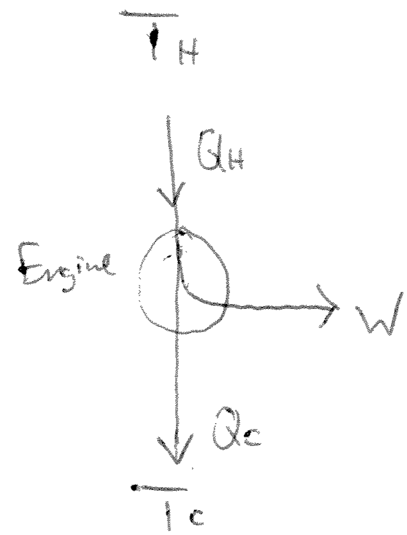
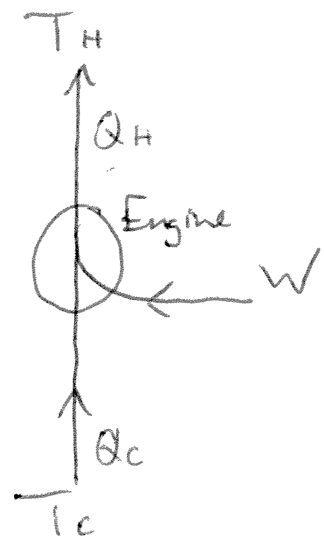
LECTURE 9

HEAT ENGINE



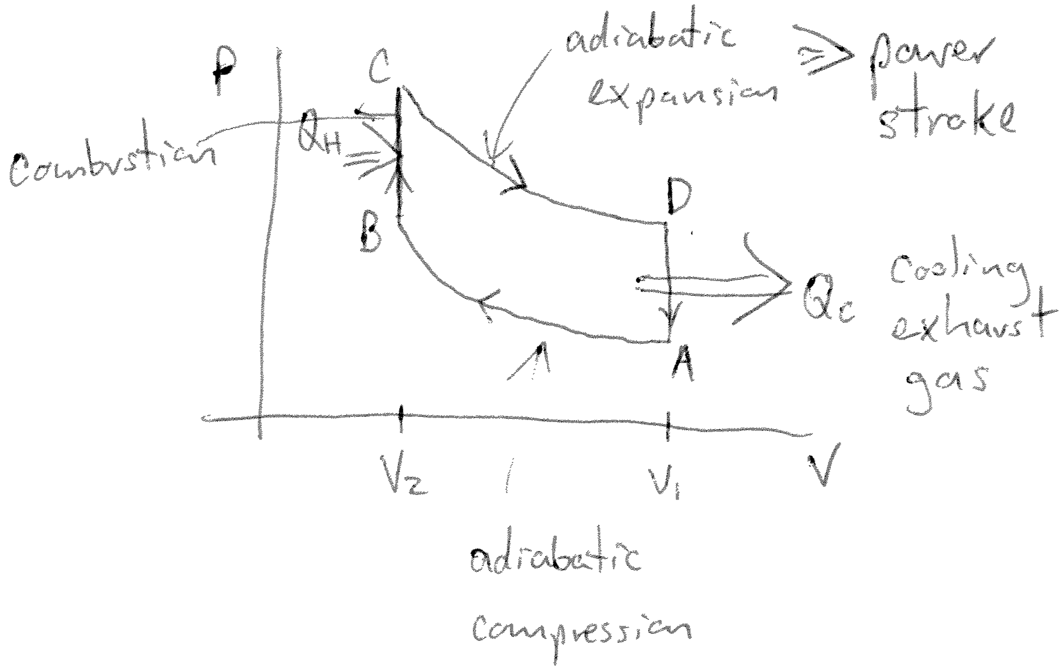
HEAT PUMP



	HEAT ENGINE	HEAT PUMP
Q_H	+'ve, input	−'ve, output
Q_C	−'ve, output	+'ve, input
W	+'ve, work is being done	−'ve, done on system
Energy Balance	$\lvert Q_H \rvert = \lvert Q_C \rvert + \lvert W \rvert$	$\lvert Q_C \rvert + \lvert W \rvert = \lvert Q_H \rvert$
Efficiency/Performance	$e = \dfrac{W}{Q_H}$	$K = \dfrac{\lvert Q_C \rvert}{\lvert W \rvert}$
	$e = 1 - \left\lvert \dfrac{Q_C}{Q_H} \right\rvert$	$K = \dfrac{\lvert Q_C \rvert}{\lvert Q_H \rvert - \lvert Q_C \rvert}$

Gas leaves engine as exhaust and doesn't re-enter "engine", but since equivalent gas/air mixture enters again we may consider the process cyclic.

MODEL → Otto



What is the efficiency of this engine?

→ Need W for each cycle

→ Treat "working substance" as an ideal gas.

$B \to C$ & $D \to A$: const. V, $W = 0$

$A \to B$ and $\cancel{B} \to \cancel{C}$ $C \to D$

$$\Delta U = 0 = Q - W$$

cyclic

$$W = Q = Q_H - Q_c$$

$$\begin{cases} Q_H = nC_v(T_c - T_B) > 0 \\ Q_c = nC_v(T_A - T_D) < 0 \end{cases}$$

$$e = 1 - \frac{|Q_c|}{|Q_H|}$$

$$= 1 - \frac{\cancel{nC_v}(T_D - T_A)}{\cancel{nC_v}(T_c - T_B)}$$

$TV^{\gamma - 1} = \text{const}$ adiabatic

$$\begin{cases} V_A = V_D = V_1 \\ V_B = V_c = V_2 \end{cases}$$

$$\begin{cases} T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \\ T_c V_c^{\gamma-1} = T_D V_D^{\gamma-1} \end{cases}$$

$$T_A V_1^{\gamma-1} = T_B V_2^{\gamma-1}$$

$$T_c V_2^{\gamma-1} = T_D V_1^{\gamma-1}$$

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$$e = 1 - \frac{\left(T_C \left(\frac{V_2}{V_1}\right)^{\gamma-1} - T_B \left(\frac{V_2}{V_1}\right)^{\gamma-1}\right)}{T_C - T_B}$$

$$\boxed{e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}}$$

$$r = \frac{V_1}{V_2}$$

$$\boxed{e = 1 - \frac{1}{r^{\gamma-1}}} \quad \text{Otto}$$

$$TV^{\gamma-1} = \text{const}$$

$$\left[\frac{V_2}{V_1}\right]^{\gamma-1} = \frac{T_A}{T_B} = \frac{T_D}{T_C} \quad D$$

$$\boxed{e = 1 - \frac{T_A}{T_B} = 1 - \frac{T_D}{T_C}} \quad \text{Otto cycle}$$

EXAMPLE $e = \frac{W}{Q_H} = 1 - \frac{Q_c}{Q_H}$

Derive an expression for the thermal efficiency of the Carnot engine in terms of reservoir temperatures only.

$a \to b$ gas expands isothermally at temperature T_H, absorbs Q_H.

$$Q_H = W_{ab} = nRT_H \ln\left(\frac{V_b}{V_a}\right) \quad dW = p\,dV$$

$c \to d$ rejection of thermal energy

$$Q_c = |W_{cd}| = nRT_c \ln\left(\frac{V_c}{V_d}\right)$$

$$\frac{Q_c}{Q_H} = \frac{T_c}{T_H}\frac{\ln(V_c/V_d)}{\ln(V_B/V_A)}$$

$$PV^{\gamma} = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

$b \to c$ $T_H V_B^{\gamma-1} = T_c V_c^{\gamma-1}$

$d \to a$ $T_H V_a^{\gamma-1} = T_c V_d^{\gamma-1}$

$$\left(\frac{V_b}{V_a}\right)^{\gamma-1} = \left(\frac{V_c}{V_d}\right)^{\gamma-1}$$

$$\frac{|Q_C|}{|Q_H|} = \frac{T_c}{T_H}$$

$$\boxed{e_c = 1 - \frac{T_c}{T_H}}$$

Thermal Efficiency of Carnot engine.

$$e_{otto} = 1 - \frac{T_A}{T_B} = 1 - \frac{T_D}{T_C}$$

Entropy and Disorder

- infinitesimal isothermal expansion of I.G.
  - dQ, let gas expand so T is constant
  - internal energy depend only on T, 1st LAW

$$dQ = dW = p\,dV = \frac{nRT}{V}\,dV$$

$$\frac{dV}{V} = \frac{dQ}{nRT}$$

After expansion, gas more disordered

$\frac{dV}{V}$ - measure of disorder

$$\frac{dV}{V} \propto \frac{dQ}{T}$$

$$dS = \frac{dQ}{T}$$ infinitesimal, reversible process

$$\Delta S = \frac{Q}{T}$$ reversible process

$$\Delta S = \int_1^2 \frac{dQ}{T}$$ reversible process, need not be isothermal