**Concordia University**

| Course | Number | Sections |
|---|---|---|
| ENGR | 233 | P, Q |

| Examination | Date | Time | Total Marks | Pages |
|---|---|---|---|---|
| Final | April 2008 | 3 hours | 100 | 2 |

| Course Coordinator | Instructors |
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**Special Instructions:** use of calculators and outside materials is NOT permitted.

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Each problem is worth 10 marks unless stated otherwise.

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**Problem 1.** For the vector field

$$\vec{F}(x,y,z) = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$$

compute –**if possible**– the following quantities. If it is not possible **explain why not**.

**(a)** $\mathrm{div}(\mathrm{curl}\,\vec{F}(x,y,z))$ , **(b)** $\mathrm{curl}(\mathrm{div}\,\vec{F}(x,y,z))$ , **(c)** $\mathrm{grad}(\mathrm{div}\,\vec{F}(x,y,z))$ , **(d)** $\mathrm{div}(\mathrm{grad}\,\vec{F}(x,y,z))$

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**Problem 2.** Find the equation of the tangent plane of the surface defined by

$$z^3 - xyz = 1$$

at the point $(4, \frac{1}{2}, -1)$.

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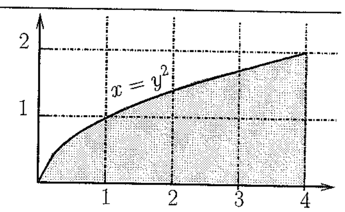
**Problem 3.**
Evaluate the following integral by reversing the order of integration

$$\int_0^2 \int_{y^2}^4 e^{\sqrt{x^3}}\, dx dy$$

[Hint: the following substitution may be of help: $u = x^{\frac{3}{2}}$]

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**Problem 4.** Find the rate of change at the point $(2, 1, 3)$ of the following function $f(x,y,z) = \frac{xy}{z^2}$ along the directions given by unit vectors parallel to
**(a)** $\mathbf{i}$; **(b)** $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

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**Problem 5.** Using **Stokes' theorem**, compute the flux of the curl of the vector field

$$\vec{F}(x,y,z) = 6yz\mathbf{i} - 24x\mathbf{j} + yze^{x^2+\arctan(z)}\mathbf{k}$$

across the surface $S$ (oriented upwards) of the paraboloid $z = y^2 + x^2$, $z \le 4$, with boundary the circle $z = 4, x^2 + y^2 = 4$.

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**Problem 6.** Find the mass $M = \iiint_{\mathcal{R}} \rho(x,y,z)dV$ of the solid in the first octant (namely $x \ge 0, y \ge 0,\ z \ge 0$) bounded by the coordinate planes and the graph of $x + y + z = 1$ if the density is given by $\rho = x + 2y$.

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**Problem 7.**
Evaluate the work done by the **conservative** force

$$\vec{F}(x,y) = ye^{xy}\mathbf{i} + (xe^{xy} + 2y)\mathbf{j}$$

along any path that joins the starting point $(0,0)$ and ending point $(1,2)$. **You must use the potential function.**

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**Problem 8.** Find the curvature $\varkappa(t)$ and the components of the acceleration $a_N(t), a_T(t)$ for the curve described by

$$\mathbf{r}(t) = (t+1)\mathbf{i} + (t^2 - t)\mathbf{j} + \mathrm{e}^{-t}\mathbf{k}$$

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**Problem 9.** Use **Green's theorem** to compute the line-integral

$$\oint_C y^2 \mathrm{d}x + x\mathrm{d}y$$

where $C$ is the boundary of the region determined by the graphs of $x = 0,\ x^2 + y^2 = 4$ and with $x \geq 0$.

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**Problem 10.** Use the **Divergence Theorem** to evaluate the outward flux $\iint_S \vec{F} \cdot \vec{n} \mathrm{d}S$ of the given vector field across the surface specified

$$\vec{F}(x,y,z) = x^3\mathbf{i} + (y^3 + xz)\mathbf{j} + (z^3 + z^2)\mathbf{k}$$

$$x^2 + y^2 + z^2 = a^2, \quad a > 0.$$

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