

McGill University
Department of Civil Engineering and Applied Mechanics
CIVE 225 Environmental Engineering
Winter 2012

Class Test 1
February 10, 2012

ID#: _____ Name: _____

Total: 110 points

- 1. Write your answers clearly with appropriate units.**
 - 2. You may write on the back of the page if you need more space.**
 - 3. Show all steps and state all assumptions for a logical derivation of your answers.**
 - 4. A majority of points will be given for stating the correct method of solution with proper explanations and stating the values of symbols to use (i.e., you will get partial points even if calculations are incorrect or incomplete).**
 - 5. This exam is CLOSED BOOK. NO FORMULA SHEETS allowed except those provided.**
 - 6. MANAGE YOUR TIME WELL. To manage your time, you can determine the maximum of time to spend on a problem using the number of points as a guide with the relationship 1 point/min.**
-

Note: $1 \text{ m}^3 = 1000 \text{ L}$

Gravity constant (g) = 9.8 m/s^2

Water Heat Capacity 4.187 kJ/kgK

Water density: 1 g/mL

Thermodynamics data on equation sheet

Grading

Question 1 (24pts):

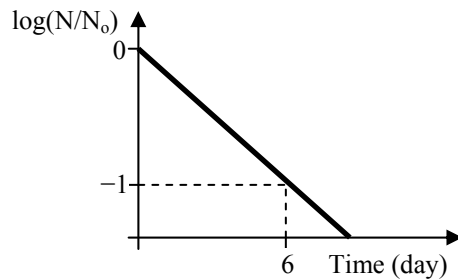
Question 2 (20 pts):

Question 3 (18 pts):

Question 4 (24 pts):

Question 5 (24 pt):

1. **(24 points)** You do the microbiological evaluation of a lake to determine the suitability as a drinking water source. The lake of interest has a volume of $25,000 \text{ m}^3$ and two streams flow into it. Stream 1 has a flow of $1,000 \text{ m}^3/\text{day}$ and carries $40.0 \text{ coliform CFU/mL}$, while Stream 2 has a flow of $1,500 \text{ m}^3/\text{day}$ and carries 15.0 CFU/mL . From surveys of wildlife in the area, animals living by the lake are likely to contribute $1.56 \text{ coliform CFU/mL/day}$. Finally, you take water from this lake, incubate it in a container within similar environmental conditions as in the lake, and you observe the following decrease in coliform counts (N , CFU/mL) over time.



Please answer the following questions to make your assessment

- What is the hydraulic retention time (HRT) of this lake?
- What type of decay rate (zero, first, second) order with respect to N does describe the decrease in coliform counts during the lab incubation?
- What is the value of this rate constant?
- What coliform counts do you expect to find in the lake on average?

Q1. Volume = 25000 m³

a) Flow : 1000 m³/day + 1500 m³/day = 2500 m³/day
HRT : Volume / Flow = 25000 m³ / 2500 m³/day
= 10 days.

4 b) First order.

Evolution of conc. (N) over time for 1st order
is $N = N_0 \exp(-kt)$ gives a straight line
for $\log(N/N_0)$ Vs time (t)

7 c) Value of k?

at $t=0$, $\log(N/N_0) = 0$
 $N/N_0 = 1$

$t=6$, $\log(N/N_0) = -1$
 $N/N_0 = 0.1$

$\therefore N/N_0 = 0.1$

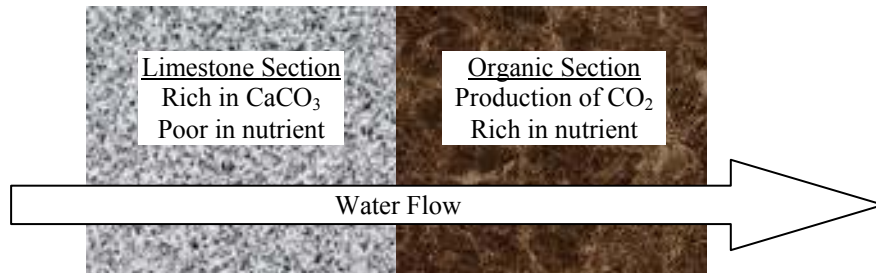
$$k = -\ln(N/N_0)/t$$

$$\therefore k = -\ln(0.1)/6 \text{ days} = 0.384 \text{ day}^{-1}$$

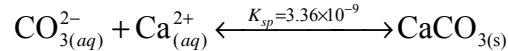
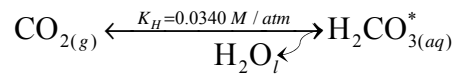
9 d) $C_3 = \frac{1000 \text{ m}^3/\text{day} \times 40 \text{ CFU/ml} + 1500 \text{ m}^3/\text{day} \times 15 \text{ CFU/ml}}{2500 \text{ m}^3/\text{day}}$
 $= 25 \text{ CFU/ml}.$

$$C_{\infty} = \frac{2500 \text{ m}^3/\text{day} \times 25 \text{ CFU/ml} + 1.56 \times 25000}{2500 + 0.384 \times 25000}$$
$$= 8.388 \text{ CFU/ml}.$$

2. (20 points) Water flows in the subsurface from a spring source penetrating the ground. It passes through a limestone where CaCO_3 dissolves and then through an organic rich section where bacteria produce a lot of CO_2 (see the schematic below). At the end of the limestone section, the pH is 8.00 and the Ca^{2+} concentration is 1.25 mg/L.



Consider the chemical equations of the carbonate system. The water flows very slowly through the subsurface; thus, this can be considered an open carbonate system.



- At what CO_2 partial pressure do you expect that calcium will start precipitating in the organic section if you assume the pH constant?
- At that point, what is the $C_{\text{T,CO}_3}$?

Solution 2)

a) (12 point)

$$K_{SP} = \frac{[Ca^{2+}][CO_3^{2-}]}{[Ca^{2+}] + [CO_3^{2-}]} \Rightarrow [CO_3^{2-}] = \frac{K_{SP}}{[Ca^{2+}]} = \frac{3.36 \times 10^{-9}}{3.125 \times 10^{-5}} = 1.075 \times 10^{-4} M$$

$$[Ca^{2+}] = 1.25 \text{ mg/l} \times \frac{1 \text{ mol}}{40 \text{ gr}} \times \frac{1 \text{ gr}}{1000 \text{ mg}} \Rightarrow [Ca^{2+}] = 3.125 \times 10^{-5} \text{ mol/l}$$

4
(1 point)

$$[CO_3^{2-}] = C_{T,CO_3} \alpha_2$$

$$P_{CO_2} = \frac{[H_2CO_3^*]}{K_H} = \frac{C_{T,CO_3} \alpha_0}{K_H} \Rightarrow [CO_3^{2-}] = \frac{\alpha_0}{\alpha_2} P_{CO_2}$$

$$P_{CO_2} = \frac{[CO_3^{2-}][H^+]^2}{K_{a1}K_{a2} \cdot K_H}$$

$$P_{CO_2} = 1.075 \times 10^{-4} \times \frac{[10^{-8}]^2}{[10^{-6.35}][10^{-10.35}][0.0340]} = 0.0157 = 0.016 \text{ atm}$$

(8 point)

b) (8 point)

$$C_{T,CO_3} = \frac{P_{CO_2} K_H}{\alpha_0}$$

$$\alpha_0 = \frac{[10^{-8}]^2}{([10^{-8}]^2 + [10^{-8}][10^{-6.35}] + [10^{-6.35}][10^{-10.35}])} = 0.0218$$

$$C_{T,CO_3} = \frac{[0.016] \times 0.0340}{0.0218} = 0.025 M$$

3. (18 points) For the following, provide brief but complete answers.

a) Name 3 differences between *Bacteria* and *Eukarya*.

Bacteria: no nucleus, no organelles, smaller cells, DNA/RNA processing machinery-
Bacterial type, fix N_2 to NH_3 and can return NH_3 to N_2 , evolutionarily more divers

Eukarya: nucleus, organelles, larger cells, DNA/RNA processing machinery-
Eukaryal/Archeal type, cannot covert nitrogen between N_2 and NH_3 , evolutionarily less
divers.

b) What do we mean by strong acid and what proportion of dissociation do you expect of
such an acid at pH 4.5?

Strong acid is an acid with low pKa, usually lower than 3. At 4.5, the dissociation will tend
to be 100%.

c) Give two examples of disinfection agents used in the water and wastewater industry?

Ozone (O_3), hypochlorous acid (HOCl), chloramine, UV-radiation

d) Define cytoplasm.

The cytoplasm is the interior volume of the cell where enzymes (including ribosomes) and
organelles (for Eukarya) floats. It is surrounded by the cell membrane.

e) What is the difference between true and opportunistic pathogens?

True pathogene: needs a host to grow and reproduce.

Opportunistic pathogen: grows and reproduces without a host or causing diseases, but can
cause diseases in special conditions.

f) What does *ppb* and *ppt* mean? Express them in SI units (mass/volume). What does *ppm_v*
mean? How many molecules of a gas at 1 *ppm_v* is there in 10^7 molecules of air?

ppb: parts per billion ($\mu\text{g/L}$ or mg/m^3)

ppt: parts per trillion (ng/L or $\mu\text{g/m}^3$)

ppm_v: parts per million per volume. Since a molecule of any gas occupies the same volume
in ideal conditions, you would have 10 molecules of this gas in 10^7 at 1 *ppm_v*.

4. **(24 points)** During a winter blackout, a friend of yours thinks that it is a good idea to bring the barbecue grill inside the your apartment of 250 m^3 . Your friend thought that everything would be ok if the window is open letting $10 \text{ m}^3/\text{hour}$ of fresh air in the apartment. He did not factor in that the barbecue produces $28 \text{ g}/\text{hour}$ of CO. You know that the CO degrades in the air at a first order rate of $0.01/\text{hour}$.
- a) How long would it take to reach 95% of the steady-state?
- b) Would this steady-state be above or below the maximum fetal exposure of 800 ppm_v ?

Q 4.

$$a) Q = 10 \text{ m}^3/\text{hr}$$

$$V = 250 \text{ m}^3$$

$$k_d V = 28 \text{ gV/hr} \quad (-2)$$

$$k_d = 0.01 \text{ hr}^{-1}$$

assumptions $C_{in} = 0$

28
 $Q C_{in}$ instead of $k_d V$

(-2)

$$C_{ss} = \frac{Q C_{in} + k_d V}{Q + k_d V} \quad \text{at } t \rightarrow \infty \quad (4)$$

$$C_{ss} = \frac{10 \text{ m}^3/\text{hr} \times 0 + 28 \text{ gV/hr}}{10 \text{ m}^3/\text{hr} + 0.01 \text{ hr}^{-1} \times 250 \text{ m}^3} = 2.24 \text{ gV/m}^3$$
$$= 1957.23 \text{ m}^3 C_0 \quad (2)$$
$$= 1957.23 \frac{\text{m}^3}{\text{m}^3} \quad (2)$$

$$C_t = C_{ss} + (C_0 - C_{ss}) e^{-\left(\frac{Q}{V} + k_d\right)t} \quad (6)$$

$$C_t = 0.95 C_{ss} \Rightarrow C_t = 0.95 \times 2.24 = 2.128 \text{ gV/m}^3$$

$$= 1359.72 \text{ m}^3 C_0 = 0.01 \text{ hr}^{-1} \times 250 \text{ m}^3$$

$$2.128 = 2.24 + (0 - 2.24) e^{-\left(\frac{10}{250} + 0.01\right)t}$$

$$\Rightarrow -2.9957 = -0.05 t \Rightarrow t = 59.91 \text{ hr} \quad (4)$$

(1)

$$b) \frac{800 \text{ m}^3 \text{ CO}}{10^6 \text{ m}^3 \text{ air}} = 800 \text{ PPM}_V$$

assumptions
Ideal gas
 $T = 25^\circ\text{C}$, $P = 1.013 \times 10^5 \text{ Pa}$

$$PV = nRT \Rightarrow nT = \frac{PV}{R} \Rightarrow \frac{1.013 \times 10^5 \cdot V}{8.314} = 1 \text{ mol} \times 298.15 \quad (2)$$

$$\Rightarrow V = 0.02447 \text{ m}^3 \quad (1)$$

$$\frac{800 \text{ m}^3 \text{ CO}}{10^6 \text{ m}^3 \text{ air}} \times \frac{28 \text{ g CO}}{1 \text{ mol CO}} \times \frac{1 \text{ mol CO}}{0.02447 \text{ m}^3 \text{ CO}} = 0.915 \frac{\text{g CO}}{\text{m}^3 \text{ air}}$$

$$(4) \quad \text{Ans } \frac{28 \text{ g}}{2.24} \rightarrow 0.915 \text{ g/m}^3 \quad (4)$$

The same if they convert $\text{g/m}^3 \rightarrow \text{PPM}_V$
or the other way around (conversion of mass
to $\frac{\text{m}^3 \text{ CO}}{\text{m}^3 \text{ air}}$)

$$C_t(\text{PPM}_V) = C_t(\text{mg/m}^3) \times \frac{1 \text{ m}^3}{1000 \text{ L}} \times \frac{22.414 \text{ L/mol}}{\text{MW}(\text{mg/mol})} \times \frac{T(\text{K})}{273.15 \text{ K}} \times \frac{10^6}{10^3}$$

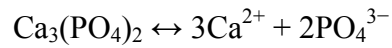
$$\text{PPM} \frac{\text{g}}{10^6 \text{ g}} \quad (3)$$

if they use R_{water} instead of the ideal gas
(-2)

if C_t is used instead of C_o (-2)

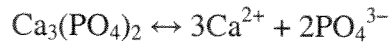
(2)

5. **(24 points)** Calcium phosphate $[\text{Ca}_3(\text{PO}_4)_2]$ dissolves in water that does not contain calcium or phosphate, and with a pH such that PO_4^{3-} remains in this form (i.e, does not undergo further acid-base reactions).



- What is the K_{SP} of calcium phosphate at 25°C ?
- If calcium phosphate is allowed to reach dissolution equilibrium, what would be the final temperature? The temperature remains constant at 25°C .
- At that temperature, would you have more or less calcium phosphate in solution? Show calculations.
- If you design a reactor, propose a solution to adjust your design to ensure optimal dissolution of calcium phosphate.

5. (24 points) Calcium phosphate $[Ca_3(PO_4)_2]$ dissolves in water that does not contain calcium or phosphate, and with a pH such that PO_4^{3-} remains in this form (i.e., does not undergo further acid-base reactions).



- What is the K_{sp} of calcium phosphate at $25^\circ C$?
- If calcium phosphate is allowed to reach dissolution equilibrium, what would be the final temperature? ~~The temperature remains constant at $25^\circ C$.~~
- At that temperature, would you have more or less calcium phosphate in solution? Show calculations.
- If you design a reactor, propose a solution to adjust your design to ensure optimal dissolution of calcium phosphate.

7) a) $K_{sp} = \exp \frac{-\Delta G^\circ}{RT}$ $\Delta G^\circ = \sum \Delta G^\circ_{products} - \sum \Delta G^\circ_{reactants}$ 2

$$\Delta G^\circ = \left(3(-553.04) + 2(-1013) \right) - (-3899) \text{ kJ/mol}$$

$$= 213.88 \text{ kJ/mol}$$

$$K_{sp} = \exp \left(\frac{-213.88}{8.314 \times 10^{-3} \cdot 298} \right) = \boxed{3.23 \times 10^{-38}}$$

12) b) $K_{sp} = (3x)^3 (2x)^2$

$$= 108 x^5$$

$$\Delta H^\circ = (3(-542.96) + 2(-1266)) - (-4138)$$

$$\Delta H^\circ = -22.88 \text{ kJ/mol}$$

$$x = \left(\frac{3.23 \times 10^{-38}}{108} \right)^{1/5}$$

$$x = 1.24 \times 10^{-8} \text{ M}^3$$

$$= 1.24 \times 10^{-8} \frac{\text{mol}}{\text{kg H}_2\text{O}}$$

$$C_p = \frac{\Delta H}{m \Delta T}$$

$$\Delta T = \frac{\Delta H}{m C_p}$$

$$= \frac{-22.88 \text{ kJ/mol} \times 1.24 \times 10^{-8} \frac{\text{mol}}{\text{kg}}}{4.187 \frac{\text{kJ}}{\text{kg K}}}$$

$$\Delta T = -6.77 \times 10^{-8} \text{ K}^3$$

$$T_F = 24.999 \dots \rightarrow \boxed{25^\circ\text{C}}$$

c)

Because the temperature drop is so little, the concentration would be the same.

3

Can prove this with Van't Hoff equation.

d)

Because the reaction is exothermic (ΔH° is negative) keeping the reactor cool will optimize dissolution.

2

OR

(2) LONGER DISSOLUTION TIME COULD ALSO ACHIEVE SIMILAR RESULTS.