PROBLEM 1

From Equation 5.5:

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$C_x = 0.35$ $C_0 = 0.2$ $C_s = 1.35$ and

$x = 2.5\text{mm} = 2.5 \times 10^{-3}\text{m}$

$$\frac{C_x - C_0}{C_s - C_0} = \frac{0.35 - 0.2}{1.35 - 0.2} = 0.1304 = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 1 - 0.1304 = 0.8696$$

By linear interpolation from Table 5.1

z	erf(z)
1.0	0.8427
z	0.8696
1.1	0.8802

$$\frac{z - 1.0}{1.1 - 1.0} = \frac{0.8696 - 0.8427}{0.8802 - 0.8427}$$

$$z = 1.07 = \frac{x}{2\sqrt{Dt}}$$

From Table 5.2, at 1050°C (1323 K)

$$D = D_0 \exp\left(-\frac{Q_d}{RT}\right)$$

For α-Fe; $D_0 = 6.2 \times 10^{-7}\ \text{m}^2/\text{s}$ $Q_d = 80\ \text{kJ/mol}$

$R = 8.31\ \text{J/mol.K}$ $T = 1323\text{K}$

$$D = (6.2 \times 10^{-7}\text{m}^2/\text{s}) \exp\left[-\frac{80{,}000\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1323\ \text{K})}\right]$$

$$D = 4.29 \times 10^{-10} \ m^2/s$$

$$\therefore \quad 1.07 = \frac{2.5 \times 10^{-3}}{2\sqrt{(4.29 \times 10^{-10})(t)}}$$

$$t = \frac{(2.5 \times 10^{-3})^2}{(1.07)^2 (4)(4.29 \times 10^{-10})}$$

$$t = 3{,}181 \ s = \underline{\underline{53 \text{ minutes}}}$$

PROBLEM 2

Diffusion flux, $J = -D \frac{\Delta C}{\Delta x}$

$$D = D_0 \exp\left(-\frac{Q_d}{RT}\right)$$

$$\Rightarrow J = -D_0 \frac{\Delta C}{\Delta x} \exp\left(-\frac{Q_d}{RT}\right)$$

For $J = 5.6 \times 10^{-10}$ kg/m²-s, $T = 760°C$ (1033 K),

$$\frac{\Delta C}{\Delta x} = -325 \text{ kg/m}^4$$

$$Q_d = 120{,}000 \text{ J/mol}$$

$$\Rightarrow D_0 = -\frac{J}{\frac{\Delta C}{\Delta x}} \exp\left(\frac{Q_d}{RT}\right)$$

$$= -\left(\frac{5.6 \times 10^{-10} \text{ kg/m}^2\text{-s}}{-325 \text{ kg/m}^4}\right) \exp\left[\frac{120{,}000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1033 \text{ K})}\right]$$

$$D_0 = 2.03 \times 10^{-6} \text{ m}^2/\text{s}$$

For the value of the diffusion flux at 1027°C (1300 K):

$$\therefore J = -(2.03 \times 10^{-6})(-325) \exp\left[\frac{-120{,}000}{(8.31)(1300)}\right]$$

$$J = 9.89 \times 10^{-9} \text{ kg/m}^2\text{-s}$$

PROBLEM 3

Plane strain fracture toughness, $K_{IC} = Y\sigma\sqrt{\pi a}$

$K_{IC} = 45\ MPa\sqrt{m}$ $\sigma_c = 380 MPa$ $a = 2.3\ mm = 2.3 \times 10^{-3} m$

Solving for Y for the this condition of fracture;

$$Y = \frac{K_{IC}}{\sigma\sqrt{\pi a}} = \frac{45\ MPa\sqrt{m}}{(380 MPa)\sqrt{\pi\left(\frac{2.3 \times 10^{-3}}{2}\right)}}$$

$$Y = 1.97$$

NB: a = half of the length of an internal crack, 2.3 mm

Stress level (σ_c) for a critical internal crack length of 3.80 mm

$$\sigma_c = \frac{K_{IC}}{Y\sqrt{\pi a}}$$

$$= \frac{45\ MPa\sqrt{m}}{(1.97)\sqrt{\pi\left(\frac{3.8 \times 10^{-3}}{2}\right)}}$$

$$\sigma_c = 295.7\ MPa$$

PROBLEM 4

A cylindrical red brass rod, diameter = 7.8mm

Maximum tensile and compressive loads (+7450 N and −7450 N) respectively

Stress Amplitude, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

$$\sigma_{max} = \frac{F_{max}}{A_0} = \frac{F_{max}}{\pi \left(\frac{d_0}{2}\right)^2}$$

$$\sigma_{max} = \frac{7450\ N}{\pi \left(\frac{7.8 \times 10^{-3} m}{2}\right)^2} = 155.9 \times 10^6\ N/m^2 = 156\ MPa$$

$$\sigma_{min} = \frac{-7450\ N}{\pi \left(\frac{7.8 \times 10^{-3} m}{2}\right)^2} = -155.9 \times 10^6\ N/m^2 = -156\ MPa$$

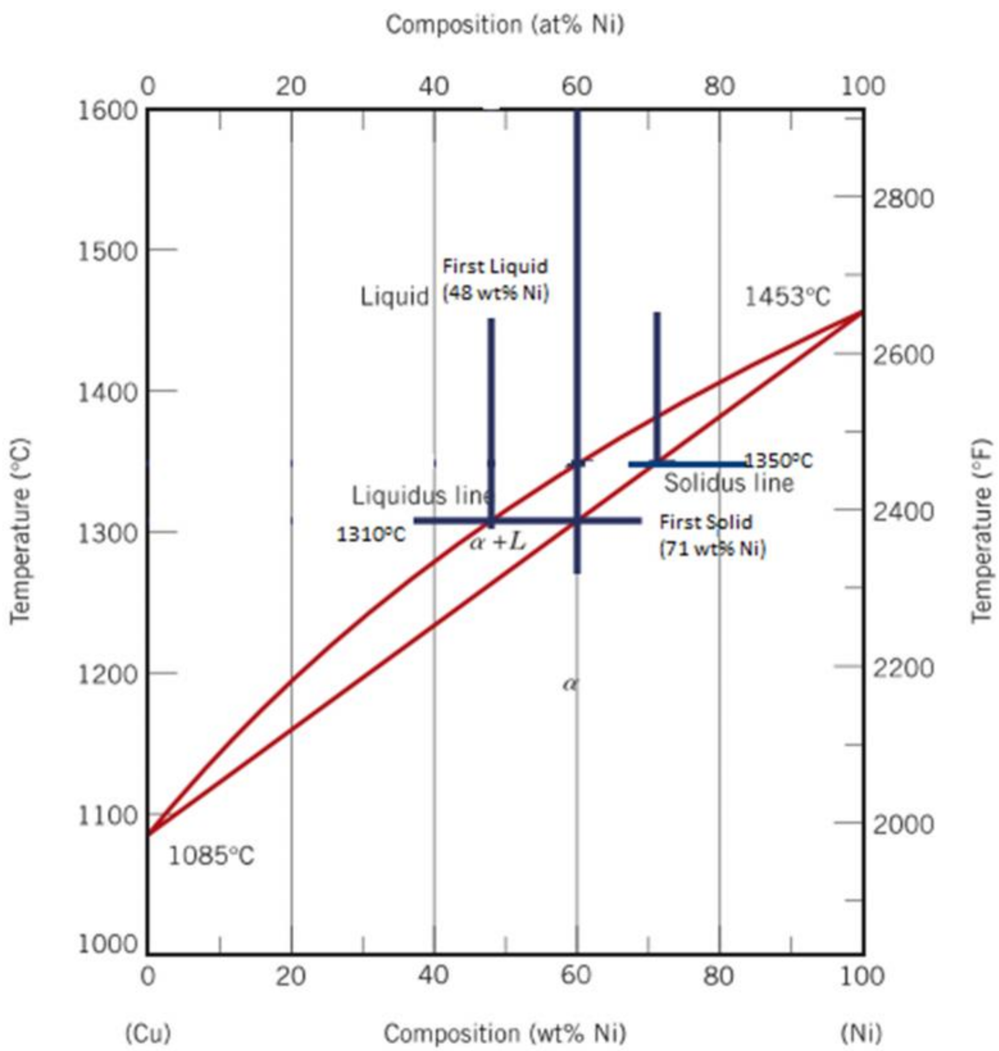
$$\Rightarrow \quad \sigma_a = \frac{156 - (-156)}{2} = 156\ MPa$$

From figure 8.34 for the red brass, the number of cycles to failure at this stress amplitude is:

$$N \approx 7 \times 10^4 \text{ cycles}$$

PROBLEM 5

Cu-Ni composition of 60 wt% Ni – 40 wt% Cu and T=1300^{0}C



a) Upon heating from 1300^{0}C, the first liquid phase forms at the temperature at which the 60 wt% Ni vertical line intersects the $\alpha \rightarrow (\alpha + L)$ phase boundary i.e. about **1310 ^{0}C**

b) The composition of this liquid phase corresponds to the intersection with the $\alpha \rightarrow (\alpha + L)$ phase boundary, of a tie line constructed across the $(\alpha + L)$ phase region at 1310^{0}C i.e. **48 wt% Ni**

c) Complete melting of the alloy occurs at the intersection of this same vertical line at 60 wt% Ni with the $(\alpha + L) - L$ phase boundary i.e. about **1350 ^{0}C**

d) The composition of the last solid remaining prior to complete melting corresponds to the intersection with $\alpha - (\alpha + L)$ phase boundary, of the tie line constructed across the $(\alpha + L)$ phase region at 1350^{0}C i.e. **71wt% Ni**