1107

1. Which of the following subsets of $\mathbf{F}(\mathbf{R})$ are subspaces of $\mathbf{F}(\mathbf{R})$?

$$\begin{aligned} S &= \{f \in \mathbf{F}(\mathbf{R}) \mid f(-x) + f(x) = 0\} \checkmark \\ T &= \{f \in \mathbf{F}(\mathbf{R}) \mid f(-1)f(0) = 0\} \times \\ U &= \{f \in \mathbf{F}(\mathbf{R}) \mid f(0) \geq 0\} \times \\ V &= \{f \in \mathbf{F}(\mathbf{R}) \mid f(0) + f(1) = 0\} \checkmark \end{aligned}$$

A. V and S

B. U and S

C. S and T

E. V and T

E. U and T

F. U and V.

Note:

- If $f(x) = x-1$ & $g(x) = x$, then f & $g \in T$ but $-1 = f - g \notin T$ $\therefore$ T is not a s.s of $F(R)$

- U is not closed under mult^n by -1: eg. If $h(x) = 1$, then $h \in U$ but $-h(x) = -1$ shows $-h \notin U$.

Hence, by elimination, A is correct.

2. If a, b and c are scalars and u, v and w are vectors in some vector space V, which of the following statements are always true?

I. The set $\{u, v, w\}$ of vectors is linearly independent if $au + bv + cw = 0$ when $a = b = c = 0$.
This is true $\forall \{u,v,w\} \subset V$

✓ II. The set $\{u, v, w\}$ of vectors is linearly independent if $au + bv + cw = 0$ only if $a = b = c = 0$.
This is the def^n.

III. The set $\{u, v\}$ spans V if $\{u, v\}$ is linearly independent. No: eg $\{(1,0,0), (0,1,0)\}$ is l.i. in $\mathbb{R}^3$ but doesn't span $\mathbb{R}^3$

✓ IV. The set $\{u, v, w\}$ spans V if every vector in V is a linear combination of $\underbrace{u - 2v}_{u_1}, \underbrace{u + v + w}_{v_1}$ and $\underbrace{v - 3w}_{w_1}$.

A. Only I & II are true.

B. Only II & IV are true.

C. Only II & III are true.

D. Only I & III & IV are true.

E. Only II & III & IV are true.

F. Only I is true.

Clearly, $V = \text{span}\{u_1, v_1, w_1\} \subseteq \text{span}\{u, v, w\}$, since each of u_1, v_1 and w_1 is a l.c. of $\{u, v, w\}$

Thus $\{u, v, w\}$ spans V.

3. Let $A = \begin{bmatrix} -1 & 2 & -1 \\ -5 & 7 & -3 \\ 3 & -4 & 2 \end{bmatrix}$. Which one of the following is true?

A. The second column of A^{-1} is $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.

B. The third column of A^{-1} is $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$.

C. The third row of A^{-1} is $[1 \quad 2 \quad 3]$.

D. The first row of A^{-1} is $[2 \quad 0 \quad 1]$.

E. The matrix A is not invertible.

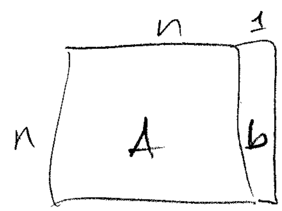
F. The second row of A^{-1} is $[-1 \quad -1 \quad 2]$.

$$\left[\begin{array}{ccc|ccc} -1 & 2 & -1 & 1 & 0 & 0 \\ -5 & 7 & -3 & 0 & 1 & 0 \\ 3 & -4 & 2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & -1 & 0 & 0 \\ 0 & -3 & 2 & -5 & 1 & 0 \\ 0 & 2 & -1 & 3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & -2 & 1 & 1 \\ 0 & 2 & -1 & 3 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 1 & -1 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 & 2 & 3 \end{array}\right]$$

4. Suppose $n \geq 2$. In a linear system $Ax = b$, with n equations and n unknowns, the rank of A is $n-1$ and the rank of the augmented matrix $[A \mid b]$ is also $n-1$. Which one of the following statements is true?

A. The system has no solution.
B. The system has a unique solution.
C. The system has infinitely many solutions.
D. The system has exactly $n-1$ solutions.
E. The determinant of A is non-zero.
F. Such a system cannot exist.



If rank A = rank$[A|b]$ $= n-1 < n$ $=$ # variables. ∴

Thus the system is consistent with ∞'ly many sols

5. Compute $\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]^{2012}$

$$\left[\begin{array}{c|c} I_2 & P \\ \hline 0 & N \end{array}\right]\left[\begin{array}{c|c} I & P \\ \hline 0 & N \end{array}\right] = \begin{bmatrix} I & P+PN \\ 0 & N^2 \end{bmatrix}$$

A. $\begin{bmatrix} 1 & 0 & 2012 & 0 \\ 0 & 1 & 0 & 0 \\ 2012 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 2012 & 2012 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2012 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ E. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ F. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$N^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad PN = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = N$$

$$A^3 = \begin{bmatrix} I & P+PN \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I & P \\ 0 & N \end{bmatrix} = \begin{bmatrix} I & P+PN+\cancel{PN^2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & P+PN \\ 0 & 0 \end{bmatrix} = A^2$$

$$\therefore A^{2012} = A^2 = \begin{bmatrix} I & \begin{smallmatrix} 1 & 1 \\ 0 & 0 \end{smallmatrix} \\ 0 & 0 \end{bmatrix}$$

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6. Let $\mathbf{M}_{22}$ denote, as usual, the vector space of real 2×2 matrices and let $K = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$, and define

$$C = \{A \in \mathbf{M}_{22} \mid KA = AK\}.$$

A. C is a subspace of $\mathbf{M}_{22}$ of dimension 1

B. C is a subspace of $\mathbf{M}_{22}$ of dimension 2

C. C is a subspace of $\mathbf{M}_{22}$ of dimension 3

D. C is a subspace of $\mathbf{M}_{22}$ of dimension 4

E. C is a subspace of $\mathbf{M}_{22}$ of dimension 0

F. C is not a subspace of $\mathbf{M}_{22}$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} b & 2a \\ d & 2c \end{bmatrix} = \begin{bmatrix} 2c & 2d \\ a & b \end{bmatrix}$$

$\therefore \quad b = 2c, \quad 2a = 2d$

$\therefore \quad b = c, \; a = d \quad \therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & 2c \\ c & d \end{bmatrix} = dI_2 + cK;$

Moreover, $\{I_2, K\}$ is clearly l.i. $\therefore \dim C = 2$

11^{34}

7. Let A be a 6×4 matrix. Answer the following questions:

(1) Can the system $Ax = 0$ have a non trivial solution? it will always have ∞ly many

(2) Can the columns of A span $\mathbf{R}^6$? No, we need at least 6 vectors to span $\mathbb{R}^6$

(3) Can the columns of A be linearly independent? Yes, when rank $A = 4$, which is possible.

A. Yes, No, Yes.

B. Yes, Yes, Yes.

C. Yes, No, No.

D. No, No, Yes.

E. No, No, No.

F. No, Yes, Yes.

$\therefore$ Yes, No, Yes $\left(A = \begin{bmatrix} I_4 \\ 0 - 0 \\ 0 - 0 \end{bmatrix} \right)$

8. The set of vectors $\{(3,0,4),(0,1,0),(-4,0,3)\}$ is an orthogonal basis of $\mathbf{R}^3$. Find $c_3 \in \mathbf{R}$ such that

$$(0,1,1) = c_1(3,0,4) + c_2(0,1,0) + c_3(-4,0,3) \quad \text{for some scalars } c_1, c_2 \in \mathbf{R}.$$

That is, find the third Fourier coefficient of $(0,1,1)$ with respect to the ordered orthogonal basis of $\mathbf{R}^3$ above.

A. $\frac{3}{25}$

B. $\frac{3}{5}$

C. 3

D. 1

E. $\frac{1}{25}$

F. $-\frac{3}{25}$

We know $c_3 = \dfrac{(0,1,1)\cdot(-4,0,3)}{(-4)^2+0^2+3^2}$

$= \dfrac{3}{25}$

AB^{t}

9. Which two of the following statements are <u>false</u>?

(i) For all invertible $n \times n$ matrices A and B, $\det(A^{-1}BA) = \det B$ True

(ii) For all invertible $n \times n$ matrices A and B, $\det(A^{-1}B^{-1}AB) = 1$ True

(iii) For all $n \times n$ matrices A and B, $(A^tB^t)^t = AB$ $(A^tB^t)^t = BA$!

(iv) For all invertible $n \times n$ matrices A and B, $(ABA^{-1})^{-1} = A^{-1}B^{-1}A$

(v) For all $n \times n$ matrices A and B, $\det(A^tB) = \det(B^tA)$

$(ABA^{-1})^{-1} = AB^{-1}A^{-1}$

A. (i) and (iii)

B. (ii) and (iii)

C. (iii) and (iv)

D. (ii) and (iv)

E. (ii) and (v)

F. (i) and (v)

It's possible that $AB \neq BA$,

and $A^{-1}B^{-1}A \neq AB^{-1}A^{-1}$.

10. Let A be an $n \times n$ matrix. One of the the following statements in not equivalent to the statement:

The number 0 is not an eigenvalue of A.

Which one?

So $0 \neq \det(A - 0I)$

$= \det A$

$\therefore$ A is invertible

A. The homogeneous system $Ax = 0$ has a non-trivial solution.

B. A is invertible. ✓

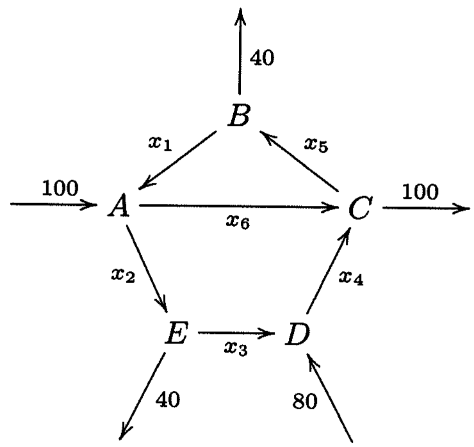
C. The determinant of A is not zero. ✓

D. The columns of A span $\mathbf{R}^n$. ✓

E. The rows of A are linearly independent. ✓

F. The rank of A is n. ✓

11. Consider the network of streets with intersections A, B, C, D and E below. The arrows indicate the direction of traffic flow along the **one-way streets**, and the numbers refer to the **exact** number of cars observed to enter or leave A, B, C, D and E during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



a) Write down a system of linear equations which describes the the traffic flow, **together with all the constraints** on the variables x_i, $i = 1, \ldots, 6$.
(Do not perform any operations on your equations: this is done for you in (b). *Do not simply copy out the equations implicit in (b). You will not get any marks if you do this.*)

Intersection	Flow IN	=	Flow OUT
A	$100 + x_1$	=	$x_2 + x_6$
B	x_5	=	$x_1 + 40$
C	$x_4 + x_6$	=	$100 + x_5$
D	$x_3 + 80$	=	x_4
E	x_2	=	$x_3 + 40$

and $x_i \geq 0$ (one-way street)

$x_i \in \mathbb{Z}$ (whole numbers of cars)

11(b). The reduced row-echelon form of the augmented matrix of the system in part (a) is

$$\begin{array}{cccccccc} & & & & s & t & & \\ \end{array}$$
$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 & -40 \\ 0 & 1 & 0 & 0 & -1 & 1 & 60 \\ 0 & 0 & 1 & 0 & -1 & 1 & 20 \\ 0 & 0 & 0 & 1 & -1 & 1 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

$$\begin{aligned} x_1 &= -40 + s \\ x_2 &= 60 + s - t \\ x_3 &= 20 + s - t \\ x_4 &= 100 + s - t \\ x_5 &= s \\ x_6 &= t \end{aligned} \quad ; \; s, t \in \mathbb{R}$$

c) If $\overline{AC}$ were closed due to roadwork, find the minimum flow along $\overline{ED}$, **using your results from (b).**

(You must justify all your answers.)

Since x_6 = flow along $\overline{AC}$, $t = 0$.

Flow along $\overline{ED}$ is $x_3 = 20 + s$.

Constraints (in light of $t=0$)

$$\begin{aligned} x_1 &\geq 0 \iff s \geq 40 \\ x_2 &= 60 + s \geq 0 \checkmark \\ x_3 &= 20 + s \geq 0 \checkmark \\ x_4 &= 100 + s \geq 0 \checkmark \\ x_5 &= s \geq 0 \checkmark \end{aligned}$$

$\therefore \quad x_3 = 20 + s \geq \underline{60}$

$$\begin{array}{ccccc} & u_1 & u_2 & u_3 & u_4 \end{array}$$

12. Let $U = \text{span}\{(1,0,0,1),(0,1,0,0),(1,1,0,0),(1,4,0,-1)\}$

a) Find a basis of U which is a subset of the given spanning set.

b) Find an orthogonal basis of U.

c) Find the best approximation to $(1, -1, 2, -1)$ by vectors in U.

d) Extend your basis in (b) to an **orthogonal** basis of $\mathbf{R}^4$.

a) Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$. Then $\text{col}\, A = U$.

$A \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\therefore$ $\{u_1, u_2, u_3\}$ is such a basis

b) Since $u_1 \cdot u_2 = 0$, we set $w_1 = u_1$, $w_2 = u_2$ and

$$w_3 = u_3 - \frac{u_3 \cdot w_1}{\|w_1\|^2} w_1 - \frac{u_3 \cdot w_2}{\|w_2\|^2} w_2$$

$$= (1,1,0,0) - \frac{1}{2}(1,0,0,1) - (0,1,0,0)$$

$$= \left(\tfrac{1}{2}, 0, 0, -\tfrac{1}{2}\right).$$ $\therefore$ $\{w_1, w_2, w_3\}$ is an orthogonal basis for U. (Or $w_3' = (1,0,0,-1)$ will do too)

c) $$\text{proj}_U (1,-1,2,-1) = \frac{(1,-1,2,-1)\cdot w_1}{\|w_1\|^2} \cdot w_1 + \frac{(1,-1,2,-1)\cdot w_2}{\|w_2\|^2} w_2 + \frac{(1,-1,2,-1)\cdot w_3'}{\|w_3\|^2}$$

$$= 0 + \frac{(-1)}{1}(0,1,0,0) + \frac{2}{2}(1,0,0,-1)$$

$$= (1,-1,0,-1)$$

12(d) We know that $\begin{bmatrix} w_1 \\ w_2 \\ w_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$,

so $\{w_1, w_2, w_3', e_3\}$ is a basis of $\mathbb{R}^4$; we apply G-S

to obtain $w_4 = e_3 - \frac{e_2 \cdot w_1}{\|w_1\|^2} w_1 - \frac{e_3 \cdot w_2}{\|w_2\|} w_2 - \frac{e_3 \cdot w_3'}{\|w_3\|^2} w_3'$

$= e_3 - 0 - 0 - 0$

(or: simply note that e_3 is orthogonal to $w_1, w_2, \& w_3'$.

1151

13. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

a) Compute $\det(A - \lambda I_3)$ and hence show that the eigenvalues of A are 0 and 2.

b) Find a basis of $E_0 = \{v \in \mathbf{R}^3 \mid Av = 0\}$.

c) Find a basis of $E_2 = \{v \in \mathbf{R}^3 \mid Av = 2v\}$.

d) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. Explain why your choice of P is invertible.

a) $\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} \overset{\text{row 2}}{=} (2-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (2-\lambda)\{(\lambda-1)^2 - 1\}$

$= (2-\lambda)\{\lambda^2 - 2\lambda + 1 - 1\}$

$= (2-\lambda)\lambda(\lambda-2)$

$= -(2-\lambda)^2\lambda$

$\therefore \det(A-\lambda I_3) = 0 \iff \lambda = 0$ or $\lambda = 2$

b) $E_0 = \ker(A - 0I) = \ker\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \ker\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \operatorname{Span}\{(-1,0,1)\}$

$\therefore \{(-1,0,1)\}$ is a basis for E_0

c) $E_2 = \ker(A - 2I) = \ker\begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \ker\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= \operatorname{span}\{(0,1,0),(1,0,1)\}$ $\therefore \{(0,1,0),(1,0,1)\}$ is a basis for E_2.

d) Set $P = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Then $P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. We know P is invertible because $\dim E_0 + \dim E_2 = 1+2 = 3$; OR :

$\det P = \begin{vmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \overset{\text{row 2}}{=} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2 \neq 0.$

11 50

14. Let $u = (1, -1, 1)$ and define a linear transformation $S : \mathbf{R}^3 \to \mathbf{R}^3$ by

$$S(v) = u \times v, \qquad v \in \mathbf{R}^3,$$

where "×" denotes the cross product. *(You do not have to prove that S is linear.)*

a) If $(x, y, z) \in \mathbf{R}^3$, show that $S(x, y, z) = (-y - z, x - z, x + y)$.

b) Find the standard matrix of S.

c) Find a basis for $\ker S$ and describe it geometrically.

d) Find a basis for $\operatorname{im} S$ and describe it geometrically.

a) $\begin{vmatrix} \hat{i} & \hat{j} & \vec{k} \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = (-z-y, -(z-x), y+x) = (-y-z, x-z, x+y)$

as required.

b) $A = [Se_1 \; Se_2 \; Se_3] = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$, (by (a)) is

the standard matrix of S

c) $\ker S = \ker \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} = \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix} = \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$= \operatorname{Span}\{(1,-1,1)\}$ $\therefore \{u\}$ is a basis for $\ker S$

(as we expect). Thus $\ker S$ is the line through O with direction $(1,-1,1)$.

d) $\operatorname{Im} S = \operatorname{col} A = \operatorname{Span}\{\overset{u_2}{(0,1,1)}, \overset{u_3}{(-1,0,1)}\}$; hence $\{u_2, u_3\}$

is a basis of $\operatorname{Im} S$. Thus $\operatorname{im} S$ is the plane through O

with normal $u_2 \times u_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = (1, -1, 1) = u$. (As expected)

1203

15. State whether the following are true or false.

If you say the statement may be false, you must give an explicit example - with numbers! If you say the statement is true, you must give a clear explanation -e.g. by quoting a theorem from class.

i) Every diagonalizable matrix is invertible.

$\boxed{\text{FALSE}}$

eg $A = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ is diagonal (and so diag'ble) but rank $A = 1 < 2$ so A is not invertible.

ii) If a 2 by 2 matrix A satisfies $A^2 = 0$, then $A = 0$.

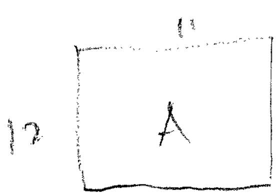
$\boxed{\text{FALSE}}$

If $A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, then $A \neq 0$

but $A^2 = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

iii) The rows of a 12×11 matrix are always linearly dependent.

$\boxed{\text{TRUE}}$



12 vectors in $\mathbb{R}^{11}$ are always l.d. since $\dim \mathbb{R}^{11} = 11 \geq$ size of any l.i. set

1208