University of Guelph

Prof. Pascal Matsakis

CIS1910

STEP 1. Basics

Based on Sections 2.1, 2.3, 4.1 and 4.2 of the Textbook (ISBN: 0073383090 © 2012)

THE THREE COMMANDMENTS

tuples and sets

functions

numeral systems

THE THREE COMMANDMENTS 1.3

1 *Thou shalt tell the truth, the whole truth, and nothing but the truth.*

I’m a billionaire.

If I were a billionaire then I would go to the moon every day.

7×3+1=15

It is not true that 7×3+1=15.

It is false that 7×3+1=15.

It is wrong to say that 7×3+1=15 is true.

Do we have 7×3+1=15?

THE THREE COMMANDMENTS 1.4

2 *Thou shalt designate all things with appropriate names and symbols.*

- ❑ Follow standards.
- ❑ Keep it short, simple, and clear.

Your instructor Pascal Matsakis (or Pascal, Dr. M, kfst, Fathead, ☼, 1)

How many digits in the hand? 5 (or 五, )

There is a student in this class, let’s call him John (or Joe, Jim, s)

There is this integer, let’s call it i (or j, k, m, n)

Consider two integers, i and j. Consider ten other integers, i_1, i_2, …, i_{10}.

a, b, c, …, z, α, β, γ, …, ϖ, 0, 1, 2, …, 9, +, −, ×, ÷, <, ≤, ∧, ∨, ❖, ★…

THE THREE COMMANDMENTS 1.5

3 *Thou shalt be courteous and introduce each symbol before using it.*

n is odd. ✗

Let n be Pascal's neighbour. n is odd.

Let n be Pascal. n is odd. ✗

There exists an integer n such that n is odd.

7n+1=15 ✗

Is there an integer n such that 7n+1=15?

Consider an integer n:
7n+1=15
n=2 ✗

Consider an integer n:
if 7n+1=15 then n=2.

For any integer n, we have: 2n is even.

For all integers n, we have: n+1 is even. ✗

the three commandments

TUPLES AND SETS

functions

numeral systems

TUPLES AND SETS: Tuples 1.7

Consider a nonnegative integer n.
An ***n-tuple***, or ***tuple*** of ***length*** n, is a collection of n objects where order and multiplicity have significance.

(u) is a 1-tuple, (0,1) is a 2-tuple, (Pascal,©,Guelph) is a 3-tuple.
We have (0,1)≠(1,0) and (0,0,1)≠(0,1).

The objects in a tuple are the ***terms*** of the tuple.

The first term of ((x,y),z) is (x,y) and the second term is z.

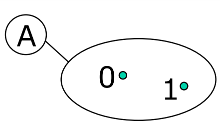
The 0-tuple is the ***empty tuple***; a 1-tuple is a ***singleton***;
a 2-tuple is a ***pair***; a 3-tuple is a ***triple***; etc.

() is the empty tuple, (u) is a singleton, (0,1) is a pair.

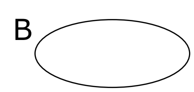
TUPLES AND SETS: Sets 1.8

set*:* a collection of objects; order and multiplicity have NO significance.

A={0,1}={1,0}={0,1,0,0,1}, B={0,1,2,3,...,99}, C={1,1/2,1/3,1/4,...}
D={(0,1),3.1,Dumbo,B}≠((0,1),3.1,Dumbo,B)



Venn diagrams



The objects in a set are called the ***elements*** of the set.
The notation e∈S (or S∋e) denotes that e is an element of the set S (read “e is an element of S” or “e belongs to S” or “S contains e”).

0∈A, 2∉A, A∉B, 78∈B, 0.125∈C, B∈D, 0∉D, {}∉A, {}∉{}, {}∈{{}}

The set with no elements is the ***empty set***; it is denoted by {} or ∅.
A set with exactly one element is a ***singleton*** (***set***);
a set with two elements is a ***pair*** (***set***); etc.

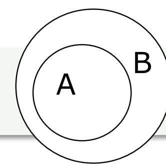
TUPLES AND SETS: Subsets and Supersets 1.9

Let A and B be two sets.

We say that A is a ***subset*** of B
(or that A is included in B; B includes A; B is a ***superset*** of A)
and we write A⊆B (or B⊇A),
iff every element of A is also an element of B.

We say that A is a ***proper subset*** of B
(or that B is a ***proper superset*** of A),
and we write A⊂B (or B⊃A),
iff A⊆B and A≠B.

A={0,1}, B={0,1,2,3,…,99}, D={Pascal,Dumbo,3.1,B}
A⊆A, A⊄A, A⊆B, A⊂B, B⊄D, ∅⊆{}, {}⊂A



TUPLES AND SETS: Cartesian Product 1.10

Let A and B be sets.
The ***Cartesian product*** of A and B, denoted by AxB,
is the set of all pairs (a,b), where a∈A and b∈B.
AxA is also denoted by A^2.

Let A, B, C be sets.
The ***Cartesian product*** of A, B, C, denoted by AxBxC,
is the set of all triples (a,b,c), where a∈A, b∈B, c∈C.
AxAxA is also denoted by A^3.

......

{0,1}x{}={}, $\{0,1\}^2$={(0,0),(0,1),(1,0),(1,1)},
{0,1}x{u,v,w}={(0,u),(0,v),(0,w),(1,u),(1,v),(1,w)}

TUPLES AND SETS: Common Number Sets 1.11

$\mathbb{N}$ is the set $\{0,1,2,\ldots\}$ of natural numbers.
$\mathbb{Z}$ is the set $\{\ldots,-2,-1,0,1,2,\ldots\}$ of integers.
$\mathbb{Z}^+$ is the set $\{1,2,3,\ldots\}$ of positive integers.
$\mathbb{R}$ is the set of real numbers.
$\mathbb{R}^-$ is the set of negative real numbers.
$\mathbb{R}^*$ is the set of nonzero real numbers.
......

TUPLES AND SETS: Integer Intervals 1.12

Let m and n be two integers.

m..n is the set of all the integers
that are greater than or equal to m and less than or equal to n.

m..+∞ is the set of all the integers
that are greater than or equal to m.

−∞..n is the set of all the integers
that are less than or equal to n.

......

$0..9 = \{0,1,2,3,4,5,6,7,8,9\}$
$-\infty..+\infty = \mathbb{Z}$
$1..+\infty = \mathbb{Z}^+$

TUPLES AND SETS: Real Intervals 1.13

Let u and v be two real numbers.

[u,v] is the set of all the real numbers
that are greater than or equal to u and less than or equal to v.

]u,v[is the set of all the real numbers
that are greater than u and less than v.

[u,v[is the set of all the real numbers
that are greater than or equal to u and less than v.

[u,+∞[is the set of all the real numbers
that are greater than or equal to u.

]−∞,v[is the set of all the real numbers
that are less than v.

......

$]-\infty,0[= \mathbb{R}^-$
$[4,4]=\{4\}$
$[4,4[=\varnothing$

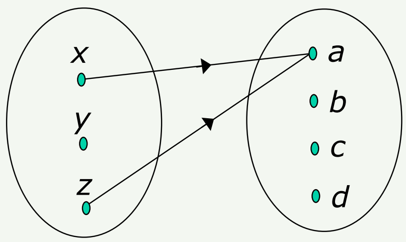
FUNCTIONS: Domain, Codomain, Graph 1.15

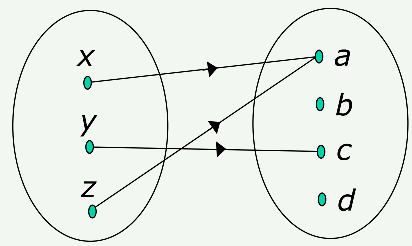
Let U and V be two sets. A ***function*** from U to V is a triple (U,V,G) where G is a subset of UxV such that for any u∈U, v_1∈V and v_2∈V:

if $(u,v_1)\in G$ and $(u,v_2)\in G$ then $v_1=v_2$.

U is the ***domain*** of the function, V the ***codomain***, G the ***graph***.

Domain is {x,y,z},
codomain is {a,b,c,d},
graph is {(x,a),(z,a)}.





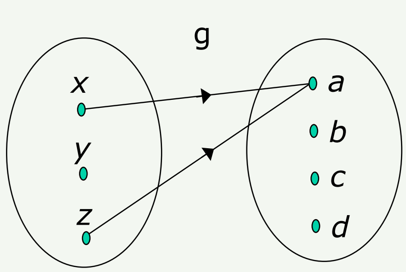
Domain is {x,y,z},
codomain is {a,b,c,d},
graph is {(x,a),(y,c),(z,a)}.

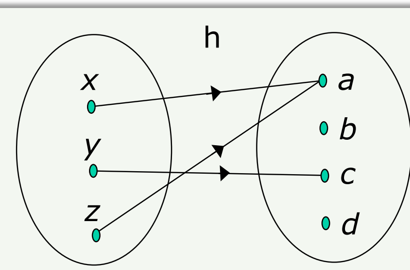
FUNCTIONS: Images, Preimages 1.16

Consider a function f=(U,V,G).

If (u,v) belongs to G then v is denoted by f(u), i.e., **f(u)=v**.
It reads "f of u is v", "the ***image*** of u under f is v"
or "u is a ***preimage*** of v under f".

g(x)=a, g(~~y~~), g(z)=a





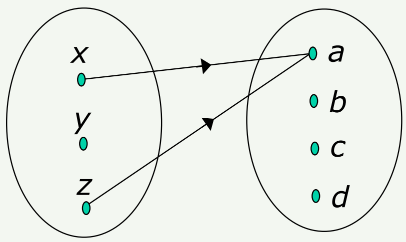
h(x)=a, h(y)=c, h(z)=a

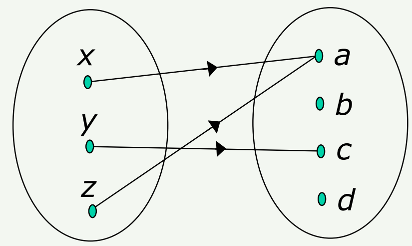
FUNCTIONS: Domain of Definition 1.17

Consider a function f=(U,V,G).

The ***domain of definition*** of f is the subset of U defined as follows:
u of U belongs to the domain of definition iff it has an image under f
(we then say that f is ***defined at*** u).

Domain of definition is {x,z}.





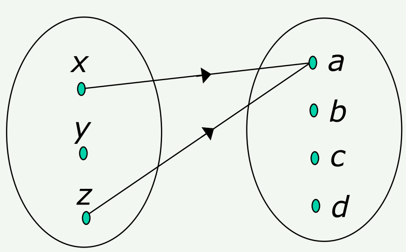
Domain of definition is {x,y,z}.

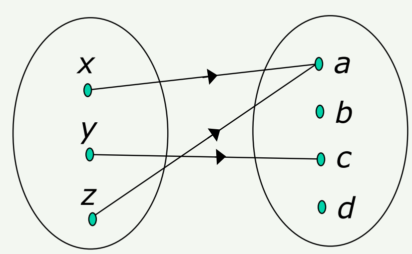
FUNCTIONS: Range 1.18

Consider a function f=(U,V,G).

The ***range*** of f is the subset of V defined as follows:
v of V belongs to the range iff it has a preimage under f.

Range is {a}.





Range is {a,c}.

FUNCTIONS: Examples and Notation (1/2) 1.19

Consider the function $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto 2-\sqrt{x}$

Domain is $\mathbb{R}$ and codomain is $\mathbb{R}$.
Domain of definition is $[0,+\infty[$ and range is $]-\infty,2]$.
For any x in $[0,+\infty[$, $f(x)=2-\sqrt{x}$.
~~$f(-1)$~~, $f(0)=2$, $f(3)=2-\sqrt{3}$, $f(9)=-1$

Consider the function $f : [-10,10] \to [0,+\infty[$, $x \mapsto 2-\sqrt{x}$

Domain is $[-10,10]$ and codomain is $[0,+\infty[$.
Domain of definition is $[0,4]$ and range is $[0,2]$.
For any x in $[0,4]$, $f(x)=2-\sqrt{x}$.
~~$f(-1)$~~, $f(0)=2$, $f(3)=2-\sqrt{3}$, ~~$f(9)$~~

FUNCTIONS: Examples and Notation (2/2) 1.20

- Consider a function f
- Consider a function $f : x \mapsto f(x)$
- Consider a function $x \mapsto f(x)$
- Consider a function $f(x)$ **not allowed in 1910**

same

- Consider the* function $x \mapsto 2-\sqrt{x}$
- Consider the* function $2-\sqrt{x}$ **not allowed in 1910**

same

* Domain is $\mathbb{R}$?
Codomain is $\mathbb{R}$?

Consider the* function $f : x \mapsto 2-\sqrt{x}$

Consider a function $f : \mathbb{R} \to [0,+\infty[$

the three commandments
tuples and sets
functions

NUMERAL SYSTEMS

NUMERAL SYSTEMS: Base b Expansion 1.22

For any $b \in \mathbb{N}$, $n \in \mathbb{N}$,
with $b>1$, $n>0$,

there exist $k \in \mathbb{N}$, $a_k \in \mathbb{N}$, $a_{k-1} \in \mathbb{N}$, ... $a_0 \in \mathbb{N}$,
with $a_k<b$, $a_{k-1}<b$, ... $a_0<b$ and $a_k>0$
such that $n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_0 b^0$.

k, a_k, a_{k-1}, ... a_0 are unique.

This representation of n is the ***base b expansion of n***. It is denoted by $(a_k a_{k-1} \dots a_0)_b$.

a_k, a_{k-1}, ... a_0 are ***base b digits***.

11 in terms of powers of 2: $11=1\text{x}2^3+0\text{x}2^2+1\text{x}2^1+1\text{x}2^0$
$k=3$, $a_3=1$, $a_2=0$, $a_1=1$, $a_0=1$
$11=(1011)_2$

197 in terms of powers of 3: $197=2\text{x}3^4+1\text{x}3^3+0\text{x}3^2+2\text{x}3^1+2\text{x}3^0$
$k=4$, $a_4=2$, $a_3=1$, $a_2=0$, $a_1=2$, $a_0=2$
$197=(21022)_3$

NUMERAL SYSTEMS: Common Bases 1.23

b=10: ***decimal*** expansion
b=16: ***hexadecimal*** expansion
b=8: ***octal*** expansion
b=2: ***binary*** expansion

The decimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
The hexadecimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.
The octal digits are 0, 1, 2, 3, 4, 5, 6, 7.
The binary digits, or ***bits***, are 0, 1.

hexadecimal, octal and binary representation of the integers 0 through 15

b=10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b=16	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
b=8	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
b=2	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

NUMERAL SYSTEMS: Quotient and Remainder 1.24

For any $a \in \mathbb{N}$, $d \in \mathbb{N}$, with $d>0$,

there exist $q \in \mathbb{N}$, $r \in \mathbb{N}$, with $r<d$, such that $a = dq + r$.

q and r are unique.

a is the ***dividend***,
d is the ***divisor***,
q is the ***quotient***,
r is the ***remainder***.

q is denoted by a ***div*** d
and r is denoted by a ***mod*** d.

How many times does 3 “fit” into 7? 2 times, and 7=2x3+1
2 = 7 div 3 and 1 = 7 mod 3

NUMERAL SYSTEMS: Base 10 to b and Vice Versa 1.25

10 to b

$n = (a_k b^{k-1} + a_{k-1} b^{k-2} + \dots + a_1)b + a_0$, where $q_0 = a_k b^{k-1} + a_{k-1} b^{k-2} + \dots + a_1$

q_0 → $n = q_0 b + a_0$ and $q_0 = n$ **div** b and $a_0 = n$ **mod** b

$q_0 = (a_k b^{k-2} + a_{k-1} b^{k-3} + \dots + a_2)b + a_1$, where $q_1 = a_k b^{k-2} + a_{k-1} b^{k-3} + \dots + a_2$

q_1 → $q_0 = q_1 b + a_1$ and $q_1 = q_0$ **div** b and $a_1 = q_0$ **mod** b

$q_1 = (a_k b^{k-3} + a_{k-1} b^{k-4} + \dots + a_3)b + a_2$, where $q_2 = a_k b^{k-3} + a_{k-1} b^{k-4} + \dots + a_3$

q_2 → $q_1 = q_2 b + a_2$ and $q_2 = q_1$ **div** b and $a_2 = q_1$ **mod** b

....

$q_{k-3} = (a_k b + a_{k-1})b + a_{k-2}$, where $q_{k-2} = a_k b + a_{k-1}$

q_{k-2} → $q_{k-3} = q_{k-2} b + a_{k-2}$ and $q_{k-2} = q_{k-3}$ **div** b and $a_{k-2} = q_{k-3}$ **mod** b

b to 10: $q_{k-2} = a_k b + a_{k-1}$, where $q_{k-1} = a_k$

q_{k-1} → $q_{k-1} = q_{k-2}$ **div** b and $a_{k-1} = q_{k-2}$ **mod** b

$a_k = q_{k-1}$

$(a_k a_{k-1} \dots a_1 a_0)_b$

NUMERAL SYSTEMS: Base 2 to 8/16 and Vice Versa 1.26

2 to 8 and 8 to 2:
consider blocks of
3 binary digits

2 to 8

$n = (a_k a_{k-1} \dots a_1 a_0)_2$

$n = \dots + a_6 \times 2^6 + a_5 \times 2^5 + a_4 \times 2^4 + a_3 \times 2^3 + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0$

$n = \dots + a_6 \times 2^6) + (a_5 \times 2^5 + a_4 \times 2^4 + a_3 \times 2^3) + (a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)$

$n = \dots + a_6 \times 2^0) \times 2^6 + (a_5 \times 2^2 + a_4 \times 2^1 + a_3 \times 2^0) \times 2^3 + (a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)$

$n = \dots + a_6 \times 2^0) \times 8^2 + \underbrace{(a_5 \times 2^2 + a_4 \times 2^1 + a_3 \times 2^0)}_{\alpha_1} \times 8 + \underbrace{(a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)}_{\alpha_0}$

$n = (\dots \alpha_2 \alpha_1 \alpha_0)_8$

8 to 2

2 to 16 and 16 to 2:
consider blocks of
4 binary digits

NUMERAL SYSTEMS: When 10+10=100 1.27

Base 8:

```
  ¹ ¹7 5 1
+     7 4 3
= 1 7 1 4
```

Base 2:

```
          1 1 1 0
x           1 0 1
  ¹  ¹  ¹ 1 1 1 0
      1 1 1 0 0
= 1 0 0 0 1 1 0
```

Based on Sections 2.1, 2.3, 4.1 and 4.2 of the Textbook (ISBN: 0073383090 © 2012)

the three commandments
tuples and sets
functions
numeral systems

END