Why should C_p and C_v be different?

For a given temperature increase, the internal energy change ΔU of an ideal gas has the same value no matter what the process.

→ for an ideal gas, U depends only on T, not p or V

Rearrange first law $Q = \Delta U + W$

constant volume: $\Delta V = \emptyset$, so $p\Delta V = W = \emptyset$

constant pressure: $W > 0$, work done by gas, additional energy must be supplied

so $C_p > C_v$ for I.G.

[if volume decreases during heating $C_p < C_v$]

CONSTANT V

$dQ = nC_v\,dT$

$W = 0$

$dU = dQ - \cancel{dW}^{\,0}$

$= dQ$

$dU = nC_v\,dT$

CONSTANT P

$dQ = nC_p\,dT$

$dW = p\,dV$

$p\,dV = nR\,dT = dW$

$dQ = dU + dW$

$\cancel{n}C_p\,\cancel{dT} = \cancel{n}C_v\,\cancel{dT} + \cancel{n}R\,\cancel{dT}$

$$\boxed{C_p = C_v + R}$$

$\gamma = \dfrac{C_p}{C_v} \leftarrow \dfrac{3}{2}R$ (monatomic gas)

$C_p = \dfrac{5}{2}R$

$\gamma = 1.67$

diatomic $\gamma = \dfrac{7/2\,R}{5/2\,R} = 1.40$

EXAMPLE

A typical office has ≈ 2500 moles of air. Find ΔU when office is cooled from 25.0°C to 0.0°C.

$$\Delta U = n C_v \Delta T$$

$$C_v = \frac{5}{2} R$$

$$\Delta U = (2500 \text{ mol}) \left(\frac{5}{2} R \right) (0 - 25)°C$$

$$= -1.3 \times 10^6 \text{ J}$$

FIRE PISTON

What do we know?

Variables: p_1, V_1, T_1, p_2, V_2, T_2

p_1: P_{atm}

T_1: room temperature

T_2: ? Autoignition $T = 407\,°C$

Can we simplify the problem by categorizing its type?

isochoric? ✓ X

isobaric X

isothermal X

adiabatic $Q = 0$

Relate V, T, and p for an adiabatic process concerning an ideal gas, see what we can do with all of the unknowns.

First law: $dU = dQ - dW$

$$dU = nC_v\,dT$$

$$dW = p\,dV$$

$$nC_v\,dT = -p\,dV$$

$$nC_v dT = -\frac{nRT}{V} dV$$

$$\frac{dT}{T} + \frac{R}{C_v}\frac{dV}{V} = 0$$

$$\frac{R}{C_v} = \frac{C_p - C_v}{C_v} = \frac{C_p}{C_v} - 1$$

$$= \gamma - 1$$

$$\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0 \quad .$$

$$\ln T + (\gamma - 1)\ln V = \text{constant}$$

$$\ln T + \ln V^{\gamma - 1} = \text{constant}$$

$$\ln\left(TV^{\gamma-1}\right) = \text{constant}$$

$$\underline{TV^{\gamma-1} = \text{constant}}$$

adiabatic, I.G. $\boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$$

$$\frac{407^\circ C + 273}{25 + 273} = \left(\frac{V_1}{V_2} \right)^{\gamma - 1}$$

$$\ln \left(\frac{680}{298} \right) = 0.4 \ln \left(\frac{V_1}{V_2} \right)$$

$$\ln \left(\frac{V_1}{V_2} \right) = 2.06$$

$$\boxed{\frac{V_1}{V_2} = 7.75}$$

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p & V

$$T V^{\gamma - 1} = \text{constant}$$

$$\left(\frac{pV}{nR} \right) V^{\gamma - 1} = \text{constant}$$

$$pV^{\gamma} = \text{constant}$$

$$\boxed{p_1 V_1^{\gamma} = p_2 V_2^{\gamma}}$$ adiabatic, I.G.