Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2013

TEST 2

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name : _____

Student Number: _____

Tutorial Section (A1, A4, ...): _____

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Given that $ye^{xy} = 1$ and that y may be defined as a differentiable function of x calculate its derivative y', when $x = 0, y = 1$.
   (a) -1, (b) 0, (c) 1, (d) -0.5.

2. **[2 marks]** Calculate the derivative, $f'(0)$, of f where $f(x) = \sin(e^{2x})$ at $x = 0$.
   (a) 1, (b) -1, (c) 0, (d) 2. e) $2\cos 1$, none of these!

3. **[2 marks]** Let f be a differentiable function with $f'(2) = 1/3$ whose inverse, F, satisfies $F(0) = 2$. What is the value of $F'(0)$?
   (a) 0, (b) 3, (c) -1, (d) 2.

4. **[2 marks]** Evaluate the following limit using any method: $\lim_{x\to 0} \dfrac{\sin x}{e^x - 1}$.
   (a) -2, (b) 1, (c) 0, (d) e.

5. **[2 marks]** Evaluate the following limit using any method: $\lim_{x\to\infty} \dfrac{2^x - 1}{3^x + 1}$.
   (a) $\ln 3$, (b) $\ln 2$, (c) 0, (d) 1.

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[5+5 marks]** a) Let $f(x) = \text{Arctan}(3x + 4^x)$. Find its derivative $f'(x)$. No need to simplify your answer.

   b) Evaluate $\lim_{x\to 0^+} x \ln x$. (Hint: Use L'Hospital's Rule after you l'Hospitalize it!)

a) $f'(x) = \dfrac{1}{1 + (3x + 4^x)^2}\,(3 + 4^x \ln 4)$ ← (2), (2), (1), (2)

b) $x \ln x = \dfrac{\ln x}{\left(\frac{1}{x}\right)}$ ← (2)

$\lim_{x\to 0^+} \dfrac{\ln x}{\left(\frac{1}{x}\right)} = \lim_{x\to 0^+} \dfrac{1/x}{-1/x^2}$ ← (1), (1)

$= \lim_{x\to 0^+} (-x)$

$= 0$ ← (1)