atm}$

Assume: $V_1 = 0$

$A_2 = \pi(13)^2 = 531\,ft^2$

$V_2 = Q/A_2 = 10000\,ft^3/s / 531\,ft^2$
$= 18.8\,ft/s$

$H_{DYN} = V^2/2g$

$$H_L = 0.016\,H_{DYN}\frac{L}{D} = 0.016\left(\frac{V^2}{2g}\right)\frac{L}{D} = \frac{0.016(18.8\,ft/s)^2(585\,ft)}{2(32.2\,ft/s^2)(26\,ft)}$$
$= 1.98\,ft$

This is taken into account in H_L

$$\cancel{P_1/\rho g} + \cancel{V_1^2/2g} + z_1 = \cancel{P_2/\rho g} + V_2^2/2g + z_2 + H_L + h_t$$

$h_t = z_1 - h_L$
$= 448.02\,ft$

$$P_t = \rho g Q h_t = (1.94\,slug/ft^3)(32.2\,ft/s^2)(10000\,ft^3/s)(448.02\,ft)\cdot\frac{lbf\,s^2}{slug\,ft}\cdot\frac{hp\,s}{550\,lbf\,ft}$$
$= 508853\,hp$ $P_t(0.94) = \boxed{P_{t\,new} = 478321\,hp}$

6.7) Known

SG = 1.26

$\mu = 0.4\ Pa\cdot s$

$Q = 20\ l/s = 0.02\ m^3/s$

$d = 0.1\ m \quad r = 0.05\ m$

$L = 45\ m$

$\theta = 15°$

$P_1 = 590\ kPa(g)$

Smooth Pipe

$z_2 = 45\sin 15 = 11.65\ m$

$z_1 = 0$

$A_1 = \pi r^2 = 7.85\times10^{-3}\ m^2$

$V_1 = Q/A = 0.02\ m^3/s / 7.85\times10^{-3}\ m^2 = 2.55\ m/s$

$V_1 = V_2$

$Re = \frac{\rho V D}{\mu} = \frac{1.26(1000\ kg/m^3)(0.1\ m)(2.55\ m/s)}{0.9\ Pa\cdot s} = 357$

$Re < 2000$ $\boxed{\therefore \text{Laminar Flow}}$

$f = 64/Re = 0.1793$

$h_f = \frac{f v^2 L}{2gD} = \frac{0.1793(2.55\ m/s)^2(45\ m)}{2(9.81\ m/s^2)(0.1\ m)} = 26.7\ m$

$P_1/\rho g + \cancel{V_1^2/2} + z_1 = P_2/\rho g + \cancel{V_2^2/2} + z_2 + h_f$

$\longrightarrow P_2 = P_1 - \rho g(z_2 + h_f) = 590000\ Pa - (1.26)(1000\ kg/m^3)(9.81\ m/s^2)(11.65\ m + 26.7\ m)$

$\boxed{P_2 = 116\ kPa}$

$h_f = 4\tau_w L/\rho g D \rightarrow \tau_w = h_f \rho g D/4L = 26.7\ m(1.26)(1000\ kg/m^3)(9.81\ m/s)(0.1\ m)/4(45\ m)$

$\boxed{\tau_w = 183\ Pa}$

6.8) Known

$\nu = 0.0002\ ft^2/s$

$gh_f = 0.4\ ft\ lbf/lbm \cdot \frac{32.2\ lbm\ ft}{lbf\ s^2} = 12.88\ ft^2/s^2$

$Q = 8.0\ ft^3/s$

$\varepsilon = 0.00015 \quad \varepsilon/D = 0.00015/D$

Assume: $\Delta z = 0$ negligible

$v = Q/A = 4Q/\pi D^2 = \frac{4(8.0\ ft^3/s)}{\pi D^2} = 10.2\ ft^3/s/D^2 \quad \nu = \mu/\rho$

$Re = \frac{\rho v D}{\mu} = \frac{4Q\cancel{D}}{\nu \pi D^{\cancel{2}}} = \frac{4(8.0\ ft^3/s)}{(0.0002\ ft^2/s)\pi D} = 5.09\times10^4\ ft/D$

$gh_f = \frac{f v^2 L}{2D} = \frac{f v^2(100\ ft)}{2D} = \frac{f v^2(50\ ft)}{D}$

D(ft)	Re	ε/D	f	v (ft/s)	gh_f	
0.5	1.0×10^5	0.0003	0.028	40.80	4161.6	high
2.0	2.5×10^4	0.000075	0.026	2.55	4.23	low
1.5	3.4×10^4	0.00010	0.024	4.53	16.40	low
1.6	3.2×10^4	0.00009	0.024	4.00	12.00	closest

$\boxed{D_{min} = 1.6\ ft}$

for square pipes

6.9) Known

$h = 0.4\,m$

$w = 0.6\,m$

$Q = 200\,m^3/h$

$P_1 = 101\,kPa$

$T_H = 35°C = 308K$

$T_C = 12°C = 285K$

$\varepsilon = 0.00015\,m$

$L = 100\,m$

Use only for Re $\rightarrow \boxed{D_H = \frac{4A}{P}} = \frac{4hw}{2(h+w)} = \frac{4(0.4)(0.6)m^2}{2(0.4+0.6)m} = 0.48\,m$ $r_H = 0.24\,m$

perimeter

$v = \frac{Q}{A} = \frac{200\,m^3/h}{(0.4m)(0.6m)} \cdot \frac{1\,h}{3600\,s} = 0.231\,m/s$

$\mu = 1.88\times10^{-5}\,kg/ms$

$\mu = 1.77\times10^{-5}\,kg/ms$

Heating	Cooling
$\rho = \frac{P}{RT_H} = \frac{101000\,Pa}{(287\,J/kgK)(308K)} = 1.14\,kg/m^3$	$\rho = \frac{P}{RT_c} = \frac{101000\,Pa}{(287\,J/kgK)(285\,K)} = 1.23\,kg/m^3$
$Re = \frac{\rho v D_H}{\mu} = \frac{(1.14\,kg/m^3)(0.231\,m/s)(0.48\,m)}{(1.88\times10^{-5}\,kg/ms)} = 6.7\times10^3$	$Re = \frac{\rho v D_H}{\mu} = \frac{(1.23\,kg/m^3)(0.231\,m/s)(0.48\,m)}{(1.77\times10^{-5}\,kg/ms)} = 7.7\times10^3$
$h_f = \frac{fv^2L}{2gD_H} = \frac{0.035(0.231\,m/s)^2(100\,m)}{2(9.81\,m/s^2)(0.48\,m)} = 0.0198\,m$	$h_f = \frac{fv^2L}{2gD_H} = \frac{0.0335(0.231\,m/s)^2(100\,m)}{2(9.81\,m/s)(0.48\,m)} = 0.0190\,m$
$h_f = \cancel{\Delta z} + \frac{\Delta P}{\rho g}$	$h_f = \cancel{\Delta z} + \frac{\Delta P}{\rho g}$
$\Delta P = h_f \rho g = (0.0198\,m)(1.14\,kg/m^3)(9.81\,m/s^2) = 0.221\,kg/ms^2$	$\Delta P = h_f \rho g = (0.0190\,m)(1.23\,kg/m^3)(9.81\,m/s) = 0.0229\,kg/ms^2$
$\boxed{\Delta P = 0.221\,Pa}$	$\boxed{\Delta P = 0.0229\,Pa}$

6.10) Known

$D_1 = 8in = 0.\bar{6}\,ft$

$r_1 = 0.\bar{3}\,ft$

$D_2 = 24in = 2ft$

$r_2 = 1ft$

$L_1 = 50\,ft$

$L_2 = 300\,ft$

$T = 60°F$

$v_1 = 18\,ft/s$

v_2

$\rho = 62.4\,lbm/ft^3 = 1.94\,slug/ft^3$

$\mu = 23.7\times10^{-6}\,lbf\,s/ft^2$

$Q = v_1A_1 = (18\,ft/s)(\pi(0.\bar{3}\,ft)^2) = 6.28\,ft^3/s$

$V_2 = Q/A_2 = (6.28\,ft^3/s)/(\pi(1ft)^2) = 2\,ft/s$

Assume: Smooth Pipes, Δz negligible

$k = \left(1 - \frac{A_1^2}{A_2^2}\right)^2 = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 = \left(1 - \frac{(0.\bar{6})^2}{(2)^2}\right)^2 = 0.79$

$h_f = \frac{kv^2}{2g} = \frac{0.79(18\,ft/s)^2}{2(32.2\,ft/s^2)} = 3.97\,ft$

$Re_1 = \frac{\rho v_1 D_1}{\mu} = \frac{(1.94\,slug/\cancel{ft^3})(18\,\cancel{ft/s})(0.\bar{6}\,\cancel{ft})}{(23.7\times10^{-6}\,\cancel{lbf\,s/ft^2})} \cdot \frac{\cancel{lbf\,s^2}}{slug\,ft}$
$= 9.8\times10^5$

$h_{f_1} = \frac{f_1 v_1^2 L_1}{2g D_1} = \frac{0.0125(18\,\cancel{ft/s})^2(50\,ft)}{2(32.2\,\cancel{ft/s^2})(0.\bar{6}\,\cancel{ft})}$
$= 4.72\,ft$

$Re_2 = \frac{(1.94\,slug/ft^3)(2\,ft/s)(2\,ft)}{(23.7\times10^{-6}\,lbf\,s/ft^2)}$
$= 3.3\times10^5$

$h_{f_2} = \frac{0.014\,(2\,ft/s)^2(300\,ft)}{2(32.2\,ft/s^2)(2\,ft)}$
$= 0.13\,ft$

$h_{f\,total} = h_{f_1} + h_{f_2} + h_f = 4.72\,ft + 0.13\,ft + 3.97\,ft \boxed{= 8.8\,ft}$

6.11) Known

$Q = 1\,ft^3/s$
$\mu = 2.11\times10^{5}\,lbf\,s/ft^2$
$D = 3\,in = 0.25\,ft \quad r = 0.125\,ft$
$L = 200\,ft$
$P_1 = 14.7\,psia = 2116.8\,lbf/ft^2$
$P_2 = 20\,psia = 2880\,lbf/ft^2$
$\Delta z = 0$
$\varepsilon = 0.006\,in = 0.0005\,ft$
$\varepsilon/D = 0.002$

$v = Q/A = (1\,ft^3/s)/\pi(0.125\,ft)^2 = 20.37\,ft/s$

$Re = \frac{\rho v D}{\mu} = \frac{(1.94\,slug/\cancel{ft^3})(20.37\,\cancel{ft/s})(0.25\,ft)}{2.11\times10^{-5}\,\cancel{lbf\,s/ft^2}} \cdot \frac{lbf\,s^2}{slug\,ft}$
$= 4.68\times10^5$

$h_f = \frac{f v^2 L}{2gD} = \frac{0.024\,(20.37\,ft/s)\,(200\,ft)}{2(32.2\,ft/s)(0.25\,ft)} = 123.7\,ft$

$h_L = \frac{P_2 - P_1}{\rho g} = \frac{(2880 - 2116.8)\,lbf/ft^2}{(1.94\,slug/ft^3)(32.2\,ft/s^2)} = 12.2\,ft$

$h_{p_{pump}} = h_L + h_f = 135.9\,ft$

$P_p = \rho Q g h_{p_{pump}} = (1.94\,slug/\cancel{ft^3})(1\,\cancel{ft^3/s})(32.2\,\cancel{ft/s^2})(135.9\,\cancel{ft})(lbf\,s^2/slug\,ft)(hp\,s/550\,lbf\,ft)$
$= 15.4\,hp$

$\boxed{P_{pump} = 15.4\,hp}$

6.12) Known

$\varepsilon = 0.0005\,ft$
$K = 0.54 \quad \varepsilon/D = 0.002$
$n = 0.70$
$D_0 = 2\,in = 0.167\,ft$
$C_c = 0.6$
$D_v = 0.6(0.167\,ft)$
$= 0.1\,ft \quad r_v = 0.05\,ft$
$Q = 1\,ft^3/s$
$D_1 = 3\,in \quad r_1 = 0.125\,ft$
$\mu = 2.11\times10^{-5}\,lbf\,s/ft^2$

$\bar{V}_1 = Q/A = (1\,ft^3/s)/[\pi(0.125\,ft)^2] = 20.37\,ft/s$

$\bar{V}_v = Q/A_v = (1\,ft^3/s)/[\pi(0.05\,ft)^2] = 127.32\,ft/s$

$R_{ED_1} = \frac{\rho \bar{V}_1 D_1}{\mu} = \frac{(1.94\,slug/\cancel{ft^3})(20.37\,\cancel{ft/s})(0.25\,\cancel{ft})}{2.11\times10^{-5}\,\cancel{lbf\,s/ft^2}} \cdot \frac{lbf\,s^2}{slug\,ft}$
$= 468220.4$

$f = 0.023$ from chart

6.12...) $h_{f_1} = \frac{f\bar{v}_1^2 L}{2gD} = \frac{0.023(20.37 ft/s)^2(200 ft)}{2(32.2 ft/s^2)(0.25 ft)} = 118.55 ft$

$h_{f_2} = \frac{[2\times(0.54)+0.6](20.37 ft/s)^2}{2(32.2 ft/s^2)} = 10.8 ft$ $\quad h_f = h_{f_1} + h_{f_2} = 129.37 ft$

$h_p = h_f + h_L = 12.2 ft + 129.37 ft = 141.57 ft$

$P_p' = \rho Q g h_p = (1.94 slug/ft^3)(1 ft^3/s)(32.2 ft/s^2)(141.57 ft)(lbf\, s^2/slug\, ft)(hp\, s/550\, lbf\, ft)$
$= 16.1\ hp$

$P_p = P_p'/0.7 = \boxed{23.0\ hp}$ ← I am uncertain of the accuracy of this

---

6.13) Known

$SG = 0.9$
$\mu = 0.0008\ lbf\, s/ft^2$
$Q = 1 ft^3/s$
$L = 3000\ ft$
$\Delta z = 40\ ft$
$\Delta P = 0$
$D = ?$
$\varepsilon = 0.00085$
$K = 0.5$

$h_{f_i} = \frac{K\bar{v}^2}{2g} = \frac{0.5v^2}{2(32.2 ft/s^2)} = 0.00776 v^2$

$\bar{v} = Q/A = 4Q/\pi D^2 = 4(1 ft/s^3)/\pi D^2 = 1.27 ft^2/D^2$

$Re = \frac{\rho\bar{v}D}{\mu} = \frac{(1.27 ft^2)\rho}{\mu D} = \frac{(1.94 slug/ft^3)(0.9)(1.27 ft^2)}{(0.0008\ lbf/ft^2)D} \cdot \frac{lbf\ s^2}{slug\ ft}$

$Re = 2771.78/D$

$h_f = \frac{f\bar{v}^2 L}{2gD} = \frac{f(1.27 ft^2/D^2)^2 L}{2gD} = \frac{f(1.27 ft^2)^2(3000 ft)}{2(32.2 ft/s^2) D^5}$ $\quad h_f = 75.1/D^5$

D	Re	ε/D	f	h_{fi} (ft)	h_f (ft)	h_t (ft)	
0.5	5.5×10^3	0.0003	0.038	0.2	91.4	91.6	high
0.7	4×10^3	0.00021	0.040	0.1	17.9	18.0	low
⟶ 0.6	4.6×10^3	0.00025	0.040	0.2	38.5	38.7	$\approx \Delta z = 40 ft$

$\boxed{D = 0.6 ft}$

6.14) Known

$V = 100\ m^3$

$L = 50 m$

$D = 0.02 m$

$r = 0.01 m$

$P_1 = 400\ kpa\ (g)$

$= 501\ kpa\ (a)$

$P_2 = 101\ kpa\ (a)$

$T = 15°C = 288 k$

$\mu = 1.136 \times 10^{-3}\ kg/m\,s$

$\Delta z = 0$

$\rho \approx 1000\ kg/m^3$

$A = \pi(0.01m)^2 = 0.000314\ m^2$

Rubber hose $\varepsilon = 0.0015\ mm$

$= 1.5 \times 10^{-6} m$

$\varepsilon/D = 0.000075$

$$Re = \frac{\rho \bar{v} D}{\mu} = \frac{(1000 kg/m^3)\,\bar{v}\,(0.02m)}{(1.136\times10^{-3} kg/m s)}$$

$$\longrightarrow \bar{v} = \frac{Re}{17605.6}\ m/s$$

$$h_f = \frac{f \bar{v}^2 L}{2gD} = \frac{f v^2 (50m)}{2(9.81 m/s^2)(0.02m)} \rightarrow h_f = 127.4 f \bar{v}^2$$

$$\frac{P_2}{\rho g} + \cancel{z_2} + h_f = \frac{P_1}{\rho g} + \cancel{z_1} \qquad h_f = \frac{P_1 - P_2}{\rho g} = \frac{(501000 - 101000) Pa}{(1000 kg/m^3)(9.81 m/s^2)} = 40.8 m$$

f	Re	$\bar{v}$ (m/s)	h_f ft	Result
0.025	2.5×10^4	1.42	6.42	low
0.015	3.5×10^5	19.88	755.3	high
0.020	6.5×10^4	3.69	26.02	low
0.019	8.0×10^4	4.54	49.89	high
0.0195	7.0×10^4	3.98	39.27	Close enough

$t = V/Q$

$= V/\bar{v}A$

$= (100 m^3)/\bar{v}(0.000314 m^2)$

$= 318471\ m/\bar{v}\,(3600\ s/h)$

$= 88.46/\bar{v}\ h$

$= 88.46/(3.98)$

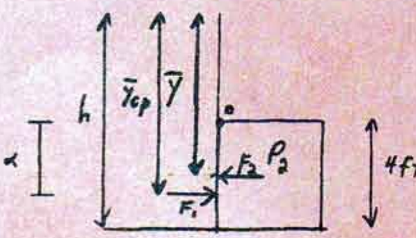
$= 22.2\ h$

$$\boxed{t = 22.2 h}$$

Problem Set 7

7.1) Known

$P_2 = 4\,psig = 576\,psfg$



a) Square $\quad A_s = 4\cdot 4 = 16\,ft^2 \qquad \bar{y} = h-2$

$I_{xx} = bh^3/12 = (4)(4)^3(12) = 21.\bar{3}\,ft^4 \qquad \bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A_s} + \bar{y} = \frac{21.\bar{3}ft^4}{(h-2)(16ft^2)} + (h-2)$

$P_1 = \rho g \bar{y} \quad F_1 = P_1 A_s \quad F_2 = P_2 A_s \qquad = \frac{1.\bar{3}}{(h-2)} + (h-2)$

$\Sigma M = 0 \quad F_1 \alpha = F_2\, 2 \qquad \alpha = 4 - (h - \bar{y}_{cp})$

$\rho g \bar{y} \cancel{A_s}(4 - h + \bar{y}_{cp}) = 2P_2 \cancel{A_s}$

$\rho g(h-2)\left(4 - \cancel{h} + \frac{1.\bar{3}}{h-2} + \cancel{h} - 2\right) = 2P_2 \Rightarrow (1.94\,slug/ft^3)(32.2\,ft/s^2)\left(\frac{1.\bar{3}}{h-2} + 2\right)(h-2) = 2(576\,psfg)$

$\Rightarrow \left(\frac{1.\bar{3}}{h-2} + 2\right)(h-2) = 18.44 \Rightarrow$ Calculator Equ^n Solver $\longrightarrow$ $\boxed{h = 10.6\,ft}$

---

b) Triangle $\nabla\ {}^{1/3 h}$ $\quad A_T = \frac{1}{2}(4\cdot 4) = 8\,ft^2 \qquad \bar{y} = h - \frac{2}{3}(4) = h - \frac{8}{3}$

$I_{xx} = bh^3/36 = 4\cdot 4^3/36 = 7.\bar{1}\,ft^4 \qquad \bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A_T} + \bar{y} = \frac{7.\bar{1}ft^4}{(h-\frac{8}{3})8ft^2} + (h - \frac{8}{3})$

$\qquad = \frac{0.\bar{8}}{(h-\frac{8}{3})} + (h - 8/3) \qquad \alpha = 4 - (h - \bar{y}_{cp})$

$\Sigma M = 0 \quad F_1\alpha = \frac{4}{3}F_2 \qquad = 4 - \cancel{h} + \frac{0.\bar{8}}{(h-8/3)} + (\cancel{h} - 8/3)$

$\rho g \bar{y} \cancel{A_T}\alpha = \frac{4}{3}P_2 \cancel{A_T} \qquad = \frac{0.\bar{8}}{(h-8/3)} + 4/3$

$\rho g(h - 8/3)\left(\frac{0.\bar{8}}{h-8/3} + 4/3\right) = \frac{4}{3}(576)$

$(1.94\,slug/ft^3)(32.2\,ft/s)(h - 8/3)\left(\frac{0.\bar{8}}{h-8/3} + 4/3\right) = 768$

$(h - 8/3)\left(\frac{0.\bar{8}}{h-8/3} + 4/3\right) = 12.29 \Rightarrow$ Calculator Equ^n Solver $\longrightarrow$ $\boxed{h = 11.2\,ft}$

7.2) Known

$P_1 = 100\,kPa(g)$

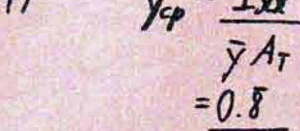
Water

$\rho = 998\,kg/m^3$

$A_c = \frac{1}{2}\pi r^2$

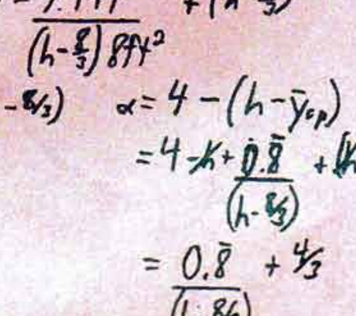
$= \pi/2(1)^2$

$= 1.5708\,m^2$



$\bar{y} = 2 + \frac{4(1)}{3\pi}$

$= 2.424$



r = 1

7.2...) $I_{xx} = 0.1098 r^4 = 0.1098(1)^4 = 0.1098 m^4$ $\qquad F_2 = P_2 A_c$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A} + \bar{y} = \frac{0.1098 m^4}{(2.424)(1.5708)m^2} + 2.424 m = 2.453 m$$

$P_1 = \rho g \bar{y}$

$F_1 = \rho g \bar{y} A_c$

$\Sigma M = 0 \quad F_1 \bar{y}_{cp} = F_2 \bar{y}'$

$(P_1 \rho g \bar{y}) \cancel{A_c} \bar{y}_{cp} = P_2 \cancel{A_c} (2.424 m)$

$[100000 Pa + (958 kg/m^3)(9.81 m/s^2)(2.424 m)](2.453 m) = P_2(2.424 m) \longrightarrow \boxed{P_2 = 124.2 kPa(g)}$

7.3) Known

$S.G = 1.5$

Displaced Volume $\rightarrow V = l \cdot w \cdot h - \frac{1}{2}(\pi r^2) l$

$= (1m)(2m)(2m) - \frac{1}{2}(\pi(1m)^2)(1m)$

$= 4m^3 - \frac{\pi}{2} m^3 = 2.429 m^3$

$F = \rho g V = 1.5(1000 kg/m^3)(9.81 m/s^2)(2.429 m^3)$

$\longrightarrow \boxed{F = 35.7 kN} \quad \boxed{F_{latch} = F/2 = 17.9 kN}$

7.4) Known

$\gamma = 8 kN/m^3$

$\gamma = \rho g$

$\bar{y} = 5 + (1 + 0.707)/2$

$= 5.854 m$

$A_y = (1.5)(1.707) = 2.561 m^2$

$I_{xx} = bh^3/12 = 1.5(1.707)^3/12 = 0.622 m^4$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A_y} + \bar{y} = \frac{0.622 m^4}{(5.854 m)(2.561 m^2)} + 5.854 m$$

$= 5.895 m$

$P_y = \gamma \bar{y}$

$F_y = \gamma \bar{y} A_y$

$\bar{x} = 0.707/2$

$= 0.3535 m$

$A_x = (0.707 m)(1.5 m)$

$= 1.061 m^2$

$P_x = \gamma h \quad F_x = \gamma h A_x$

$h = (5 + 1 + 0.3535)$

$= 6.3535$

$\circlearrowleft \Sigma M = 0 \quad F_y(\bar{y}_{cp} - 5m) + F_x(\bar{x}) = F_j(1.0m)$

$F_j \cdot 1m = \gamma \bar{y} A_y(\bar{y}_{cp} - 5m) + \gamma h A_x \bar{x}$

$= \gamma[\bar{y} A_y(\bar{y}_{cp} - 5m) + h A_x \bar{x}]$

$= 8 kN/m^3[(5.854 m)(2.561 m^2)(5.895 - 5)m + (6.3535 m)(1.061 m^2)(0.3535 m)]$

$F_j = 8 kN/m^3 (15.8 m^4)/1m$

$= 126.4 kN \qquad \boxed{F_j = 126.4 kN}$

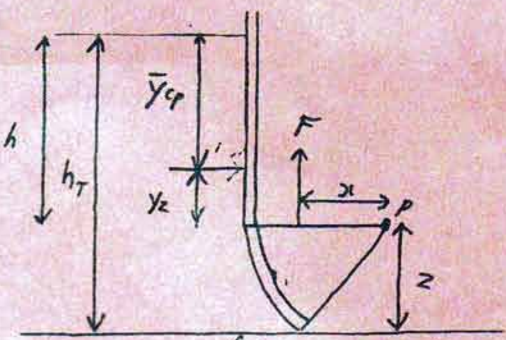
7.5) Note: Force acting on a circle are normal to the surface and pass through the hinge, thus creating no moment about the hinge.

7.5...) Known

$z = 6\sin 45 \approx 4.24\,ft$ $\qquad$ $\rho = 62.4\,lbm/ft^3$

$x = 6-2 = 4\,ft$ $\qquad$ $F = 30000\,lbf$

$\bar{y}_{cp} = \frac{2}{3}h \quad y_2 = h - \bar{y}_{cp} = \frac{1}{3}h$



$\circlearrowleft \sum M_p = 0 \quad (30000\,lbf)(4\,ft) - F_A y_2 = 0$

$120000\,lbf \cdot ft = (\frac{1}{2}\rho g h)(wh)(\frac{1}{3}h)$

$h^3 = \frac{6(120000\,lbf \cdot ft)}{\rho g w} = \frac{720000\,\cancel{lbf}\cdot ft}{(62.4\,\cancel{lbm}/ft^3)(32.2\,ft/\cancel{s^2})(30\,\cancel{ft})} \cdot \left(\frac{32.2\,\cancel{lbm}\,\cancel{ft}}{\cancel{lbf} \cdot s^2}\right)$

$h^3 = 384.6\,ft^3 \rightarrow h = 7.27\,ft$

$h_T = h + z = (4.24 + 7.27)\,ft = 11.5\,ft$ $\quad \boxed{h = 11.5\,ft}$

7.6) Known

$w = 1\,ft$

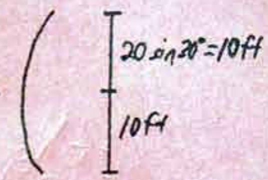
$r = 20\,ft$

$\rho = 1.94\,slug/ft^3$

$I_{xx} = bh^3/12 = (1\,ft)(20\,ft)^3/12 = 666.7\,ft^4$

$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A} + \bar{y} = \frac{666.7\,ft^4}{(10\,ft)(20\,ft^2)} + 10\,ft = 13.3\,ft$

$\rightarrow$



a) $F_x = \rho g \bar{y} A = (1.94\,slug/\cancel{ft^3})(32.2\,ft/s^2)(10\,ft)(20\,ft^2)$

$= 12493.6\,slug\,ft/s = 12493.6\,lbf$ $\quad \boxed{F_x = 12493.6\,lbf}$

b) Buoyancy $\quad V = [\frac{1}{6}\pi r^2 - 2(\frac{1}{2}bh)]w$

$= [\frac{1}{6}\pi(20\,ft)^2 - (20\cos 30°)(20\sin 30°)](1\,ft)$

$= 36.2\,ft^3$

$F_{up} = \rho V g = (1.94\,slug/\cancel{ft^3})(36.2\,\cancel{ft^3})(32.2\,ft/s^2) = 2261\,lbf$ $\quad \boxed{F_{up} = 2261\,lbf}$

c,d) 0, due to all forces acting through the center

7.7) Known

$\sigma_{max} = 100\,MPa$

$SG = 1.5$

$h = 20$

$D = 6m$

$h_{ring} = 2m$

$\sigma = PD/2t \rightarrow t = \frac{PD}{2\sigma}$

$t = \frac{\rho g h D}{2\sigma} = \frac{1.5(1000\,\cancel{kg/m^3})(9.81\,\cancel{m/s^2})(h)(6\,\cancel{m})}{2(100000000\,\cancel{N/m^2})} \cdot \left(\frac{\cancel{N}\,\cancel{s^2}}{\cancel{kg}\,\cancel{m}}\right) \cdot \left(\frac{1000\,mm}{m}\right)$

$= 0.44145\,h \cdot mm/m$

$P = \rho g h$ $\quad$ top $h = 2m \rightarrow t = 0.89$

middle $h = 10m \rightarrow t = 4.42$

bottom $h = 20m \rightarrow t = 8.84 \rightarrow \boxed{t_{min} = 8.84}$

Note: I believe the manual adds a safety factor

7.8) Known

$V = 4000\,m^3$

$h = 2000\,m$

$T = 275.16\,K$ } tables

$P = 79510\,Pa$ }

$\rho_a = 1.0066\,kg/m^3$

$R_{H_2} = 4125\ J/kgK$

$R_{He} = 2077\ J/kgK$

$R_a = 287\ J/kgK$

$T_{Ha} = 40°C = 313\,K$

$\rho_{H_2} = P/RT = 79510\,Pa / (4125\,J/kgK)(275.16\,K)$
$= 0.070\ kg/m^3$

$\rho_{He} = 79510\,Pa / (2077\ J/kgK)(275.16\,K)$
$= 0.139\ kg/m^3$

$\rho_{Ha} = 79510\,Pa / (287\ J/kgK)(313\,K)$
$= 0.885\,kg/m^3$

i) $F = \rho_a g V - \rho_{H_2} g V = gV(\rho_a - \rho_{H_2}) = (9.8\,m/s^2)(4000\,m^3)(1.0066 - 0.070)\,kg/m^3 = 36752\ kg\,m/s^2$

$\boxed{= 36.8\ kN}$

ii) $F = gV(\rho_a - \rho_{He}) = (9.81\,m/s^2)(4000\,m^3)(1.0066 - 0.139)\,kg/m^3 = 34044\ kg\,m/s^2$

$\boxed{= 34.0\ kN}$

iii) $F = gV(\rho_a - \rho_{Ha}) = (9.81\,m/s^2)(4000\,m^3)(1.0066 - 0.885)\,kg/m^3 = 4771.6\ kg\,m/s^2$

$\boxed{= 4.8\,kN}$

7.9) 

$r = 0.5\,m$

$l = 1.0\,m$

$A = r \cdot r + 3/4\,\pi r^2$
$= (0.5\,m)(0.5\,m) + 3/4(\pi)(0.5)^2$
$= 0.839\,m^2$

$V = A \cdot l = (0.839\,m^2)(1.0\,m) = 0.839\,m^3$

$V_L = \pi r^2 l = \pi (1.0\,m)(0.5\,m)^2 = 0.785\ m^3$

$F_B = \rho g V = (1000\,kg/m^3)(9.81\,m/s)(0.839\,m^3)$
$= 8230.59\ kg\,m/s^2$

$W_L = \rho_L g V_L \qquad W_L = F_B$

$\rho_L = \dfrac{W_L}{g V_L} = \dfrac{8230.59\ kg\,m/s^2}{(9.81\,m/s^2)(0.785\,m^3)}$
$= 1069\,kg/m^3$

$\boxed{\rho_L = 1069\ kg/m^3}$

$F_x = \rho g h A = \rho g h r l = (1000\,kg/m^3)(9.81\,m/s^2)(0.25\,m)(0.5\,m)(1.0\,m)$
$= 1226.25\ kg\,m/s^2$

$\boxed{F_x = 1226\ N}$

7.10) Known

$h_{oil} = 4\,m$

$SG_{oil} = 0.75$

$SG_w = 1.03$

$w = 10\,m$

$h = 5\,m$

$\rho_w = 1000\,kg/m^3$

$V_b = hb \cdot l$
$= (5\,m)(10\,m)\,l = 50\,m^2\,l$

$V_{oil} = b \cdot h_{oil}\,l$
$= (10\,m)(4\,m) \cdot l = 40\,m^2 \cdot l$

$W = W_b + W_{oil} = W_b + \rho_{oil} V_{oil}\,g$

$\Sigma F_y = 0 \quad \rho_w V_b g - (W_b + \rho_{oil} V_{oil} g) = \rho_w SG_w V_b g - W_b - \rho_w SG_{oil} V_{oil} g$

$W_b = \rho_w g\,(V_b SG_w - V_b SG_{oil})$
$= (1000\ kg/m^3)(9.81\,m/s^2)(1.03 \cdot 50\,m^2 \cdot l - 0.75 \cdot 40\,m^2 \cdot l)$
$= 210915\ kg/s^2 \cdot l$

7.10...) $\Sigma F_y = 0 \quad \rho_w SG_w V_{b2} g - W_b = 0$ Without oil

$V_{b2} = \dfrac{W_b}{\rho_w SG_w g} \rightarrow \cancel{\ell} w h_2 = \dfrac{210\,915\ \cancel{kg}/s^2 \cdot \cancel{\ell}}{(1000\ \cancel{kg}/m^3)(9.81 m/\cancel{s^2})(1.03)}$

$h_2 = 20.87\ m^2 / (10 m)$ $\quad \boxed{h_2 = 2.09 m}$

$= 2.09\ m$

$\curvearrowleft + \Sigma M = 0 \quad F_w d_w - F_o d_o - F_d d_d = 0$ $\quad d_d = 6.0m \quad h_w = 5 \quad \bar{y}_w = 2.5m \quad \bar{y}_{cpw} = \frac{1}{3} h_w = 1.67m$

$F_d = (F_w d_w - F_o d_o) / d_d$

$= [\rho_w g SG_w \bar{y}_w \bar{y}_{cpw} A_w - \rho_w g SG_o \bar{y}_o \bar{y}_{cpo} A_o] / d_d$ $\quad h_o = 4 \quad \bar{y}_o = 2m \quad \bar{y}_{cpo} = \frac{1}{3} h_o = 1.33m$

$\rightarrow = (1000 kg/m^3)(9.81 m/s^2)[(1.03)(2.5m)(1.67m)(5m) - (0.75)(2m)(1.33m)(4m)] / 6m$

$= 22107\ kg/s^2 = 22107\ kg/s^2 \cdot m/m \quad \boxed{= 22.11\ kN/m}$

7.11) known

$h_1 = 0.3m$

$r_1 = 0.05m$

$r_2 = r_1 \cos 45° = 0.03535m$

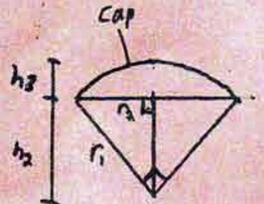
$h_2 = 0.03535$

$h_3 = r_1 - h_2 = 0.01465$

$V_{cap} = V_{dome} = \frac{1}{3}\pi h_3^2 (3r_1 - h_3)$

$= \frac{\pi}{3}(0.01465m)^2(3(0.05) - 0.01465)$

$= 3.042 \times 10^{-5} m^3$



$V_1 = (h_1 + h_3)\pi r_2^2 - V_{cap}$

$= (0.3 + 0.01465)m \cdot \pi (0.03535)^2 - 3.042 \times 10^{-5} m^3 = 1.205 \times 10^{-3} m^3$

$V_2 = V_{sphere} - V_{center} = 4\pi r^3/3 - 2V_{cap} - \pi r_2^2 (2h_1)$

$= 4\pi (0.05m)^3/3 - 2(3.042 \times 10^{-5} m^3) - \pi (0.03535m)^2 (2)(0.03535m)$

$= 1.85 \times 10^{-4} m^3$

$F = \rho g (V_1 - V_2)$

$= (1000 kg/m^3)(9.81 m/s^2)(1.205 \times 10^{-3} m^3 - 1.85 \times 10^{-4} m^3)$

$= 10.01\ N$

$\boxed{F = 10.0\ N}$