MATH 2008A – Intermediate Calculus – Fall 2013
Test 3

Date: 20 November 2013; 16:35–17:25
Instructor: Prof. L. Campbell

NAME: SOLUTIONS **ID#:** ________

This exam has 4 questions (total 20 marks). Answer all 4 questions
No calculators allowed.

1. Given that $xe^{xz} + yz = 0$, use implict differentiation to find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ and $\frac{\partial x}{\partial z}$. **6 marks**

$$F(x,y,z) = xe^{xz} + yz$$

$$F_x = (xz+1)e^{xz}, \quad F_y = z, \quad F_z = x^2e^{xz} + y$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = -\frac{(xz+1)e^{xz}}{x^2e^{xz}+y}$$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-z}{x^2e^{xz}+y}$$

$$\frac{\partial x}{\partial z} = \frac{-F_z}{F_x} = -\frac{(x^2e^{xz}+y)}{(xz+1)e^{xz}}$$

2. Find an equation of the tangent plane to the surface $F(x,y,z) = xe^{xz} + yz = 0$ at the point $(1,1,0)$. **4 marks**

$$F_x(1,1,0) = 1, \quad F_y(1,1,0) = 0, \quad F_z(1,1,0) = 2$$

Equation of tangent plane is:

$$1(x-1) + 0(y-1) + 2(z-0) = 0$$

$$\Rightarrow \quad x - 1 + 2z = 0$$

OR $\quad x + 2z = 1$

3. Find the directional derivative of the function $f(x, y) = x^2 - 2xy - y^2 - 8x - 4y$ at the point (0,-2) in the direction that makes an angle of $\theta = \pi/4$ with the positive x-axis. **5 marks**

$\nabla f = \langle 2x - 2y - 8, -2x - 2y - 4 \rangle$

$\nabla f(0, -2) = \langle 4 - 8, 4 - 4 \rangle = \langle -4, 0 \rangle$

The unit vector in the direction $\theta = \pi/4$ is $\vec{u} = \langle \cos \pi/4, \sin \pi/4 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$D_{\vec{u}} f(0, -2) = \langle -4, 0 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{-4}{\sqrt{2}}$

4. Find all the critical points of the function $f(x, y) = x^2 - 2xy - y^2 - 8x - 4y$. Use the second-derivative test to classify the point(s) (local maximum, local minimum or saddle point). **5 marks**

$f_x = 2x - 2y - 8 = 0 \Rightarrow x - y = 4$

$f_y = -2x - 2y - 4 = 0 \Rightarrow x + y = -2$

Add: $2x = 2 \Rightarrow x = 1 \Rightarrow y = -3$

Critical point $(1, -3)$.

$f_{xx} = 2$, $f_{xy} = -2$, $f_{yy} = -2$

$D(1, -3) = f_{xx}(1, -3) f_{yy}(1, -3) - (f_{xy}(1, -3))^2$

$= 2(-2) - 4 = -8 < 0$

So $(1, -3)$ is a saddle point