Solutions

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2013

TEST 1

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** What value of x will make the two vectors $(x, 3)$ and $(2, 12)$ orthogonal?

   (a) $x = 12$, (b) $x = -18$, (c) $x = 0$, (d) $x = -4$.

2. **[2 marks]** Find an equation of the line through the points $(1, 2, 4)$ and $(3, -1, 6)$.

   (a) $x = 1 + 2t, y = 2 - 3t, z = 4 + 2t$, (b) $x = 1 + 2t, y = 1 - t, z = -1 + 2t$, (c) $x = 1 + t, y = 1 - 2t, z = -1 + 2t$, (d) $x = 1, y = 1 - t, z = -1 + 2t$.

3. **[2 marks]** Find a normal vector to the plane through the points $(1, -1, 0)$, $(2, 1, 1)$ and $(-1, 0, 1)$.

   (a) $(0, 1, 2)$, (b) $(3, 1, -4)$, (c) $(2, 0, -1)$, (d) $(1, -3, 5)$.

4. **[2 marks]** Two vectors in three dimensional space are parallel if and only if their cross product is zero.

   (a) TRUE, (b) FALSE, (c) It depends.

5. **[2 marks]** Convert the equation of the circle $(x - 1)^2 + y^2 = 1$ to polar coordinates.

   (a) $r = 2$, (b) $r = 3\sin\theta$, (c) $r = 2\cos\theta$, (d) $r = -\cos\theta$.

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[10 marks]** Find the arc length of the curve $r = 2\sin\theta$ between the rays $\theta = 0$ and $\theta = \pi/4$.

$$\text{Arclength} = \int_0^{\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad \leftarrow (5)$$

$$= \int_0^{\pi/4} \sqrt{4\sin^2\theta + 4\cos^2\theta}\, d\theta \quad (1)\ (1)$$

$$= 2\int_0^{\pi/4} \sqrt{\underbrace{\sin^2\theta + \cos^2\theta}_{1}}\, d\theta \quad (1)$$

$$= 2\int_0^{\pi/4} d\theta$$

$$= \boxed{\pi/2} \quad \leftarrow (2)$$

7. [10 marks] Find the area under the curve $r = \sqrt{\cos\theta}$ between the rays $\theta = 0$ and $\theta = \pi/2$.

(1) (3)

$$\text{Area} = \int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta \quad \left(= \frac{1}{2}\int_0^{\pi/2} f(\theta)^2 \, d\theta \right)$$

(1)

$$= \frac{1}{2}\int_0^{\pi/2} \cos\theta \, d\theta. \quad \leftarrow (2)$$

$$= \frac{1}{2} \sin\theta \Big|_0^{\pi/2}$$

$$= \frac{1}{2} \quad \leftarrow (3)$$