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COMM 225: MIDTERM REVIEW QUESTIONS

TOPIC: PROJECT MANAGEMENT

Q 1.1: Kozar International, Inc. begun marketing a new instant-developing film project. The estimates of R&D activity time (weeks) for Kozar's project are given in the table below. The project has two paths: A-C-E-F and A-B-D-F. Assume the activity times are independent.

a) What is the probability that the project will be completed between 35 and 45 days?

b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

Activity	*Time(weeks)*				*Mean*	*Variance*
	Predecessors	*Optimistic time*	*Probable time*	*Pessimistic time*		
A	*-*	*9*	*9*	*9*	*9*	*0.00*
B	*A*	*8*	*10*	*12*	*10*	*0.44*
C	*A*	*9*	*12*	*18*	*12.5*	*2.25*
D	*B*	*5*	*8*	*11*	*8*	*1.00*
E	*C*	*5*	*7*	*10*	*7.166*	*0.69*
F	*D, E*	*10*	*12*	*14*	*12*	*0.44*

Solution:

(a) What is the probability that the project will be completed between 35 and 45 days?

The project has two paths:

A-C-E-F:

- Expected Duration = 9 + 12.5 + 7.166 + 12 = 40.66 weeks (Critical Path).
- Variance $\sigma^2 = 0 + 2.25 + 0.69 + 0.44 = 3.38$, Standard Deviation = $\sigma = 1.838$

For 35 weeks, $z_{35} = \frac{\text{T−Expected Duration}}{\sigma} = \frac{35-40.66}{=1.838}$, or z_{35} = -3.08 ➔ Prob (z ≤ z_{35}) = 0.001

For 45 weeks, $z_{45} = \frac{\text{T−Expected Duration}}{\sigma} = \frac{45-40.66}{=1.838}$, or z_{45} = 2.36 ➔ Prob (z ≤ z_{45}) = 0.9909

Probability that this path will be completed between 35 and 45 days is 0.9909-0.001 = 0.9899

A-B-D-F:

- Expected Duration = 9 + 10 + 8 + 12 = 39 weeks.
- Variance = 0 + 0.44 + 1.00 + 0.44 = 1.88, Standard Deviation = 1.371

For 35 weeks, $z_{35} = \frac{\text{T−Expected Duration}}{\sigma} = \frac{35-39}{=1.371}$, or z_{35} = -2.918 ➔ Prob (z ≤ z_{35}) = 0.0018

For 45 weeks, $z_{45} = \frac{\text{T−Expected Duration}}{\sigma} = \frac{45-39}{=1.371}$, or z_{45} = 4.376 ➔ Prob (z ≤ z_{45}) = 1

Probability that this path will be completed between 35 and 45 days is 1 – 0.0018 = 0.9982

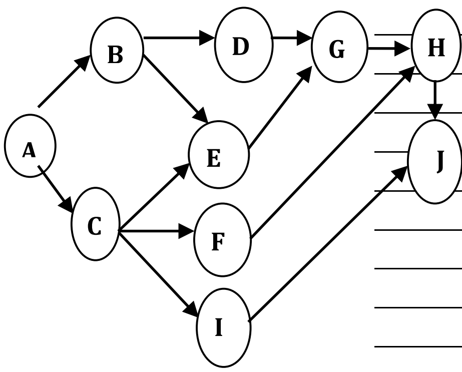
Hence, the probability of project completion between 35 and 45 days = 0.9899*0.9982= 0.9881 = 98.81%

(b) If the time to complete the path A-B-D-F is normally distributed, what is the probability that this path will take at least 38 weeks to be completed?

- The non-critical path A-B-D-F has an expected duration of 39 weeks and standard deviation of 1.371.
- This implies $Z_{38} = \frac{\text{T}-\text{Expected Duration}}{\sigma} = \frac{38-39}{1.371}$=-0.73.
- This z value corresponds to a probability of 0.2327.
- Probability that this path will take less than 38 weeks to be completed is 23.27%.
- Hence, the probability that this path will take at least 38 weeks to be completed is 76.73%.

Q 1.2: Given the following network and time & cost estimates, answer the following questions:

(a) What is the project completion time?
(b) What is the total cost required for completing this project on normal time?
(c) Crash the network the maximum amount possible and compute the total crash cost.



Activity	Predecessor	Activity duration (weeks)		Activity cost ($)		Crash Cost / Week	# Days
		Normal	Crash	Normal	Crash		
A	-	6	4	10000	16000	3000	2
B	A	28	22	5000	9200	700	6
C	A	29	27	20000	20700	350	2
D	B	10	5	4000	6000	400	5
E	B,C	10	9	2500	3000	500	1
F	C	10	9	1000	7000	6000	1
G	D,E	15	14	1500	7500	6000	1
H	G,F	10	8	600	10600	5000	2
I	C	2	1	1000	2000	1000	1
J	H,I	10	8	900	8800	3950	2
TOTAL =				**46,500**			

Solution:

Parts (a & b)

The project has the following paths and their durations:

- A-B-D-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- A-B-E-G-H-J, Duration = 6 + 28 + 10 + 15 + 10 + 10 = 79
- **A-C-E-G-H-J, Duration = 6 + 29 + 10 + 15 + 10 + 10 = 80 (critical path)**
- A-C-F-H-J, Duration = 6 + 29 + 10 + 10 + 10 = 65
- A-C-I-J, Duration = 6 + 29 + 2 + 10 = 47

The project completion time = length of critical path = 80 weeks
Normal total cost = sum of the normal cost for all the activities = **$46,500.**

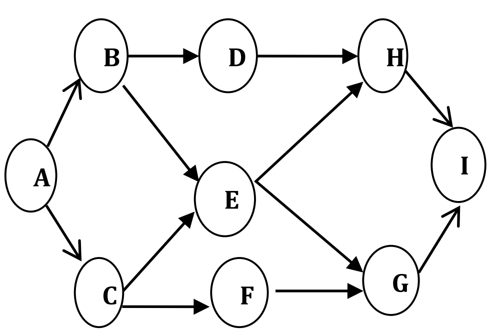
c)

Critical Path	Duration	It#1	It#2	It#3	It#4	It#5	It#6	It#7
A-B-D-G-H-J	79	79*	78*	77*	75*	73*	71*	70*
A-B-E-G-H-J	79	79*	78*	77*	75*	73*	71*	70*
A-C-E-G-H-J	80*	79*	78*	77*	75*	73*	71*	70*
A-C-F-H-J	65	64	64	63	61	59	57	57
A-C-I-J	47	46	46	45	43	41	41	41

It#	Critical path before crashing (length)	Activity crashed	Critical path after crashing (length)	Cumulative crashing cost
1	A-C-E-G-H-J (80)	C by 1	A-B-D-G-H-J (79)	$350
			A-B-E-G-H-J (79)	
			A-C-E-G-H-J (79)	
2	A-B-D-G-H-J (79)	D by 1	A-B-D-G-H-J (78)	$1,250
	A-B-E-G-H-J (79)	E by 1	A-B-E-G-H-J (78)	
	A-C-E-G-H-J (79)		A-C-E-G-H-J (78)	
3	A-B-D-G-H-J (78)	B by 1	A-B-D-G-H-J (77)	$2,300
	A-B-E-G-H-J (78)	C by 1	A-B-E-G-H-J (77)	
	A-C-E-G-H-J (78)		A-C-E-G-H-J (77)	
4	A-B-D-G-H-J (77)	A by 2	A-B-D-G-H-J (75)	
	A-B-E-G-H-J (77)		A-B-E-G-H-J (75)	$8,300
	A-C-E-G-H-J (77)		A-C-E-G-H-J (75)	
	A-C-F-H-J (77)		A-C-F-H-J (75)	
	A-C-I-J (77)		A-C-I-J (75)	
5	A-B-D-G-H-J (75)	J by 2	A-B-D-G-H-J (73)	
	A-B-E-G-H-J (75)		A-B-E-G-H-J (73)	$16,200
	A-C-E-G-H-J (75)		A-C-E-G-H-J (73)	
	A-C-F-H-J (75)		A-C-F-H-J (73)	
	A-C-I-J (75)		A-C-I-J (73)	
6	A-B-D-G-H-J (73)	H by 2	A-B-D-G-H-J (71)	
	A-B-E-G-H-J (73)		A-B-E-G-H-J (71)	$26,200
	A-C-E-G-H-J (73)		A-C-E-G-H-J (71)	
	A-C-F-H-J (73)		A-C-F-H-J (71)	
7		G by 1		
	A-B-D-G-H-J (71)		A-B-D-G-H-J (70)	$32,200
	A-B-E-G-H-J (71)		A-B-E-G-H-J (70)	
	A-C-E-G-H-J (71)		A-C-E-G-H-J (70)	

After crashing by 10 weeks, the duration of the project is **70 weeks** and the total cost for the crashed project is **<u>$46,500+ $32,200 = $78,700.</u>**

Q 1.3: The following table provides the necessary information for crashing a project. The project manager would like to crash the network by three weeks in the most economical way. Which activities should be crashed and by how many weeks?



Activity	*Activity duration (weeks)*		*Activity cost ($)*		*Crash Cost/ Week*
	Normal	*Crash*	*Normal*	*Crash*	
A	*4*	*3*	*4000*	*6000*	*2000*
B	*3*	*2*	*5000*	*6000*	*1000*
C	*2*	*1*	*2000*	*2800*	*800*
D	*5*	*3*	*4000*	*6000*	*1000*
E	*6*	*5*	*2500*	*3000*	*500*
F	*3*	*2*	*1000*	*2000*	*1000*
G	*4*	*3*	*2000*	*2900*	*900*
H	*4*	*3*	*1500*	*2600*	*1100*
I	*6*	*5*	*5000*	*12000*	*7000*

Solution:

The project has the following paths and their durations (in weeks):

- A-B-D-H-I, Duration = 22
- **A-B-E-H-I, Duration = 23**
- **A-B-E-G-I, Duration = 23**
- A-C-E-H-I, Duration = 22
- A-C-E-G-I, Duration = 22
- A-C-F-G-I, Duration = 19

The normal duration of the project is 23 weeks and the normal total cost (sum of normal costs for all the activities) is **$27,000.**

Critical Path	Duration	It#1	It#2	It#3
A-B-D-H-I	22	**22***	**21***	**20***
A-B-E-H-I	**23***	**22***	**21***	**20***
A-B-E-G-I	**23***	**22***	**21***	**20***
A-C-E-H-I	22	21	**21***	**20***
A-C-E-G-I	22	21	**21***	**20***
A-C-F-G-I	19	19	19	18

It#	CP Before Crashing	Activity Crashed	CP After Crashing	Cumulative cost
1	A-B-E-H-I (23) A-B-E-G-I (23)	E by 1	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	$ 500
2	A-B-E-H-I (22) A-B-E-G-I (22) A-B-D-H-I (22)	B by 1	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21)	$ 1500
3	A-B-E-H-I (21) A-B-E-G-I (21) A-B-D-H-I (21) A-C-E-H-I (21) A-C-E-G-I (21)	A by 1	A-B-E-H-I (20) A-B-E-G-I (20) A-B-D-H-I (20) A-C-E-H-I (20) A-C-E-G-I (20)	$ 3500

After crashing by 3 weeks, the duration of the project is **20 weeks** and the total cost of the crashed project is **$27,000 + $3500 = $30,500.**

Q 1.4: A company is planning to install a new computerized system for paying its employees. The management has determined the activities required for completing the project, the precedence relationships of the activities, and activity time estimates (in weeks) as given in the table below.

Activity	*Preceding activity*	*Optimistic Time*	*Most likely Time*	*Pessimistic Time*	*Expected Time*	*Standard Deviation*
A	--	*7*	*9*	*14*		*1.167*
B	*A*	*2*	*2*	*8*	*3*	*1.000*
C	*A*	*8*	*12*	*16*	*12*	
D	*A*	*3*	*5*	*10*	*5.5*	*1.167*
E	*B*	*4*	*6*	*8*	*6*	
F	*B*	*6*	*8*	*10*	*8*	*0.667*
G	*C, F*	*2*	*3*	*4*		
H	*D*	*2*	*2*	*8*	*3*	*1.000*
I	*H*	*6*	*8*	*16*	*9*	*1.667*
J	*G, I*	*4*	*6*	*14*	*7*	*1.667*
K	*E, J*	*2*	*2*	*5*		*0.5*

a) Calculate the missing expected times and standard deviations.
b) Find the critical path(s) and the expected project duration.
c) What is the earliest start time for activity E? What is the latest finish time for activity J?

Solution:

(a) Calculate the missing expected times and standard deviations.

Activity	Preceding activity	Optimistic Time	Most likely Time	Pessimistic Time	Expected Time	Standard Deviation	Variance
A	--	7	9	14	9.5	1.167	1.36
B	A	2	2	8	3	1	1
C	A	8	12	16	12	1.333	1.78
D	A	3	5	10	5.5	1.167	1.36
E	B	4	6	8	6	0.667	0.44
F	B	6	8	10	8	0.667	0.44
G	C, F	2	3	4	3	0.333	0.11
H	D	2	2	8	3	1	1
I	H	6	8	16	9	1.667	2.78
J	G, I	4	6	14	7	1.667	2.78
K	E, J	2	2	5	2.5	0.5	0.25

(b) Find the critical path(s) and the expected project duration.

1. A-B-E-K: 9.5+3+6+2.5=21
2. A-B-F-G-J-K: 9.5+3+8+3+7+2.5=33
3. A-C-G-J-K: 9.5+12+3+7+2.5=34
4. A-D-H-I-J-K: 9.5+5.5+3+9+7+2.5=36.5* (critical path and expected project duration is 36.5)

(c) What is the earliest start time for activity E? What is the latest finish time for activity J?

Activity	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)
A	0	9.5	0	9.5
B	9.5	12.5	13	16
C	9.5	21.5	12	24
D	9.5	15	9.5	15
E	12.5	18.5	28	34
F	12.5	20.5	16	24
G	21.5	24.5	24	27
H	15	18	15	18
I	18	27	18	27
J	27	34	27	34
K	34	36.5	34	36.5

TOPIC: FORECASTING

Q2.1: Monthly sales for National Mixer, Inc. for a seven-month period were as follows:

Month (t)	Feb.	Mar.	Apr.	May	June	Jul.	Aug.
Sales (1000 UNITS)	19	18	15	20	18	22	20

Forecast the sales volume for September using each of the following methods:

a) 5-month moving average;
b) Weighted average, where the weights are: 0.60 (August), 0.30 (July), 0.10 (June)

c) Exponential smoothing with a smoothing constant equal to 0.20. Use the naïve approach to get the initial forecast.
d) Linear trend equation, Y = 16.86+0.5*T;

Solution:

a) 5-month moving average;

Using the 5-month moving average, the forecast for September $= \frac{15+20+18+22+20}{5} = 19$

(in thousands of units)

b) Weighted average, where the weights are - 0.60 (August), 0.30 (July), 0.10 (June)

Using the weighted moving average, the forecast for September is $0.1*18+0.3*22+0.6*20$ = 20.4 (in thousands of units)

c) Exponential smoothing with a smoothing constant equal to 0.20

To get the method started, we use the naïve approach and we set the forecast for March to be equal to the actual demand in the previous period (i.e., February).

MONTH	*SALES (1000 UNITS)*	*Forecast (alpha=0.2)*	*Error = (Sales-Forecast)*
Feb.	*19*	-	
Mar.	*18*	*19*	-1
Apr.	*15*	*18.8*	-3.8
May	*20*	*18.04*	1.96
June	*18*	*18.432*	-0.432
Jul.	*22*	*18.3456*	3.6544
Aug.	*20*	*19.07648*	0.92352
September		*19.26118*	

The forecast for September = 19.2612 (in thousands of units)

*d) Linear trend equation, Y = 16.86+0.5*T;*

For the month of September, T = 8, as the data starts from Feb (T=1), and hence the forecast for September = 16.86+0.5*8 = 20.86 (in thousands of units).

Q 2.2: A fashion retailer buys fabric from several textile manufacturers. The demand for fabric across various locations in a city based on past historical data is shown in the table below. The manager would like to forecast the demand for the upcoming year in order to decide how many feet of fabrics they should buy.

a) Develop a trend adjusted exponential smoothing model using α=0.30 and β=0.20 and compute the adjusted exponentially smoothed forecasts for years 4 through 6.
b) Develop a trend line equation for the given data and use the resulting equation to forecast the sales for years 4 through 6.
c) Based on MAD values, which of the two forecasting models used above seems to be more accurate. Explain your answer.

Year (t)	1	2	3	4	5
Fabric (feet)	4260	4510	4050	3720	3900

Solution:

a) Trend adjusted exponential smoothing model with α=0.30 and β=0.20.

Initialization:

We will use the average of the first three periods to obtain an initial estimate for the smoothed average at the end of period 3:

$S_3 = \frac{(4260+4510+4050)}{3} = 4273.33$

The trend at the end of period 3 (T3) can be seen as the next change from period 1 to period 3, which is calculated as follows:

T_3 = (4050 - 4260)/(3-1) = - 105 (negative indicating a decreasing trend)

Hence, the trend adjusted forecast for period 4 is $TAF_4 = S_4 + T_3$ = 4273.33 – 105 = 4168.33

Updating: We use the update equations to find new values for S_t and T_t.

Period 4:

$S_4 = TAF_4 + \alpha(A_4 - TAF_4) = 4168.33 + 0.30(3720 - 4168.33) = 4033.831$

$T_4 = T_3 + \beta(S_4 - S_3 - T_3) = -105 + 0.20(4033.831 - 4273.33 + 105) = -131.90$

$TAF_5 = S_4 + T_4 = 4033.831 - 131.90 = 3901.93$

Period 5:

$S_5 = TAF_5 + \alpha(A_5 - TAF_5) = 3901.93 + 0.30(3900 - 3901.93) = 3901.351$

$T_5 = T_4 + \beta(S_5 - S_4 - T_4) = -131.90 + 0.20(3901.351 - 4033.831 + 131.90) = -132.016$

$TAF_6 = S_5 + T_5 = 3901.351 - 132.016 = 3769.335$

Hence, the trend adjusted forecast for period 6 is 3769.335 feet.

b) Trend line equation

Year (t)	Demand (y)	t.y	t^2	Forecast
1	4260	4260	1	
2	4510	9020	4	
3	4050	12150	9	
4	3720	14880	16	3937
5	3900	19500	25	3786
15	20440	59810	55	**3635**

The slop is given by: $b = \frac{5*59810-15*20440}{5*55-15^2} = -151$

The intercept is given as: $a = \frac{20440-(-151)(15)}{5} = 4541$

The equation is: $y_t = a + bt = 4541 - 151t$

Hence, the forecast for period 6 is: $y_6 = 4541 - 151 * 6 = 3635$

c) Based on MAD values, which of the two forecasting models used above seems to be more accurate. Explain your answer.

For Trend adjusted smoothing: $MAD = \frac{|3720-4168.33|+|3900-3901.93|}{2} = \frac{448.33+1.93}{2} = 225.13$

For linear trend method: $MAD = \frac{|3720-3937|+|3900-3786|}{2} = \frac{217+114}{2} = 165.5$

Based on just two observations, it seems that the linear trend method is more accurate since it has a lower MAD value.

***Q2.3*:** Two independent methods of forecasting based on the managers experience have been prepared each month for the past 10 months. The forecasts and actual sales are as follows. Which forecast seem superior? Justify your answer with appropriate calculations using MAD, MSE & MAPE.

MONTH	*FORECAST 1*	*FORECAST 2*	*ACTUAL SALES*
1	*771*	*769*	*770*
2	*785*	*787*	*789*
3	*790*	*792*	*794*
4	*784*	*798*	*780*
5	*770*	*774*	*768*
6	*768*	*770*	*772*
7	*761*	*759*	*760*
8	*771*	*775*	*775*
9	*784*	*788*	*786*
10	*788*	*788*	*790*

Solution:

Method 1

Month	Actual Sales	F1	Error (A-F1)	|Error|	Error^2	(|Error|/actual) * 100
1	770	771	-1	1	1	0.130
2	789	785	4	4	16	0.507
3	794	790	4	4	16	0.504
4	780	784	-4	4	16	0.513
5	768	770	-2	2	4	0.260
6	772	768	4	4	16	0.518
7	760	761	-1	1	1	0.132
8	775	771	4	4	16	0.516
9	786	784	2	2	4	0.254
10	790	788	2	2	4	0.253
SUM	**7784**		**12**	**28**	**94**	**3.587%**

Method 2

Month	Actual Sales	F2	Error (A-F2)	|Error|	Error^2	(|Error|/actual) * 100
1	770	769	1	1	1	0.130
2	789	787	2	2	4	0.253
3	794	792	2	2	4	0.252
4	780	798	-18	18	324	2.308
5	768	774	-6	6	36	0.781
6	772	770	2	2	4	0.259
7	760	759	1	1	1	0.132
8	775	775	0	0	0	0.000
9	786	788	-2	2	4	0.254
10	790	788	2	2	4	0.253
SUM	**7784**		**-16**	**36**	**382**	**4.622%**

Method 1: $MAD = \frac{\sum |error|}{n} = 2.8$, $MSE = \frac{\sum error^2}{n} = 9.4$,

$$MAPE = \frac{\sum \left(\frac{\text{Absolute Error}}{\text{Actual}} \right)}{n} * 100 = \frac{3.587}{10} = 0.3587\%$$

Method 2: $MAD = \frac{\sum |error|}{10} = 3.6$, $MSE = \frac{\sum error^2}{10} = 38.2$

$$MAPE = \frac{\sum \left(\frac{\text{Absolute Error}}{\text{Actual}} \right)}{n} * 100 = \frac{4.622}{10} = 0.4622\%$$

All the three measures, MAD, MSE and MAPE are **smaller** for forecasting 1 and so that is **superior.**

Q 2.4: The following data are quarterly sales of natural gas in Saskatchewan by SaskEnergy (in peta joules ≈ 1 billion cubic feet) from Q1 of 2005 to Q3 of 2009.

Year	Q1	Q2	Q3	Q4
2005	49	24	18	37
2006	42	20	20	43
2007	48	24	20	40
2008	51	25	19	43
2009	51	24	15	

(a) Compute the seasonal relative for each quarter using the centred moving average method.

(b) Deseasonalize the data, fit an appropriate model to the deseasonalized data, extend the model four quarters, and reseasonalize these in order to forecast the sales of natural gas by SaskEnergy from Q4 2009 to Q3 2010.

Solutions:

a)

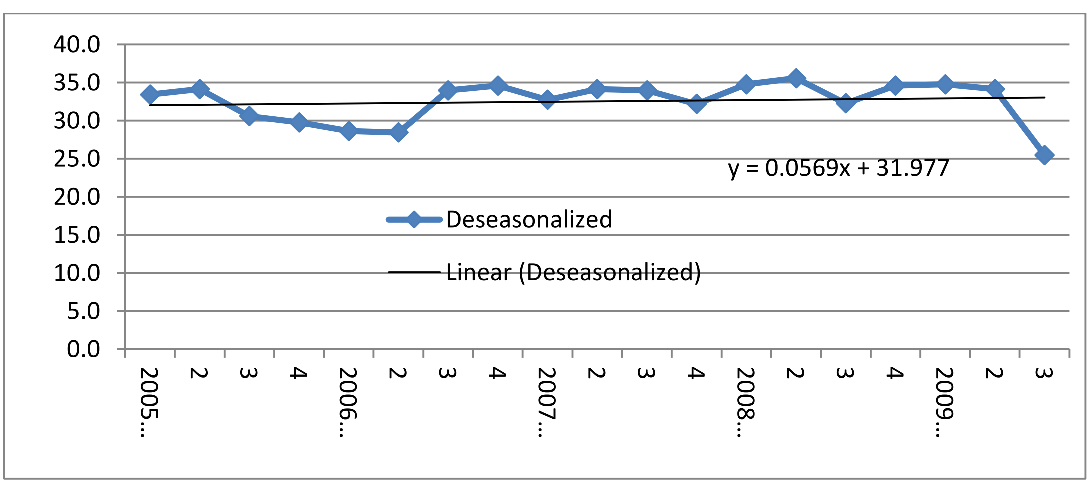
Year	Quarter	Sales	CMA_4	CMA_2	Sales/CMA_2
2005	1	49			
	2	24			
	3	18	32	31.125	0.578
	4	37	30.25	29.75	1.244
2006	1	42	29.25	29.5	1.424
	2	20	29.75	30.5	0.656
	3	20	31.25	32	0.625
	4	43	32.75	33.25	1.293
2007	1	48	33.75	33.75	1.422
	2	24	33.75	33.375	0.719
	3	20	33	33.375	0.599
	4	40	33.75	33.875	1.181
2008	1	51	34	33.875	1.506
	2	25	33.75	34.125	0.733
	3	19	34.5	34.5	0.551
	4	43	34.5	34.375	1.251
2009	1	51	34.25	33.75	1.511
	2	24	33.25		
	3	15			

Seasonal relatives	QUARTER				
	1	2	3	4	TOTAL
AVERAGE	1.466	0.702	0.588	1.242	3.999
Adjusted	**1.466**	**0.703**	**0.589**	**1.243**	4.000

b) Deseasonalizing demand through dividing the sales by the seasonal relatives for every season.

Quarter	Sales (peta joules)	Seasonal Relatives	Deseasonalized Sales
2005 1	49	1.466	33.4
2	24	0.703	34.1
3	18	0.589	30.6
4	37	1.243	29.8
2006 1	42	1.466	28.6
2	20	0.703	28.4
3	20	0.589	34.0
4	43	1.243	34.6
2007 1	48	1.466	32.7
2	24	0.703	34.1
3	20	0.589	34.0
4	40	1.243	32.2
2008 1	51	1.466	34.8
2	25	0.703	35.6
3	19	0.589	32.3
4	43	1.243	34.6
2009 1	51	1.466	34.8
2	24	0.703	34.1
3	15	0.589	25.5

Fitting the deseasonalized sales:



Using the fitted linear model to forecast for the next four quarters and then multiplying by the seasonal relatives to reseasonalize.

Quarter		Trend Forecast	Reseasonalized Forecast
2009	4	33.12	**41.1**
2010	1	33.17	**48.6**
	2	33.23	**23.4**
	3	33.29	**19.6**

TOPIC: INVENTORY MANAGEMENT

Q 3.1: The Zertex Manufacturing Company produces fertilizer to sell to wholesalers. One raw material, calcium nitrite, is purchased from a supplier located near Zertex's plant. 5,750,000 tons of calcium nitrite is forecast to be required next year to support production. If calcium nitrite costs $22.50 per ton, carrying cost is 40% of acquisition cost, and ordering cost is $595 per order:

a) In what quantities should Zertex buy calcium nitrite?
b) What annual stocking costs will be incurred if calcium nitrite is ordered at EOQ?
c) How many orders per year must take place for calcium nitrite?
d) How much time will elapse between two orders?

Solution: (BASIC EOQ MODEL)

- Annual Demand (D) = 5,750,000 tons/year
- Holding cost (H) = 40% of $22.50 = $ 9.0
- Ordering/Setup Cost (S)= $ 595/order

(a) Optimal Ordering Quantity (EOQ)

$$Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*595*5{,}750{,}000}{9.00}} = 27{,}573.135 \text{ tons}$$

(b) Total Annual Cost = Annual Holding Cost + Annual Ordering Cost

Annual Holding Cost = $\frac{Q}{2}H = \frac{27{,}573.135*9}{2} = \$124{,}079.10$

Annual Ordering Cost = $\frac{D}{Q}S = \frac{5{,}750{,}000*595}{27{,}573.135} = \$124{,}079.10$

Total Annual Cost = Annual Holding Cost + Annual Ordering Cost = $ 248,158.20

c) *Number of orders placed per year* = D/Q* = 5,750,000/27,573.135 = 208.53 orders/year

d) *Time between the placement of orders (Cycle Time)* =Q*/D = 27,573.135/5,750,000=0.00479 year

Q 3.2: A mail ordering company uses 800 boxes a year. The boxes can be purchased from either the supplier A or supplier B. Holding cost is 25% of unit cost and the ordering cost is $ 40 per order. The following quantity discounts are available.

Supplier A		*Supplier B*	
Quantity	*Unit Price*	*Quantity*	*Unit Price*
1-199	*$14.00*	*1-149*	*$14.10*
200-499	*$13.80*	*150-349*	*$13.90*
500+	*$13.60*	*350+*	*$13.70*

Which supplier should be used and what is (a) the optimal order quantity and (b) the number of orders per year if the intent is to minimize the total annual cost.

Solution: (*EOQ Model with Quantity Discounts)*
Given:

Annual Demand (D) = 800 units /year
Ordering Costs S = $40/order
Holding Costs H = (25%) x Unit price of product

Supplier A:

Range	*R*	*H*	*S*	*FEASIBLE*
1-199	*14*	*3.5*	*40*	*YES*
200-499	*13.8*	*3.45*	*40*	
500+	*13.6*	*3.4*	*40*	

The optimal ordering quantity for the first price range is

$$Q_1^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.5}} = 135.22 \text{ (This is realizable)}$$

Total annual cost of this policy is:

$$TC_1 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*14 + \frac{135.22}{2}*3.5 + \frac{800}{135.22}*40 = \$11{,}673.29$$

The optimal ordering quantity for the second price range is

$$Q_2^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.45}} = 136.20 \text{ (This is not realizable, increase it up 200 units)}$$

Total annual cost of this policy is:

$$TC_2 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*13.8 + \frac{200}{2}*3.45 + \frac{800}{200}*40 = \$11{,}545 \text{ (Lowest)}$$

The optimal ordering quantity for the third price range is

$$Q_3^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.40}} = 137.20 \text{ (This is not realizable, increase it up 500 units)}$$

Total annual cost of this policy is:

$$TC_3 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*13.6 + \frac{500}{2}*3.40 + \frac{800}{500}*40 = \$11{,}794$$

Supplier B:

Range	R	H	S	FEASIBLE
1-149	14.1	3.525	40	YES
150-349	13.9	3.475	40	NO
350+	13.7	3.425	40	

The optimal ordering quantity for the first price range is

$$Q_1^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.525}} = 134.74 \text{ (This is realizable)}$$

Total annual cost of this policy is:

$$TC_1 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*14.1 + \frac{134.74}{2}*3.525 + \frac{800}{134.74}*40 = \$11{,}754.96$$

The optimal ordering quantity for the second price range is

$$Q_2^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.475}} = 135.71 \text{ (This is not realizable, increase it up 150 units)}$$

Total annual cost of this policy is:

$$TC_2 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*13.9 + \frac{150}{2}*3.475 + \frac{800}{150}*40 = \$11{,}593.96 \text{ (Lowest)}$$

The optimal ordering quantity for the third price range is

$$Q_3^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*800*40}{3.425}} = 136.70 \text{ (This is not realizable, increase it up 350 units)}$$

Total annual cost of this policy is:

$$TC_3 = DR + \frac{Q}{2}H + \frac{D}{Q}S = 800*13.7 + \frac{3500}{2}*3.425 + \frac{800}{350}*40 = \$11{,}650.80$$

Conclusion: Since supplier A's cost of 11, 545 is less than Supplier B's 11,593.96, we should use supplier A. The optimal order quantity is 200 units. The number of orders per year to minimize cost is (800/200=4).

Q3.3: An electronic retailer stocks a popular model of alarm clock in his warehouse. The lead time has been so stable in the past year that it is safely assumed to be a constant 9 days. Past data also indicates that the probability distribution of the daily demand is approximately normal with a mean of 20 and a variance of 16.

a) What are the reordering point and the safety stock that provides a 96% service level?
b) The ordering point is arbitrarily set to 189 clocks by the manager. What is the corresponding service level?
c) Determine the safety stock that is needed to attain a 1% risk of stockout during lead-time.

Solution:

Given:

- Lead Time (*LT*) = 9 days (Constant)
- Demand (*D*) is normally distributed with mean of 20 units/day,
- Standard Deviation of demand (σ_d) is 4 units/day

Hence, use EOQ model with Stochastic Demand formulas.
From this, we will calculate the expected demand and the standard deviation of demand during lead time as follows:

- Expected demand during lead time $= \bar{d} * LT = 20*9 = 180$ units
- Standard deviation of demand during lead time $= \sigma_d \sqrt{LT} = 4*\sqrt{9} = 12$ units.

(a) From the normal distribution table, a 96% service level implies that 4% upper tail area. This corresponds to a standard normal z-value of 1.75.

- Thus, the reorder point is $ROP = \bar{d} * LT + z\sigma_d \sqrt{LT} = 180 + 1.75*12 = 201$ units.
- The amount of safety stock to be carried is $z\sigma_d \sqrt{LT} = 1.75*12 = 21$ units.

(b) If the reordering point is set to 189 units, this implies that the safety stock is 9 units as the expected demand during the lead time is 180 units.

Safety stock of 9 units implies that the service level corresponds to

$$z = \frac{9 \text{ units}}{\sigma_d \sqrt{LT}} = \frac{9}{12} = 0.75.$$

This corresponds to a service level of 77.34% (from the normal distribution table).

(c) Risk of stock out of 1% implies that the service level is 99%. For a 99% service level, the z-score is 2.33.

- Safety Stock = $z\sigma_d \sqrt{LT} = 2.33*12 = 27.96 \text{ units} \approx 28 \text{ units}$

Q 3.4: The Old Town Microbrewery makes Towside beer, which it bottles and sells in its adjoining restaurant. It costs $1700 to setup, brew, and bottle a batch of the beer. The annual cost to store the beer in inventory is $1.25 per bottle. The annual demand for beer is 21,000 bottles and the brewery has the capacity to produce 30,000 bottles annually.

(a) Determine the optimal ordering quantity, total annual inventory costs, and the number of production runs per year.

(b) If the microbrewery has only enough storage space to hold a maximum of 2500 bottles of beer in inventory, how will that effect the total inventory costs?

<u>Solution:</u>

- Demand = 21,000 bottles/year (Constant)
- Production Capacity = 30,000/year
- Carrying Cost (H) = $1.25/bottle/year
- Setup Cost (S) = $1700/order
- Assuming 360 days/year, d =21,000/360 = 58.33 bottles/day
- Assuming 360 days/year, p= 30,000/360 = 83.33 bottles/day

<u>Production Quantity Model:</u>

- Optimal Quantity= $\sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2*21{,}000*1{,}700}{1.25}}\sqrt{\frac{83.33}{83.33-58.33}} = 13{,}798.55 \approx 13{,}799\ bottles$
- Total Cost= $\frac{Q}{2p}(p-d)H + \frac{D}{Q}S = \frac{13{,}799}{2*83.33}(83.33-58.33)*1.25 + \frac{21{,}000}{13{,}799}(1{,}700) = \$\mathbf{5{,}174.64}$
- Number of runs/year = $\frac{D}{Q} = \frac{21{,}000}{13{,}799} \approx 1.52$

With a maximum storage space of 2500 bottles:

- Maximum inventory Level (I_{max}) = $\frac{Q}{p}(p-d) \Rightarrow 2500 = \frac{Q_{new}}{83.33}(83.33-58.33) \Rightarrow Q_{new}$ = 8,333.33 bottles.
- Total Cost: $TC = \frac{Q}{2p}(p-d)H + \frac{D}{Q}S = \frac{8{,}333.33}{2*83.33}(83.33-58.33)*1.25 + \frac{21{,}000}{8{,}333.33}(1{,}700) = \$5{,}846.5$
- **Total annual cost increases by $ 5,846.5- $ 5,174.64 = $671.9.**

Q 3.5*:* The Friendly Sausage Factory (FSF) can produce European Wieners at a rate of 2000 kg per week. FSF supplies wieners to local stores and restaurants at a steady rate of 150 kg per day. The cost to prepare the equipment for producing wieners is $50. Annual holding cost is $ 5 per kg of wiener. The factory operates throughout the year.

a) Determine the optimal run quantity, total annual inventory cost, the number of production runs per year.

b) If the factory has only enough storage space to hold a maximum of 300 kg in inventory, how will that effect the total inventory costs? (show with complete calculation)

<u>Solution:</u>

- Production Rate = 2000 kg/week = 104,000 kg/year (@52 weeks per year)
- Demand = 150 kg per day = 1050 kg/week = 54,600 kg/year (@7 days/week & 52 weeks/year)
- Holding Cost (H) = $5/kg/year
- Setup Cost (S) = $50/order
- $\frac{d}{p} = \frac{150}{285.7} \approx 0.525$

We will use the production quantity model:

- Optimal Run Quantity: $\sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}} = \sqrt{\frac{2*54{,}600*50}{5}}\sqrt{\frac{285.7}{285.7-150}}$ = 1,516.22 Kg
- Total Annual Cost: $\frac{Q}{2p}(p-d)H + \frac{D}{Q}S = \frac{1{,}516.22}{2*285.7}(285.7-150)*5 + \frac{54{,}600}{1{,}516.22}*50 = \$\mathbf{3,601}$
- Number of runs/year = $\frac{D}{Q}$ = $\frac{54{,}600}{1{,}516.22} \approx 36$

b) **<u>With a maximum storage space of 300 Kg, we have</u>**

- Maximum inventory Level (I_{max}) = $\frac{Q}{p}(p-d) \Rightarrow 300 = \frac{Q_{new}}{285.7}(285.7-150) \Rightarrow \ \ Q_{new} = 631.57$
- Total Cost = $\frac{Q}{2p}(p-d)H + \frac{D}{Q}S = \frac{631.57}{2*285.7}(285.7-150)*5 + \frac{54{,}600}{631.57}*50$= \$5,072.5
- **Total annual cost increases by \$5,072.5 - \$ 3,601.03 = \$1471.52.**

Q 3.6: A large producer of household products purchases a glyceride used in one of its deodorant soaps from outside of the company. It uses the glyceride at a fairly steady rate of 40 pounds per month, and the company uses a 20% annual interest rate to compute holding costs. This chemical can be purchased from two suppliers, A and B. Supplier A sells the chemical for \$1.24 per pound regardless of the quantity ordered. Supplier B, however, offers a discount schedule shown in the table below. Assume that the cost of order processing for both the suppliers is \$150. Which supplier should be used and what should be the order quantity? Justify your answer with detailed calculation.

Quantity (pound)	*Price per Pound (\$)*
0 - 499	*1.30*
500 - 999	*1.20*
1000+	*1.10*

<u>Solution:</u>

- Demand (D) = 40*12 = 480 pounds/year
- Ordering Cost (S) = \$150/order

<u>Supplier A:</u>

- Carrying Cost (H) = 0.2*1.24 = \$0.248/pound/year

$$Q_o = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*150*480}{0.248}} = 762\ pounds$$

$$TC_A = \frac{Q}{2}H + \frac{D}{Q}S + RD = \frac{762}{2}0.248 + \frac{480}{762}150 + 1.24*480 = \$784.176$$

<u>Supplier B:</u>

Range	*R*	*H*
0 - 499	1.30	*0.26*
500 - 999	1.20	*0.24*
1000+	1.10	*0.22*

<u>For R=1.3:</u>

$$Q_o = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*150*480}{0.26}} = 744.2\ pounds$$ <u>(This is not realizable and > 499, hence, ignore)</u>

<u>For R=1.2</u>

$$Q_o = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*150*480}{0.24}} = 774.597\ pounds\ \ \underline{\text{(Realizable)}}$$

$$TC\,(Q = 774.597) = \frac{Q}{2}H + \frac{D}{Q}S + RD = \frac{774.597}{2}0.24 + \frac{480}{774.597}150 + 1.2 * 480 = \$761.904$$

For R=1.1

$$Q_o = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2*150*480}{0.22}} = 809.04\ \ pounds\ \ \underline{\text{(Not realizable and <1000. Bump up to 1000)}}$$

$$TC\,(Q = 809.04) = \frac{Q}{2}H + \frac{D}{Q}S + RD = \frac{809.04}{2}0.22 + \frac{480}{809.04}150 + 1.1 * 480 = \$710$$

Conclusion: Since supplier B's cost of $710 is less than Supplier A's $784.176, we should use supplier B. The optimal order quantity is 1000 units.

Q 3.7: Pizza Pie runs the pizza concessions at the university basketball games. Personal size pizzas are assembled two hours prior to the game and cooked throughout the night in a mobile oven outside of the coliseum. Pizza sells for $5 and costs approximately $2 to make. Unsold pizzas at the end of the game are given to nearby dormitory students for a nominal $1 per pizza. Having sold out of pizza during the past two games, Pizza Pie is reevaluating its planning policy. Use the past demand data given below to determine how many pizzas should be made for each game.

No. of Pizza sold	*Frequency*
50	*15*
50	*15*
75	*30*
100	*20*
125	*10*
150	*10*

Solution

$C_s = 5\text{-}2 = 3$

$C_e = 2\text{-}1 = 1$

$$SL = \frac{C_s}{C_s + C_e} = \frac{3}{3+1} = 0.75$$

No. of Pizza sold	Frequency	Probability	Cumulative probability
50	15	0.15	0.15
50	15	0.15	0.30
75	30	0.3	0.6
100	20	0.2	0.8
125	10	0.1	0.9
150	10	0.1	1.0

SL = 0.75

Conclusion: They should make 100 pizzas for each game.