École de gestion TELFER School of Management

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Winter 2013 | **ADM 2350** | **Prof. William F. Rentz**

FINANCIAL MANAGEMENT | **Prof. Yuri Khoroshilov**

Assignment #2 Solutions

GENERAL INSTRUCTIONS: Your assignment must be **sent electronically** in **doc, docx,** or **pdf** format **to the TUTOR for your section**. Your **tutor must RECEIVE** your assignment by **no later than noon on Friday, February 15, 2013**. Late assignments will **NOT** be accepted. To ensure that your tutor receives the assignment on time, **it is STRONGLY recommended that you electronically submit your assignment before midnight on the evening of Thursday, February 14, 2013 at the latest.** Unless there are system problems with doc-depot, the professors' solution set will be posted on doc-depot by no later than 6 PM of the due date. This assignment counts 5% of your course grade. You are encouraged to work on this assignment in teams of up to 5 students **from the same section of this course**. *However,* you may turn in an individual assignment if you prefer. Each assignment must be typed and contain the student name(s) and student number(s) on each page. A scanned statement of integrity must be electronically attached to each assignment (See pages 11-12 of the course syllabus). Each individual whose name appears on the assignment must sign the statement of integrity.

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1. (10 marks) Wilhem's Camera Shop, Inc., has two bond issues outstanding. The first issue has an annual coupon rate of **8%** and **20** years to maturity. The second has an annual coupon interest rate of **6.044%** and **8** years to maturity. Both issues are semi-annual pay bonds. **To the nearest whole percent**, what yield to maturity would result in the same current price for the bonds?

   HINT #1: Conceptually, the desired yield to maturity represents the crossover discount rate or Fisher rate that generates the same PV of future cash flows for the bonds.

   HINT #2: If you are **NOT** using a financial calculator, the approximate YTM formula may be useful in first finding the common current price B and then finding the desired YTM. Note that since it is an approximation formula, it also works for semi-annual pay bonds by ignoring the semi-annual pay feature.

$$YTM = \frac{\left[I + \left(\frac{(F \quad B)}{n} \right) \right]}{\left[\frac{(2B + F)}{3} \right]}$$

Scientific Calculator Approach:

Equate the YTM approximation formula for each bond and solve for the common current price B. Since the denominators are the same, one only needs to equate the numerators.

$$\$80 + \frac{\$1,000 \quad B}{20} = \$60.44 + \frac{\$1,000 \quad B}{8}$$

$$\$80 + \$50 \quad 0.05B = \$60.44 + \$125 \quad 0.125B$$

$$0.075B = \$55.44 \qquad B = \$739.20$$

Calculate the YTM of the 20-year bond.

$$YTM = \frac{\left[\$80 + \left(\frac{\$1,000 \quad \$739.20}{20} \right) \right]}{\left[\frac{(2 \times \$739.20) + \$1,000}{3} \right]} = \frac{\$93.04}{\$826.13333333} = 0.1126 \; or \; 11.26\%$$

Calculate the YTM of the 8-year bond as a check on one's previous calculations.

$$YTM = \frac{\left[\$60.44 + \left(\frac{\$1,000 \quad \$739.20}{8} \right) \right]}{\left[\frac{(2 \times \$739.20) + \$1,000}{3} \right]} = \frac{\$93.04}{\$826.13333333} = 0.1126 \; or \; 11.26\%$$

As we will show below, the approximation formula is too approximate here because the bonds are deep discount bonds. The formula is 74 basis points below the true YTM and when rounded to the nearest whole percent is 100 basis points (i.e. 11% instead of 12%) below the true YTM.

Scientific Calculator Marking Scheme:

2 marks for correctly substituting into the equated YTMs
2 marks for correctly calculating the approximate bond value
2 marks for correctly substituting into the YTM approximation formula for one of the bonds **GIVEN** your previously calculated bond value
2 marks for correctly calculating the approximate YTM **GIVEN** previously calculated bond value
2 marks for everything correct

Financial Calculator Approach:

Equate the bond values.

$$\sum_{t=1}^{40} \frac{\$40}{(1+k_b)^t} + \frac{\$1,000}{(1+k_b)^{40}} = \sum_{t=1}^{16} \frac{\$30.22}{(1+k_b)^t} + \frac{\$1,000}{(1+k_b)^{16}}$$

$$\sum_{t=1}^{15} \frac{(\$40 \quad \$30.22)}{(1+k_b)^t} + \frac{(\$40 \quad \$30.22 \quad \$1,000)}{(1+k_b)^{16}} + \sum_{t=17}^{39} \frac{\$40}{(1+k_b)^t} + \frac{(\$40+\$1,000)}{(1+k_b)^{40}} = 0$$

Now use the cash flow menu to find the IRR, taking advantage of embedded annuities.

CF0 = 0, C01 = \$40 - \$30.22 = \$9.78, F01 = 15, C02 = \$9.78 - \$1,000 = - \$990.22, F02 = 1, C03 = \$40, F03 = 23, C04 = \$40 + \$1,000 = \$1,040, F04 = 1, IRR CPT = 6.00%. This is the effective semi-annual YTM. The nominal annual YTM = 2 x 6.00% = 12.00%.

Financial Calculator Marking Scheme:

2 marks for correctly equating the two bond values
2 marks for correctly rearranging the terms so that k_b is an IRR calculation
1 mark for correctly substituting into the cash flow registers for periods 1 – 15
1 mark for correctly substituting into the cash flow registers for period 16
1 mark for correctly substituting into the cash flow registers for periods 17 – 39
1 mark for correctly substituting into the cash flow registers for period 40
1 mark for correctly calculating the effective semi-annual YTM of 6.00%
1 mark for correctly calculating the nominal annual yield to maturity of 12.00%

2. (20 marks) The Dwindling Reserves Oil Company (DROC) expects to pay a **\$1,728.00** per share dividend **for fiscal 2013, fiscal 2014 and fiscal 2015**. Then DROC expects to pay a **\$2,488.32** per share dividend **for fiscal 2016 and fiscal 2017**. Thereafter, dividends are expected to **DECLINE** by a compound rate of **20%** per year. Investor's require a **20%** rate of return.

   a. (4 marks) What is a fair market price per share **to the nearest dollar** for DROC's stock at the **BEGINNING** of fiscal 2018?

$P_5 = D_6/(k_C - g) = (\$2,488.32 \times 0.8)/(0.20 + 0.20) = \$4.976.64$

1 mark for correctly formulating D_6
1 mark for calculating D_6 correctly
1 mark for correctly formulating P_5
1 mark for calculating P_5 correctly

   b. (4 marks) What is a fair market price per share **to the nearest dollar** for DROC's stock at the **BEGINNING** of fiscal 2016?

$P_3 = D_4/(1 + k_C) + (D_5 + P_5)/(1 + k_C)^2 = \$2{,}488.32/(1.20) + (\$2{,}488.32 + \$4{,}976.64)/(1.20)^2$

$P_3 = \$2{,}073.60 + \$5{,}184 = \$7{,}257.60$

1 mark for correctly formulating PV of dividends D_4 and D_5
1 mark for correctly formulating PV of P_5
1 mark for correctly calculating P_3 **GIVEN** your previously calculated P_5
1 mark for everything correct

NB. For those who used the annuity formula for the two dividends and then calculated the PV of P_5, one obtained $P_3 = \$3{,}801.60 + \$3{,}456 = \$7{,}257.60$.

c. (4 marks) What is a fair market price per share **to the nearest dollar** for DROC's stock at the **BEGINNING** of fiscal 2013?

$P_0 = D_1/(1 + k_C) + D_2/(1 + k_C)^2 + (D_3 + P_3)/(1 + k_C)^3$

$P_0 = \$1{,}728/(1.20) + \$1{,}728/(1.20)^2 + (\$1{,}728 + \$7{,}257.60)/(1.20)^3$

$P_0 = \$1{,}440 + \$1{,}200 + \$5{,}200 = \$7{,}840$

1 mark for correctly formulating PV of dividends D_1, D_2, and D_3
1 mark for correctly formulating PV of P_3
1 mark for correctly calculating P_0 **GIVEN** your previously calculated P_3
1 mark for everything correct

NB. For those who used the annuity formula for the three dividends and then calculated the PV of P_3, one obtained $P_0 = \$3{,}640 + \$4{,}200 = \$7{,}840$.

d. (4 marks) If an investor bought the stock at the **BEGINNING** of fiscal 2013 and sold it at the **BEGINNING** of fiscal 2014, what would be the investor's expected dividend yield, capital gains yield, and total yield?

Dividend yield = $D_1/P_0 = \$1{,}728/\$7{,}840 = 22.04\%$

Total yield = $k_C = 20\%$

Capital gains yield = Total yield – Dividend yield = 20% - 22.04% = - 2.04%

Marking Scheme:

1 mark for correct value for dividend yield
1 mark for correct value for capital gains yield
1 mark for correct total yield
1 mark for everything correct

e. (4 marks) If an investor bought the stock at the **BEGINNING** of fiscal 2017 and sold it at the **BEGINNING** of fiscal 2018, what would be the investor's expected dividend yield, capital gains yield, and total yield?

Note that the Gordon constant growth model can be used to calculate the price at the **BEGINNING** of fiscal 2017. This implies that g = - 20% = capital gains yield. Since the total yield must be 20%, the dividend yield must be 20% - (- 20%) = 40%.

Marking Scheme:

1 mark for correct value for dividend yield
1 mark for correct value for capital gains yield
1 mark for correct total yield
1 mark for everything correct

3. (25 marks) Richard and Sarah Woods want to buy a new home. The home that they wish to purchase is selling for **$500,000**. The CIBC is willing to provide Richard and Sarah with a **$400,000** mortgage. Richard and Sarah have accumulated $100,000 that they will use to pay the difference in the purchase price and the mortgage. The bank quotes them a nominal annual interest rate of **3.00 percent** that is based on **semi-annual compounding** for a **5-year term** and **30-year amortization period**. Since Richard and Sarah work for the University of Ottawa, they are both paid semi-monthly (i.e. twice a month). So, they elect to make semi-monthly payments. Thus, they will be making **EXACTLY 24** payments per year.

   a. (5 marks) What is the effective annual rate that the CIBC is offering on the mortgage? You need to calculate the effective annual rate in decimal form as 0.0xxxxxxx or in percentage form as x.xxxxxx% to ensure that you can calculate the semi-monthly payment in part c. to the nearest penny.

   b. (5 marks) What is the effective semi-monthly rate that the CIBC is offering? Calculate the effective semi-monthly rate in decimal form as 0.00xxxxxx or in percentage form as 0.xxxxxx% to ensure that you can calculate the semi-monthly payment in part c. to the nearest penny.

   c. (5 marks) To the nearest penny, what is the semi-monthly payment that Richard and Sarah must make?

   d. (5 marks) To the nearest penny, what will be the **PRINCIPAL OUTSTANDING** or **REMAINING BALANCE** after the **SECOND** payment is made?

The information below is from the Excel Loan Amortization Template in the Course Materials folder on doc-depot. See the Assignment #2 Solutions from last fall for alternative approaches to answering parts a. – d. and for the marking schemes.

LOAN AMORTIZATION SCHEDULE

INPUT DATA

$400,000.00	= PV = Loan amount or present value
3.000000%	= i_{nom} = Quoted interest rate or annual percentage rate (APR)

2	= m = # of compounding periods per year
24	= p = # of payment periods per year
30	= n = Amortization in years
5	= t = Loan term in years, t < or = n

OUTPUT DATA

$840.68	= PMT = Payment per period
$355,505.25	= BAL = Balance remaining at end of loan term
$100,881.60	= PAY = Total payments during loan term
$56,386.85	= INT = Total interest paid during term
$44,494.75	= PRN = Total principal paid during term
1.500000%	= $i_m = i_{nom}/m$ = Effective compounding period rate
3.022500%	= $i_{eff} = (1 + i_m)^m - 1$ = Effective annual rate
0.124149%	= $i_p = (1 + i_{eff})^{1/p} - 1$ = Effective payment period rate
2.979571%	= $i_{nom}^* = pi_p$ = Equivalent APR based on the pmt frequency p
720	= N = np = # of payment periods to amortize the loan
120	= T = tp = # of payment periods during term of the loan

Period	Total Payment	Interest Payment	Principal Payment	Remaining Principal
0				$400,000.00
1	$840.68	$496.60	$344.08	$399,655.92
2	$840.68	$496.17	$344.51	$399,311.41

e. (5 marks) Before purchasing their home, both Richard and Sarah become employees of the Government of Canada. Since they now will both be paid biweekly (i.e. once every two weeks), they elect to change the frequency of mortgage payments to biweekly. That is, for simplicity, assume that they will make **EXACTLY 26** payments per year. Using the concept of compound interest but without actually doing any calculation, explain whether each biweekly payment will be more than, equal to, or less than (**24/26**) times the semi-monthly payment that you previously calculated.

If Richard and Sarah make semi-monthly payments, they are committed to total payments over the term of the loan of 5 x 24 x $840.68. Suppose Richard and Sarah pay biweekly with loan payments equal to (24/26) x $840.68. Then the total payments over the term of the loan would be 5 x 26 x (24/64) x $840.68 = 5 x 24 x $840.68, which is the same as the total payments made semi-monthly. However, when one pays MORE frequently (i.e. biweekly compared to semi-monthly), LESS interest accumulates between payments as you are reducing principal MORE often. Therefore, Richard and Sarah's blended payments (i.e. principal + interest) must be LESS than (24/26) times the semi-monthly payment because LESS total interest will now be paid over the term of the loan.

Marking Scheme:

5 marks credit for a reasonable discussion of why the biweekly payment must be LESS than (24/26) times the semi-monthly payment.

4. (20 marks) The stock of Box Technologies (BT) currently sells for **$110.00** per share. The dividend just paid, D_0, was **$10** per share, and dividends are currently growing at a compound rate of **10 percent**. The coupon interest rate is **4 percent** on newly issued **20-year** Government of Canada bonds selling at par. The expected return on the market portfolio for the coming year is **14 percent**. ST's current stock beta is **1.60**. BT is considering the following 3 investments:

   Investment #1:

   BT could buy Dragon Software (DS) by exchanging BT shares for DS shares. This would **LOWER** the expected growth rate of BT shares from **10 percent** to **7 percent** and its stock beta from **1.60** to **1.10**.

   Investment #2:

   BT could enter the consumer electronics market with a 3-D handicam. This would **RAISE** the expected growth rate of BT shares from **10 percent** to **12.5 percent** and its stock beta from **1.60** to **1.85**.

   Investment #3:

   BT could start producing industrial robots. This would **LOWER** the expected growth rate of BT shares from **10 percent** to **5 percent** and its stock beta from **1.60** to **0.60**.

   a. (8 marks) Assuming that BT wants to undertake at most one of these three investments, do you recommend the status quo (i.e. no investment) or one of the investments? **Provide a justification for your recommendation**.

Req. ret. DS = Risk-free rate + Market risk premium x Beta DS = 4% + 10% x 1.1 = 15%

Price DS = D_0(1+ Growth DS)/(Req. ret. DS – Growth DS) = ($10x1.07)/(0.15 – 0.07) = $133.75

Req. ret. 3D = Risk-free rate + Market risk premium x Beta 3D = 4% + 10% x 1.85 = 22.5%

Price 3D = D_0(1+ Growth 3D)/(Req. ret. 3D – Growth 3D)

Price 3D = ($10 x 1.125)/(0.225 – 0.125) = $112.50

Req. ret. IR = Risk-free rate + Market risk premium x Beta IR = 4% + 10% x 0.60 = 10%

Price IR = D_0(1+ Growth IR)/(Req. ret. IR – Growth IR)

Price IR = ($10 x 1.05)/(0.10 – 0.05) = $210

Your objective should be to maximize shareholder wealth. Operationally, this means to maximize the share price. This occurs with the industrial robots (IR) investment that leads to a share price of $210.

Marking Scheme:

1 mark total for correctly substituting into SML for **ALL** investments
2 marks total for correct values for cost of equity for **ALL** investments
1 mark total for correctly substituting into Gordon price formula for **ALL** investments
2 marks total for correct price results for **ALL** investments
2 marks for correct conclusion for which investment should be undertaken

b. (6 marks) What would the **growth rate** for BT shares have to be under the DS acquisition for BT to be indifferent between the DS acquisition and the status quo? (Do **NOT** consider the 3-D handicam or the industrial robot investments in this analysis.)

Since the DS acquisition leads to an increase in share price with the given growth rate of 7% for DS, the growth rate must be ***less*** than the given rate for one to be indifferent.

From the Assignment #2 Solutions for the Fall 2012, the appropriate formula is as follows:

Growth DS = [(Price status quo x Req. ret. DS) – D_0]/[D_0 + Price status quo]

Growth DS = [($110 x 0.15) – $10]/[$10 + $110] = $6.50/$120 = 5.4167%

1 mark for recognizing that ST share price for Investment #1 DS must be the $110 price under the status quo
1 mark for correctly rearranging Gordon constant growth model to solve for Growth DS
1 mark for correctly substituting into Gordon constant growth model **GIVEN** your previous results
1 mark for solving for correct value of Growth DS **GIVEN** your previous results
2 marks for **ALL** values correct

c. (6 marks) What would the **beta** for BT shares have to be under the 3-D handicam investment for BT to be indifferent between the 3-D handicam and the industrial robot investments? (Do **NOT** consider the DS acquisition or the status quo in this analysis.)

The price under the 3-D handicam investment would have to rise from $112.50 to the industrial robots price of $210 for one to be indifferent. For the price to rise, both the required rate of return and the beta must ***fall***.

First, use the Gordon constant growth model in yield form to find the required rate of return.

Req. ret. 3D = [D_0 x (1 + Growth 3D)]/[Price IR] + Growth 3D

Req. ret. 3D = [$10 x 1.125]/$210 + 0.125 = 5.3571% + 12.5% = 17.8571%

From the Assignment #2 Solutions for the Fall 2012, the beta formula is as follows:

Beta 3D = 3D risk premium/market risk premium

Beta 3D = (17.8571% - 4%)/(14% - 4%) = 13.8571%/10% = 1.38571

Marking Scheme:

1 mark for correctly substituting into the Gordon yield expression
1 mark for correct value of Required return 3D
1 mark for correctly substituting into the SML **GIVEN** your result for Req. ret. 3D
1 mark for correct value of Beta 3D **GIVEN** your result for Req. ret. 3D
2 marks if **ALL** values correct

5. (20 marks) The expected return of MM is **15 percent** and the expected return of NG is **25 percent**. Their standard deviations are **10 percent** and **20 percent**, respectively. The correlation coefficient between these two stocks is **0.20**.
   a. (5 marks) What is the **expected return** and **standard deviation** of a portfolio composed of **20 percent MM** and **80 percent NG**?

$$ER_P = (0.20x15\%) + (0.80 \quad 25\%) = 3\% + 20\% = 23\%$$

$$_P = \sqrt{(0.2^2 \quad 10\%^2) + (0.8^2 \quad 20\%^2) + (2 \quad 0.2 \quad 0.8 \quad 0.2 \quad 10\% \quad 20\%)}$$

$$_P = \sqrt{4\%^2 + 256\%^2 + 64\%^2} = \sqrt{324\%^2} = 18\%$$

Marking Scheme:

1 mark for the correctly substituting into the correct expression for ER_P
1 mark for the correct calculation of ER_P
1 mark for the correctly substituting into the correct expression for σ_P
1 mark for the correct calculation of σ_P
1 mark for everything correct

NB. Some students may have used the following expression for ER_P:

$$ER_P = ER_{NG} + w(ER_{MM} \quad ER_{NG}) = 25\% + (0.2)(15\% \quad 25\%) = 25\% \quad 2\% = 23\%$$

   b. (5 marks) What is the **expected return** and **standard deviation** of a portfolio composed of **80 percent MM** and **20 percent NG**?

$$ER_P = (0.80x15\%) + (0.20 \quad 25\%) = 12\% + 5\% = 17\%$$

$$_P = \sqrt{(0.8^2 \quad 10\%^2) + (0.2^2 \quad 20\%^2) + (2 \quad 0.8 \quad 0.2 \quad 0.2 \quad 10\% \quad 20\%)}$$

$$_P = \sqrt{64\%^2 + 16\%^2 + 64\%^2} = \sqrt{144\%^2} = 16\%$$

Marking Scheme:

1 mark for the correctly substituting into the correct expression for ER_P

1 mark for the correct calculation of ER_P
1 mark for the correctly substituting into the correct expression for σ_P
1 mark for the correct calculation of σ_P
1 mark for everything correct

c. (10 marks) What is the **expected return** and **standard deviation** of a portfolio composed of the **fraction w in MM** and **(1 – w) in NG** that has **minimum standard deviation**?

Derivation of Formula Approach:

To find the minimum variance portfolio, take the derivative of the variance with respect to the fraction of investment in asset MM and set the derivative equal to zero.

$$\frac{d(\ ^2_p)}{dw} = \frac{d}{dw}\left[w^2\ ^2_{MM} + (1\quad w)^2\ ^2_{NG} + 2w(1\quad w)\ _{MM,NG}\ _{MM}\ _{NG}\right]$$

$$\frac{d(\ ^2_p)}{dw} = 2w\ ^2_{MM}\quad 2(1\quad w)\ ^2_{NG} + 2(1\quad w)\ _{MM,NG}\ _{MM}\ _{NG}\quad 2w\ _{MM,NG}\ _{MM}\ _{NG} = 0$$

$$w\left[2\ ^2_{MM} + 2\ ^2_{NG}\quad 4\ _{MM,NG}\ _{MM}\ _{NG}\right] = 2\ ^2_{NG}\quad 2\ _{MM,NG}\ _{MM}\ _{NG}$$

$$w = \frac{^2_{NG}\quad _{MM,NG}\ _{MM}\ _{NG}}{\left[\ ^2_{MM} + \ ^2_{NG}\quad 2\ _{MM,NG}\ _{MM}\ _{NG}\right]} = \frac{20\%^2\quad (0.2\times 10\%\times 20\%)}{10\%^2 + 20\%^2\quad (2\times 0.2\times 10\%\times 20\%)}$$

$$w = \frac{360\%^2}{420\%^2} = 0.8571 \Rightarrow 1\quad w = 0.1429$$

The portfolio expected return is (0.8571)(15%) + (0.1429)(25%) = 16.429%. The portfolio variance is $(0.8571^2 \times (10\%)^2) + (0.1429^2 \times (20\%)^2) + (2 \times 0.8571 \times 0.1429 \times 0.2 \times 10\% \times 20\%) = 73.462041\%^2 + 8.168164\%^2 + 9.798367\%^2 = 91.428572$. The standard deviation is $\sqrt{}91.428572\%^2 = 9.5618\%$.

Note that this formula for w is a generalization of the formula demonstrated in class for the minimum standard deviation portfolio when the correlation coefficient is – 1. To see this, substitute – 1 for the correlation coefficient in the above formula and then simplify.

$$w = \frac{^2_{NG} + \ _{MM}\ _{NG}}{\left[\ ^2_{MM} + \ ^2_{NG} + 2\ _{MM}\ _{NG}\right]} = \frac{_{NG}\left(\ _{NG} + \ _{MM}\right)}{\left(\ _{MM} + \ _{NG}\right)^2} = \frac{_{NG}}{\left(\ _{MM} + \ _{NG}\right)}$$

Marking Scheme:

1 mark for realizing that the minimum standard deviation (i.e. minimum variance) portfolio can be found by taking the derivative of the variance expression with respect to w and setting the derivative equal to zero
1 mark for the correct expression for the variance of the portfolio

1 mark for correctly taking the derivative
1 mark for correctly finding the expression for w
1 mark for correctly substituting into the expression for w
1 mark for correctly calculating the value of w
1 mark for correctly calculating the value of (1-w) **GIVEN** the student's value for w
1 mark for correctly calculating the portfolio expected return **GIVEN** the student's value for w
1 mark for correctly calculating the portfolio standard deviation **GIVEN** student's value for w
1 mark for everything correct

NB. Some students may find the appropriate formula in another textbook or online. So long as the formula is correct students may earn full marks if they do the problem correctly.

Brute Force Excel Approach:

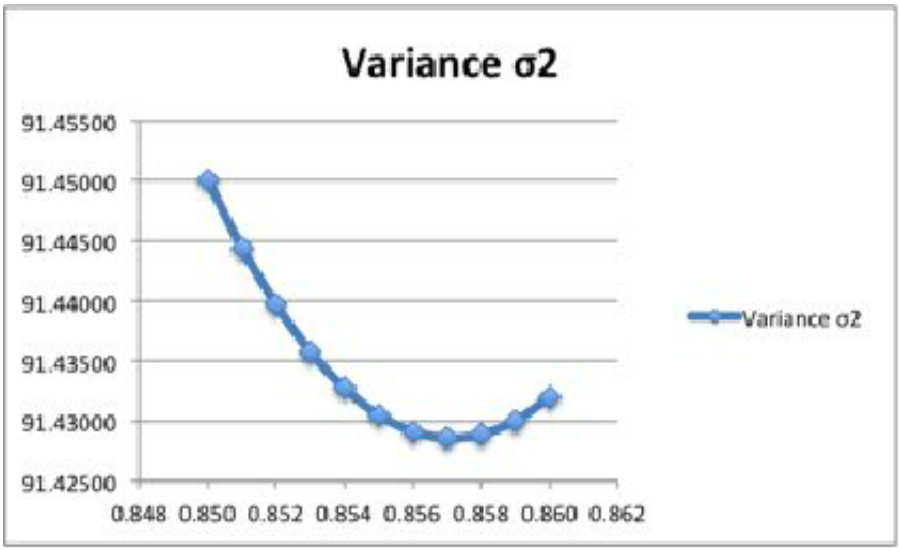
Instead of deriving the general expression for the minimum standard deviation portfolio, a student could try to find it by trial and error. The student would substitute different values of w into the variance expression.

A good starting point for the trial and error process is to calculate the optimal w for perfect negative correlation, *viz.*, $w = \sigma_{NG}/(\sigma_{MM} + \sigma_{NG}) = 20\%/(10\% + 20\%) = 2/3$. This weight represents a lower bound for w. That is, when the correlation coefficient is greater than minus one, we want to use more of the less risky asset and less of the more risky asset in the minimum variance (i.e. standard deviation) portfolio. This is because there are less risk reduction benefits by adding the risky asset when it is more correlated with the less risky asset. The minimum variance portfolio, of course, will no longer be zero when the correlation coefficient is greater than minus one.

One could vary the weight w in increments of 0.01 from 0.67 up to 1.00 using an Excel spreadsheet. If one does this, it appears that w is somewhere between 0.85 and 0.86. To further refine the result, one can go in 0.001 increments from 0.85 to 0.86. The table and graph below for the refined results show that the minimum variance of 91.42858 occurs at w = 0.857. This implies a standard deviation of $\sqrt{91.42858} = 9.5618\%$. The portfolio expected return is (0.857 x 15%) + (0.143)(25%) = 16.43%.

Weight w	Variance σ2
0.850	91.45000
0.851	91.44442
0.852	91.43968
0.853	91.43578
0.854	91.43272
0.855	91.43050
0.856	91.42912
0.857	91.42858
0.858	91.42888
0.859	91.43002

0.860 91.43200



Marking Scheme:

If a student finds the correct weights for the zero-variance portfolio with the correlation coefficient equal to minus one, then the student at least shows some understanding of the concepts that are involved in this problem. The student can be awarded 4 marks.

To receive full marks, the student must calculate the weights w = 0.857 and 1 – w = 0.143.

If the student's results are more approximate, say w is between 0.85 and 0.86, then up to 9 out of 10 marks can be awarded.

Using the Solver Add-In of Excel:

A faster way to solve this problem with Excel is to use the Solver Add-In. Enter the information below in cells A1 … D4 to use this approach. Note that the cell C4 is displaying the results for the variance calculation. What is entered into the cell C4 is the variance formula in terms of the sigmas, weight, and correlation coefficient based on the data in cells A1…A4. This formula is

=A4^2*(A1)^2+(1-A4)^2*(A2)^2+2*A4*(1-A4)*A3*A1*A2

	A	B	C	D
1	10.00	sigma MM		
2	20.00	sigma NG		
3	0.20	correlation coefficient		
4	0.67	weight	106.1380	variance

Invoke the Solver Add-In. Set cell C4 as the cell to minimize (i.e. the variance), set cell A4 as the cell allowed to vary, and restrict the value in cell A4 to be greater than or equal to zero (or better yet 0.67) but less than or equal to one.

6. (5 marks) The current price of a share of stock is **$50**. The price is expected to rise to **$55** in one year and pay an annual dividend of **$2** during the year. The **RF** is **5 percent** and the $\mathbf{ER_M}$ is **15 percent**. The beta of the stock is **1.50**. Determine whether the stock is overvalued, undervalued, or properly valued. That is, does the stock lie below, above, or on the **SML**?

Expected return = Dividend yield + Capital gains yield = $2/$50 + ($55 - $50)/$50

Expected return = 4% + 10% = 14%

Req. ret. = RF + Market risk premium x beta = 5% + (15% - 5%) x 1.5 = 20%

The expected return is ***less*** than what is required. Therefore, the stock is overvalued and plots below the SML.