LECTURE 5

ROUGH FIREWALKING CALCULAION

Thermal Conductivity

$$k_{steel} = 50.2 \ \frac{W}{m \cdot K}$$

$$K_{brick} = 0.6 \ \frac{W}{m \cdot K}$$

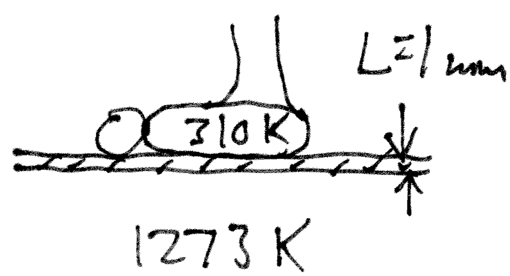
$$K_{burnt} = 0.1 \ \frac{W}{m \cdot K} = 1 \times 10^{-3} \ \frac{J}{cm \cdot s \cdot K}$$

What current of heat is needed to cause a second degree burn.

$$16.4 \ \frac{J}{cm^2}$$ J. Occup. Med., 32, 215 (1990).

$$T_{campfire} \approx 1273 \ K$$

$$T_{body} \approx 310 \ K$$



$$\frac{dQ}{dt} = KA \frac{T_{campfire} - T_{body}}{L}$$

$$dt = \frac{dQ}{A} \frac{L}{K (T_c - T_b)}$$

$$= \frac{(16.4 \ J/cm^2)(0.10 \ cm)}{(1 \times 10^{-3} \ J/cm \cdot s \cdot K)(1273 - 310) K} = 1.7 s$$

EXAMPLE

Compressing gas

What is the final temperature of the compressed gas if its initial $T_i = 27°C$ and the initial and final pressures are 1.00 atm and 21.7 atm respectively?

$$p_1 V_1 = n_1 R T_1$$

$$p_2 V_2 = n_2 R T_2 \qquad n_1 = n_2$$

$$nR = \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{9}{1}$$

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1}$$

$$T_2 = \frac{(300\,K)(21.7\,atm)}{(1.00\,atm) \times 9} = 723\,K = 450°C$$

Variation of atmospheric pressure with elevation

as $y \uparrow$, $p(y) \downarrow$, $\rho(y) \downarrow$

2 unknown functions of y,
need 2 equations to solve

① $pV = nRT \Rightarrow pV = \frac{m_{total}}{M} RT \Rightarrow \rho = \frac{pM}{RT}$

② $\frac{dp}{dy} = -\rho g$

combine and solve

$$\frac{dp}{dy} = -\frac{pM}{RT} g$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{Mg}{RT} \int_{y_1}^{y_2} dy$$

$$\ln \frac{p_2}{p_1} = -\frac{Mg}{RT} (y_2 - y_1)$$

$$\frac{p_2}{p_1} = e^{-Mg(y_2-y_1)/RT}$$

let $y = 0$ at sea level

$$p_1 = 101.3 \text{ kPa}$$

$$\Rightarrow p = p_0 e^{-Mgy/RT}$$

pressure at one mile high

$$\frac{Mgy}{RT} = \frac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(1609 \text{ m})}{(8.314 \text{ J/mol}\cdot\text{K})(283 \text{ K})}$$

$$\doteq 0.19$$

$$p = p_0 e^{-0.19}$$

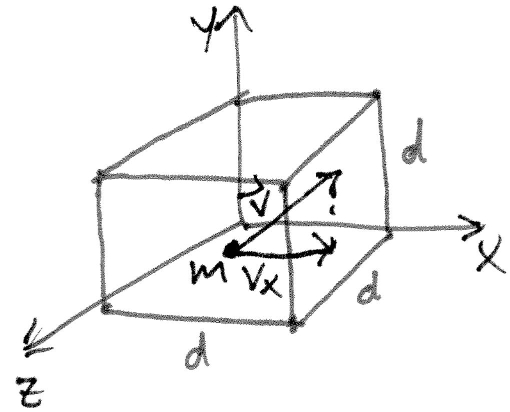
$$= (1.013 \times 10^5 \text{ Pa}) e^{-0.19}$$

$$= 0.84 \times 10^5 \text{ Pa}$$

$$= 0.83 \text{ atm}$$

$$p \rightarrow \rho = \frac{pM}{RT}$$

- Kinetic Molecular Model of an Ideal Gas -



Use Newton's Laws

N molecules

$V = d^3$

elastic collision with wall

v_x changes direction

v_y, v_z remain unchanged

momentum of molecule $\quad \Delta p_x = p_x^f - p_x^i = -mv_x - (mv_x)$

$= -2mv_x$

momentum of wall $= 2mv_x$ (p conserved)

impulse-momentum theorem, force exerted by molecule on wall is F_1 $\quad F_1 \Delta t = \Delta p = 2mv_x$

Time interval between two collisions with the same wall $\Delta t = \frac{2d}{v_x}$

force imparted to wall by a single collision $F_1 = \frac{2mv_x}{\Delta t} = \frac{2mv_x}{2d/v_x}$

$$= \frac{mv_x^2}{d}$$

All molecules

$$F_{TOTAL} = \frac{m}{d}\left(v_{x_1}^2 + v_{x_2}^2 + \ldots + v_{x_N}^2\right)$$

for N molecules, average value of square of velocity in x-direction is

$$\overline{v_x^2} = \frac{v_{x_1}^2 + v_{x_2}^2 + \ldots + v_{x_N}^2}{N}$$

TOTAL FORCE $F = \frac{Nm}{d}\overline{v_x^2}$

All 3 v components

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

average of all molecules

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

So $\overline{v^2} = 3\overline{v_x^2}$

Sub back into F equation

$$F = \frac{N}{3}\left(\frac{m\overline{v^2}}{d}\right)$$

Total pressure

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\left(\frac{N}{d^3} m\overline{v^2}\right)$$

$$= \frac{1}{3}\left(\frac{N}{V}\right) m\overline{v^2}$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2} m\overline{v^2}\right)$$

Molecular Interpretation of Temperature

Ideal Gas Law — **KMM**

$$pV = nRT \qquad pV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$nRT = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$\frac{n}{N} = \frac{1}{N_A}$$

$$\underbrace{\frac{R}{N_A}}\,T = \frac{2}{3}\left(\frac{1}{2}m\overline{v^2}\right)$$

k_B - Boltzmann Constant

$= 1.381 \times 10^{-23}$ J/molecule $\cdot$ K

$$\boxed{\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T}$$

Speed

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_BT}{m}}$$

$$= \sqrt{\frac{3RT}{M}}$$