MAT 1332, Winter 2014, Assignment 4
Due Friday February 7 by 3:00pm.
Late assignments will not be accepted; nor will unstapled assignments.
Professors in the math department will not lend you a stapler; do not ask for one.

Instructor (circle one): Robert Smith? Frithjof Lutscher Catalin Rada

DGD (circle one): 1 2 3 4

Last Name ______________________________ First Name ______________________________

Student Number ______________________________(please write clearly)

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ______________________________

QUESTION 1. Find the solution $y(x)$ of the following separable initial value problem

$$\frac{dy}{dx} = y(x)\ln(x), \qquad y(1) = 1.$$

We separate variables and get: $\frac{dy}{y} = \ln(x)dx$. Then we integrate: $\int \frac{dy}{y} = \int \ln(x)dx = \int 1 \times \ln(x)dx$. By integration by parts on the right hand side one has that:

$$\ln|y| = x\ln(x) - \int x\frac{1}{x}dx = x\ln(x) - \int 1dx = x\ln(x) - x + c,$$

where c is a number.

From $1 = y(1)$ one gets that $0 = -1 + c$, hence $c = 1$.

It follws that $\ln|y| = x\ln(x) - x + 1$, thus $|y| = e^{x\ln(x)-x+1}$, so we obtain that $y(x) = \pm e^{x\ln(x)-x+1}$.

Since $y(1) = 1 > 0$, one obtains: $y(x) = e^{x\ln(x)-x+1}$.

QUESTION 2. Find the solution $y(x)$ of the following separable initial value problem

$$\frac{dy}{dx} = (4 + y^2(x))\sin(x), \qquad y(0) = 0.$$

We separate and get: $\frac{dy}{4+y^2} = \sin(x)dx$. Then we integrate: $\int \frac{dy}{4+y^2} = \int \sin(x)dx$ (by using a substitution in the left integral) as follows:

$\frac{1}{2}\arctan(\frac{y}{2}) = -\cos(x) + c$, where c is a number.

From $0 = y(0)$, one gets that $0 = -1 + c$, hence $c = 1$, so $\frac{1}{2}\arctan(\frac{y}{2}) = -\cos(x) + 1$, or $\arctan(\frac{y}{2}) = -2\cos(x) + 2$.

Since tan is the inverse function of arctan, one obtains that $\frac{y}{2} = \tan(-2\cos(x) + 2)$, hence $y = 2\tan(-2\cos(x) + 2)$.

QUESTION 3. Find the steady states, and draw the phase-line diagram of the dynamical system given by

$$\frac{dx}{dt} = \cos(x), \qquad 0 < x < 2\pi.$$

Determine the stability of each steady state by using the stability criterion form class.

From $\cos(x) = 0$ one obtains that either $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$ in $(0, 2\pi)$.

Note that $f'(x) = -\sin(x)$ (where $f(x) = \cos(x)$), and thus:

(i) $f'(\frac{\pi}{2}) = -\sin(\frac{\pi}{2}) = -1 < 0$, and thus $\frac{\pi}{2}$ is STABLE.

(ii) $f'(\frac{3\pi}{2}) = -\sin(\frac{3\pi}{2}) = 1 > 0$, and thus $\frac{3\pi}{2}$ is UNSTABLE.

The phase-line diagram is attached.

QUESTION 4. Find the steady states, and draw the phase-line diagram of the dynamical system given by

$$\frac{dx}{dt} = x(1-x)(x^2-4).$$

Determine the stability of each steady state by using the stability criterion form class.

From $x(1-x)(x^2-4) = x(1-x)(x-2)(x+2) = 0$ one obtains that the equilibia is formed by: $x = 0, x = 1, x = 2, x = -2$.

If $f(x) = x(1-x)(x^2-4) = (x-x^2)(x^2-4)$ one has that $f'(x) = -4x^3 + 3x^2 + 8x - 4$.

Note that:

(i) $f'(0) = -4 < 0$, hence 0 is stable;

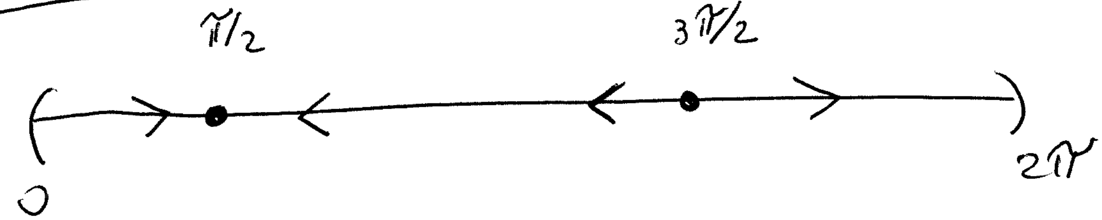
(ii) $f'(1) = 3 > 0$, hence 1 is unstable;

(iii) $f'(2) = -12 < 0$, hence 2 is stable;

(iv) $f'(-2) = 24 > 0$, hence -2 is unstable;

The phase-line diagram is attached.

Q3 Phase-Line diagram:



Q4 Phase-Line diagram: