LECTURE 6

$$V_{rms} = \sqrt{\overline{V^2}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

lighter molecules move, on average, faster than heavier molecules

e.g. @ $20°C$ $\quad V_{rms}^{H_2} = 1902 \text{ m/s}$

$V_{rms}^{CO_2} = 408 \text{ m/s}$

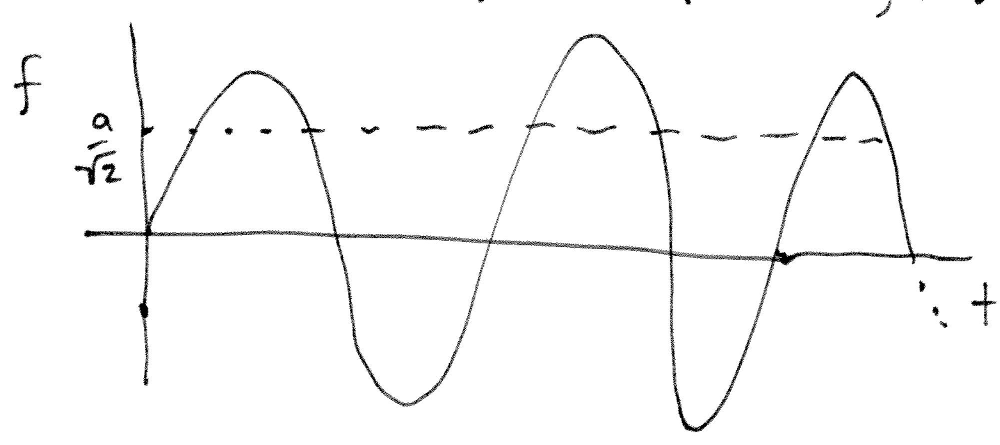
rms → Root Mean Square

$$V_{rms} = \sqrt{\overline{V^2}} = \sqrt{\frac{1}{n}\left(V_1^2 + V_2^2 + \dots + V_n^2\right)}$$

$f(t)$ over time interval $t_1 \leq t \leq t_2$

$$f_{rms} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f(t)]^2 \, dt}$$

$f = a\sin(2\pi f t)$, $f_{rms} = \frac{a}{\sqrt{2}}$



EXAMPLE

A tank of volume $0.300\ m^3$ contains 2.00 mol He gas at $20.0°C$. Assuming the He behaves like an ideal gas, find the total energy of this system.

$$E_{TOT} = N\left(\frac{1}{2} m\overline{v^2}\right) = \frac{3}{2} nRT$$

$$= \frac{3}{2}(2.00\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(293\ \text{K})$$

$$= 7.30 \times 10^3\ \text{J}$$

- Collisions between molecules -

- how often do they collide?
- how far do they travel between collisions?

N - # spheres

V - volume

MEAN FREE PATH

$$\boxed{\lambda = v t_{mean} = \frac{V}{4\pi\sqrt{2}\, r^2 N}}$$

In terms of macroscopic properties

$$pV = N k_B T$$

$$\boxed{\lambda = \frac{k_B T}{4\pi\sqrt{2}\, r^2 p}}$$

Air at 27°C and 1 atm?

$r = 2.0 \times 10^{-10}$ m

$\lambda = 58$ nm

→ mean time between collisions

$$t_{mean} = \frac{\lambda}{v_{rms}} = \frac{58 \text{ nm}}{484 \text{ m/s}}$$

$$= 1.2 \times 10^{-10} \text{ s}$$

EXAMPLE 2

At 20°C and 750 torr pressure, the mean free paths for Ar gas and N_2 gas are $\lambda_{Ar} = 9.9 \times 10^{-6}$ cm

$$\lambda_{N_2} = 27.5 \times 10^{-6} \text{ cm}$$

a) Find ratio of diameter of an Ar atom to that of N_2 molecule.

b) What is the mean free path of Ar @ 20°C and 150 torr.

$$\frac{\lambda_{Ar}}{\lambda_{N_2}} = \frac{k_B T}{4\pi\sqrt{2}\, r_{Ar}^2\, p} \; \frac{4\pi\sqrt{2}\, r_{N_2}^2\, p}{k_B T}$$

$$= \left(\frac{r_{N_2}}{r_{Ar}}\right)^2$$

$$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}} = \sqrt{\frac{27.5 \times 10^{-6}\text{ cm}}{9.9 \times 10^{-6}\text{ cm}}}$$

$$= 1.7$$

b) $$\frac{\lambda_1}{\lambda_2} = \frac{k_B T_1}{4\pi\sqrt{2}\,r^2 p_1} \frac{4\pi\sqrt{2}\,r^2 p_2}{k_B T_2}$$

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{p_2}{p_1}\right)$$

$$\lambda_2 = \frac{p_1}{p_2}\lambda_1$$

$$= \frac{750}{150}\left(9.9 \times 10^{-6}\,\text{cm}\right)$$

$$= 5 \times 10^{-5}\ \text{cm}$$

18.4 Heat Capacities

C_V - molar heat capacity at constant volume

$$Q = n C_V T$$

$$KE_{tr} = \frac{3}{2} nRT$$

translational

$$d\left(KE_{tr}\right) = \frac{3}{2} nR\,dT$$

$$dQ = nC_V\,dT$$

KE_{tr} represents total energy of gas

Δ total energy $= \Delta$ heat energy

$$\rightarrow \frac{3}{2} nR\,dT = nC_v\,dT$$

$$C_v = \frac{3}{2} R = 12.47 \text{ J/mol}\cdot\text{K}$$

Maxwell - Boltzmann

$$V_{mp} = \sqrt{\frac{2k_B T}{m}}$$

$$V_{av} = \sqrt{\frac{8k_B T}{\pi m}}$$

$$V_{rms} = \sqrt{\frac{3k_B T}{m}}$$

EXAMPLE

The most probable speed of molecules in a gas when it has uniform T_2 is the same as the rms speed of the molecules in this gas when it has uniform temperature T_1.

Calculate T_2/T_1

$$\frac{V_{mp}}{V_{rms}} = \frac{\sqrt{\frac{2k_B T_2}{m}}}{\sqrt{\frac{3k_B T_1}{m}}} = \sqrt{\frac{2T_2}{3T_1}}$$

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{V_{mp}}{V_{rms}}\right)^2 = \frac{3}{2}$$