ECONOMICS 325 – Sample Final Exam – Instructor: Whistler

The exam includes:
- the normal distribution table
- the t-distribution table
- the F-distribution table

Question 1 Answer the following.

(a) [**5 points**] For a random variable X state the result required to get: $Var(X) = E(X^2)$

(b) [**5 points**] For a random variable X that takes the values 0 or 1 show that
$E(X^2) = E(X^3)$

(c) [**5 points**] For random variables X and Y state the result required to get:
$E(X\,Y) = E(X)\,E(Y)$

(d) [**5 points**] For random variables X and Y that each take the values 0 or 1 show that
$E(X\,Y) = P_{X,Y}(1,1)$

Question 2 A random sample $X_1, X_2, \ldots, X_n$ is from a population with mean μ and variance σ^2. Two estimators of the population mean are $\overline{X}$ and X_1.

(a) [**5 points**] Show that both estimators are unbiased. Use correct notation.

(b) [**5 points**] Find the relative efficiency. Use correct notation.

Question 3 [**5 points**] For two independent random samples with $n_X = 17$ and $n_Y = 25$ consider testing:

$$H_0 : \sigma_X^2 = \sigma_Y^2 \qquad \text{against} \qquad H_1 : \sigma_X^2 < \sigma_Y^2$$

For the first sample the sample variance is $s_x^2 = 100$. With a significance level of 5% find the minimum value for the second sample variance s_y^2 that is required to reject the null hypothesis. Clearly show your answer.

<u>Question 4</u> A random sample of n = **16** matched pairs of observations is from populations with means μ_X and μ_Y.

Sample means are: $\bar{x} = 9$ and $\bar{y} = 6$

Sample standard deviations are: $s_x = 8$ and $s_y = 6$

The sample standard deviation for the n differences is $s_d = 8$

(a) [**5 points**] Find the sample covariance.

(b) [**5 points**] State a 90% confidence interval estimate for μ_X.

(c) [**5 points**] An economist reports a confidence interval estimate for μ_X that has a width of **8**. Give an approximate confidence level. Do not use the normal distribution table.

(d) [**5 points**] Using a significance level of 5% test the null hypothesis:

$H_0 : \mu_Y = 7$ against the one-sided alternative $H_1 : \mu_Y < 7$

State a test statistic and give your decision.

(e) [**5 points**] For the test in (d) draw a graph that shows the p-value for the test. Give clear labels to the graph.

(f) [**5 points**] Using a significance level of 5% test the null hypothesis:

$H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X \neq \mu_Y$

State a test statistic and give your decision. Do not assume independent samples.

SELECTED ANSWER GUIDE

Question 1 (a) $E(X) = 0$

(b) $E(X^2) = 0^2 P(0) + 1^2 P(1) = P(1)$

$E(X^3) = 0^3 P(0) + 1^3 P(1) = P(1)$

(c) X and Y are independent **or** have zero covariance.

(d) $E(XY) = (0)(0)P_{X,Y}(0,0) + (1)(0)P_{X,Y}(1,0) + (0)(1)P_{X,Y}(0,1) + (1)(1)P_{X,Y}(1,1)$

$= P_{X,Y}(1,1)$

Question 2 (a) $E(\overline{X}) = \mu$ and $E(X_1) = \mu$

(b) $Var(\overline{X}) = \sigma^2 / n$ and $Var(X_1) = \sigma^2$

The relative efficiency is: $\dfrac{Var(X_1)}{Var(\overline{X})} = n$

Question 3 Reject the null hypothesis if:

$$\frac{s_y^2}{s_x^2} = \frac{s_y^2}{100} > F_c$$

From the F-distribution table with (24, 16) degrees of freedom: $F_c = 2.24$

Therefore, $s_y^2 = (2.24)(100)$

Question 4 (a) $s_d^2 = s_x^2 + s_y^2 - 2s_{xy}$

Therefore: $s_{xy} = \frac{1}{2}(s_x^2 + s_y^2 - s_d^2) = \frac{1}{2}(8^2 + 6^2 - 8^2)$

(b) $\bar{x} \pm t_c \, s_x/\sqrt{n}$ gives $9 \pm t_c \, 8/4$

where from the t-distribution table with 15 degrees of freedom **t_c =1.753**

(c) To get a width of 8 use **t_c = 2**

From the t-distribution table **t_c = 2.131** for a 95% confidence level.

Therefore, the confidence level is **between 90 and 95%**.

(d) $t = \frac{\bar{y} - 7}{s_y/\sqrt{n}} = \frac{6-7}{6/4} = -\frac{4}{6}$

The critical value is **t_c =1.753**. Do not reject the null hypothesis.

(f) $t = \frac{\bar{x} - \bar{y}}{s_d/\sqrt{n}} = \frac{3}{8/4} = 1.5$

The critical value is **t_c = 2.131**. Do not reject the null hypothesis.