

MATH 2008A – Intermediate Calculus – Fall 2013
Test 2

Date: 23 October 2013; 16:35–17:25

Instructor: Prof. L. Campbell

NAME: SOLUTIONS

ID#: _____

This exam has 4 questions (for a total of 22 marks). Answer all 4 questions.
No calculators allowed.

1. Show that the following limit does not exist.

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy}{4x^2 + 3y^2 + z^2} = f(x,y,z)$$

5 marks

Along the x -axis, $f = 0 \rightarrow 0$ as $x \rightarrow 0$
 $y = z = 0$

Along the y -axis, $f = 0 \rightarrow 0$ as $y \rightarrow 0$
 $x = z = 0$

Along the z -axis, $f = 0 \rightarrow 0$ as $z \rightarrow 0$
 $x = y = 0$

Along $x = y = z$, $f = \frac{x^2}{4x^2 + 3x^2 + x^2} = \frac{1}{8} \rightarrow \frac{1}{8}$
as $x \rightarrow 0$

Along $x = y, z = 0$, $f = \frac{x^2}{4x^2 + 3x^2} = \frac{1}{7} \rightarrow \frac{1}{7}$
as $x \rightarrow 0$

Try any 2 directions that give different results, e.g. 0 and $\frac{1}{8}$, or 0 and $\frac{1}{7}$.

LIMIT DOES NOT EXIST

2. Sketch vertical and horizontal traces of

$$-x + 9y^2 + z^2 = 0.$$

Hence, sketch the surface in 3-D space.

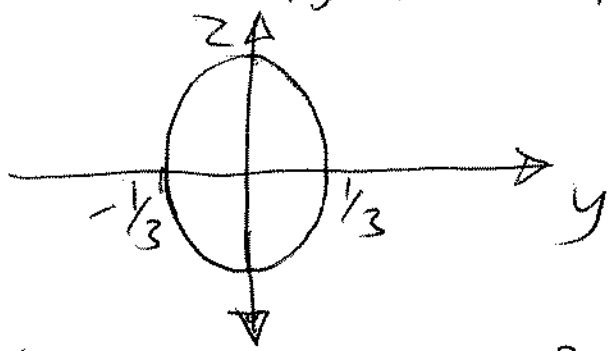
$$z=0 \Rightarrow x = 9y^2$$

$$z=k \neq 0 \Rightarrow x = 9y^2 + k^2$$

$$y=0 \Rightarrow x = z^2$$

$$y=k \neq 0 \Rightarrow x = z^2 + 9k^2$$

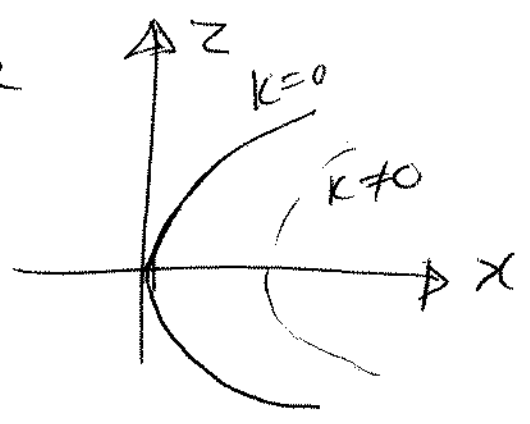
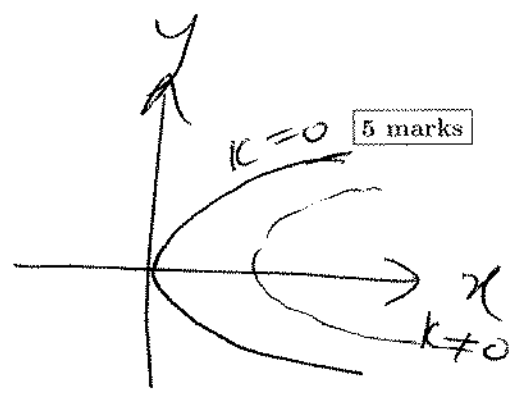
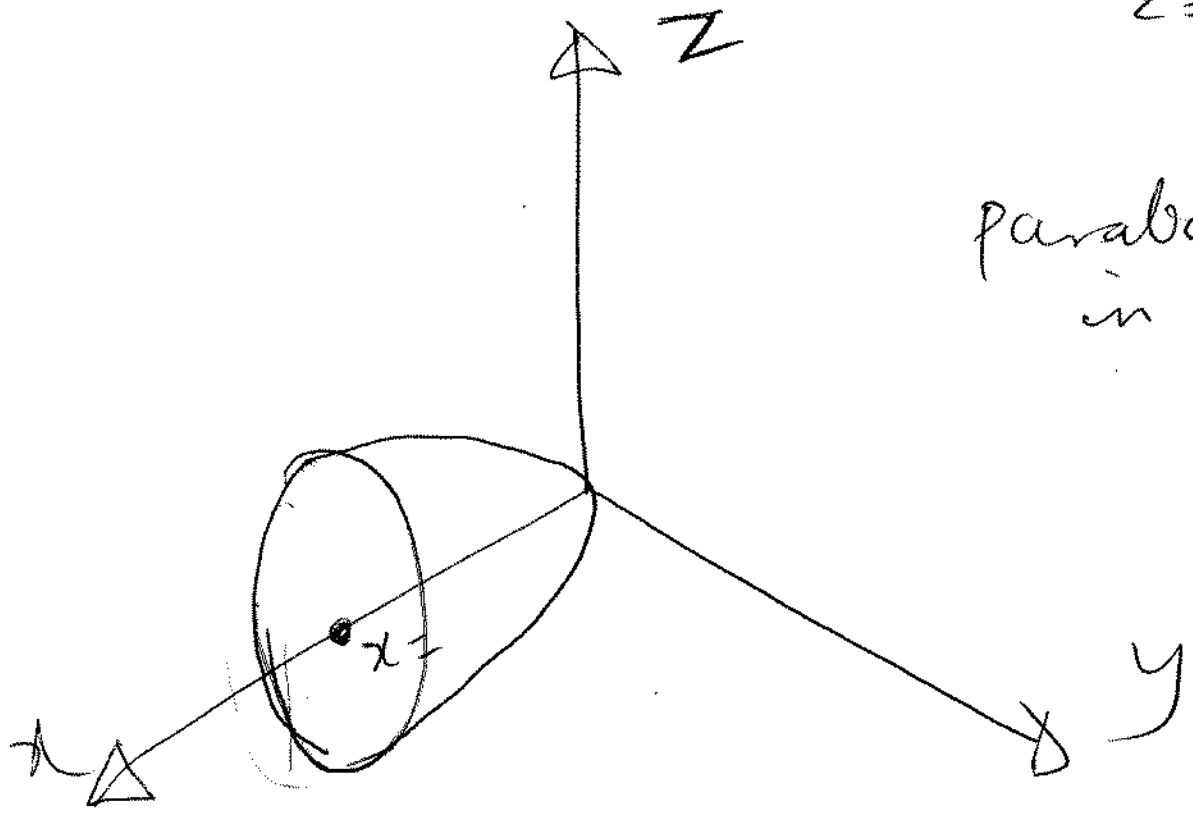
$$x=K \Rightarrow 9y^2 + z^2 = K$$



$$x=k=1 \Rightarrow 9y^2 + z^2 = 1 \quad \text{intercepts } y=0, z=\pm 1$$

$$z=0 \Rightarrow y=\pm 1/3$$

parabaloid
in 3-D
space



3. (a) Given that

$$\mathbf{r}'(t) = \langle 2e^t, \cos(t), \sqrt{t} \rangle \text{ and } \mathbf{r}(0) = \langle 1, 0, 1 \rangle,$$

find $\mathbf{r}(t)$.

5 marks

(Integrate

$$\vec{r}(t) = \langle 2e^t + c_1, \sin t + c_2, \frac{2}{3}t^{3/2} + c_3 \rangle$$

$$\langle 1, 0, 1 \rangle = \vec{r}(0) = \langle 2 + c_1, c_2, c_3 \rangle$$

$$c_1 = -1, c_2 = 0, c_3 = 1$$

$$\text{So } \vec{r}(t) = \langle 2e^t - 1, \sin t, \frac{2}{3}t^{3/2} + 1 \rangle$$

(b) Find the unit tangent vector to the curve given by the function $\mathbf{r}(t)$ at the point $(1, 0, 1)$.

2 marks

Tangent vector $\vec{r}'(0) = \langle 2, 1, 0 \rangle$

$$\text{Unit tangent vector} = \frac{\vec{r}'(0)}{|\vec{r}'(0)|}$$

$$= \frac{\langle 2, 1, 0 \rangle}{\sqrt{5}}$$

4. Given a curve in 3-D space defined by a vector-valued function $\mathbf{r}(t)$, the curvature at a point on the curve corresponding to t is

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

Consider the curve given by the function $y = x^2$. Write the function in parametric form as a vector-valued function $\mathbf{r}(t)$ and then use the above formula to find the curvature of the curve at the point $(1, 1)$.

5 marks

Write $y = x^2$ in parametric form as $\vec{r}(t) = \langle t, t^2, 0 \rangle$ fixed z ,

$$\vec{r}'(t) = \langle 1, 2t, 0 \rangle$$

$$\vec{r}''(t) = \langle 0, 2, 0 \rangle$$

$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2t & 0 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= \langle 0, 0, 2 \rangle$$

$$|\vec{r}' \times \vec{r}''| = 2$$

$$|\vec{r}'(t)| = \sqrt{1 + 4t^2}$$

The point $(1, 1)$ corresponds to $t = 1$

So curvature at $(1, 1)$

$$= \kappa(1) = \frac{2}{5^{3/2}}$$

Can be set to any constant. Set it to zero. and consider curve in the xy -plane.