



## Midterm Exam Summer 2013

ADM2302X

Last Name: \_\_\_\_\_ First Name: \_\_\_\_\_

Student #: \_\_\_\_\_

*This exam booklet contains 4 problems. If yours does not, please inform professor now. Please answer all questions in the exam booklet. Only answers in this exam booklet will be marked. Show all work.*

1. One page (8 ½ by 11 inches) review sheet, both sides, is allowed. Ruler allowed.
2. Calculator permitted for arithmetic use only.
3. **NO COMMUNICATION DEVICES** (computers, phones, etc.) **MAY BE WITHIN SIGHT.**

Question	Points	Out of
1		6
2		10
3		6
4		8
<b>TOTAL</b>		<b>30</b>

### Statement of Academic Integrity

The Telfer School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

### Statement to be signed by the student:

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: \_\_\_\_\_

Note: an examination copy or booklet without that signed statement will not be graded and will receive a midterm exam grade of zero.

1. (6 points) A company in Ottawa is trying to determine the most economical combination of sandwiches to make for a tennis club. The club has asked to company to provide 70 sandwiches in a variety that includes tuna, tuna and cheese, ham, ham and cheese, and cheese. The club has specified a minimum of 10 each of tuna and ham, and 12 each of tuna/cheese and ham/cheese. The company makes the sandwiches, using the following resources: bread, tuna, ham, cheese, mayonnaise, mustard, lettuce, tomato, packaging material, and labour hours.

The excel layout and LP sensitivity report for this problem are shown below.

	Tuna	Tuna/Cheese	Ham	Ham/Cheese	Cheese			
Number to make	10	30	10	12	8			
Cost	\$2.42	\$2.12	\$3.35	\$3.02	\$2.05	\$173.94		
<b>Constraints</b>								
Bread (slices)	2	2	2	2	2	140	<=	140
Tuna (gm)	110	80				3500	<=	4000
Ham (gm)			110	80		2060	<=	3000
Cheese (gm)		30		30	120	2220	<=	3000
Mayo (gm)	35	25	14	14	14	1520	<=	2000
Mustard (gm)			5	5		110	<=	250
Lettuce (gm)	7	7	7	7	7	490	<=	500
Tomato (gm)	14	14	14	14	14	980	<=	1000
Package (unit)	1	1	1	1	1	70	<=	70
Labour (Hours)	0.08	0.08	0.08	0.08	0.08	5.6	<=	10
Min total	1	1	1	1	1	70	>=	70
Min Tuna	1					10	>=	10
Min Tuna/Ch		1				30	>=	30
Min Ham			1			10	>=	10
Min Ham/Ch				1		12	>=	12
						<b>LHS</b>	<b>Sign</b>	<b>RHS</b>

#### Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$4	Number to make Tuna	10	0	2.42	1E+30	0.37
\$C\$4	Number to make Tuna/Cheese	30	0	2.12	1E+30	0.07
\$D\$4	Number to make Ham	10	0	3.35	1E+30	1.3
\$E\$4	Number to make Ham/Cheese	12	0	3.02	1E+30	0.97
\$F\$4	Number to make Cheese	8	0	2.05	0.07	2.05

#### Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$G\$17	Min total	70	2.05	70	0	8
\$G\$18	Min Tuna	10	0.37	10	4.545454545	6.5
\$G\$19	Min Tuna/Ch	30	0.07	30	6.25	8.666666667
\$G\$20	Min Ham	10	1.3	10	8	6.5
\$G\$21	Min Ham/Ch	12	0.97	12	8	8.666666667
\$G\$7	Bread (slices)	140	0	140	1E+30	0
\$G\$8	Tuna (gm)	3500	0	4000	1E+30	500
\$G\$9	Ham (gm)	2060	0	3000	1E+30	940
\$G\$10	Cheese (gm)	2220	0	3000	1E+30	780
\$G\$11	Mayo (gm)	1520	0	2000	1E+30	480
\$G\$12	Mustard (gm)	110	0	250	1E+30	140
\$G\$13	Lettuce (gm)	490	0	500	1E+30	10
\$G\$14	Tomato (gm)	980	0	1000	1E+30	20
\$G\$15	Package (unit)	70	0	70	1E+30	0
\$G\$16	Labour (Hours)	5.6	0	10	1E+30	4.4

The following questions represent changes made separately to the original problem (i.e. each change is applied to original problem only). Answer the following questions, each of which is independent of the others.

- a) What is the range over which the quantity of tuna could vary without changing the combination of binding constraints? (2 points)

The quantity of tuna can vary between 3500 grams and infinity.

- b) After the sandwiches are made, how many labour hours remain? (2 points)

4.40 hours of labour remain.

- c) What would be the impact on the sandwich-making plan and total cost if unit cost for tuna/cheese decreased to \$2.00 .

The decrease to \$2.00 is beyond the allowable decrease. Allowable decrease is \$0.07 and the decrease to \$2.00 is a decrease of \$0.12. Therefore, we cannot evaluate the impact of this change with the current report.

- d) What would be the impact on the sandwich-making plan and total cost if quantity of tuna available decreased to 3600 grams?

The 400 gram decrease is within the allowable decrease. The production mix will not change, albeit with different values for the non-zero decision variables. There would be no impact on the production plan or cost because the shadow price for this constraint is 0.

- e) What would be the impact on the sandwich-making plan and total cost if the company is required to deliver a minimum of 65 sandwiches?

The 5 unit decrease in total sandwiches is within the allowable decrease. The production mix will not change, albeit with different values for the non-zero decision variables. The total cost decreases by  $\$2.05 \times 5 = \$10.25$

- f) What would be the impact on the sandwich-making plan and total cost if cost of ham sandwiches and cost of ham/cheese sandwiches each decreased by \$0.35?

First, we check the 100% rule:  $(0.35/0.13) + (0.35/0.97) = 0.63 \leq 1$ . The solution remains optimal, and total cost decreases by  $\$0.35 \times (10 + 12) = \$7.70$ .

- g) An additional kilogram of tuna can be purchased for \$5.00. Should this tuna be purchased? Explain your answer.

An additional Kg of tuna would not change the optimal solution (shadow price = 0). This deal is not worthwhile.

## 2. (10 points)

Sailco Corporation must determine how many sailboats should be produced during each of the next four quarters (one quarter = three months). The demand during each of the next four quarters is as follows: first quarter, 40 sailboats; second quarter, 60 sailboats; third quarter, 75 sailboats; fourth quarter, 25 sailboats. Sailco must meet demands on time. At the beginning of the first quarter, Sailco has an inventory of 10 sailboats. At the beginning of each quarter, Sailco must decide how many sailboats should be produced during that quarter. During each quarter, Sailco can produce up to 40 sailboats with regular-time labor at a total cost of \$400 per sailboat. By having employees work overtime during a quarter, Sailco can produce additional sailboats with overtime labor at a total cost of \$450 per sailboat.

At the end of each quarter (after production has occurred and the current quarter's demand has been satisfied), a carrying or holding cost of \$20 per sailboat is incurred.

Use linear programming to determine a production schedule to minimize the sum of costs during the next four quarters. **DO NOT SOLVE.**

$x_t$  = number of sailboats produced by regular-time labor (at \$400/boat) during quarter  $t$  ( $t = 1, 2, 3, 4$ )

$y_t$  = number of sailboats produced by overtime labor (at \$450/boat) during quarter  $t$  ( $t = 1, 2, 3, 4$ )

$i_t$  = number of sailboats on hand at end of quarter  $t$  ( $t = 1, 2, 3, 4$ )

Total cost = cost of producing regular-time boats + cost of producing overtime boats + inventory costs  
 $= 400(x_1 + x_2 + x_3 + x_4) + 450(y_1 + y_2 + y_3 + y_4) + 20(i_1 + i_2 + i_3 + i_4)$

min  $z = 400(x_1 + x_2 + x_3 + x_4) + 450(y_1 + y_2 + y_3 + y_4) + 20(i_1 + i_2 + i_3 + i_4)$

s.t.

$x_1 \leq 40, x_2 \leq 40, x_3 \leq 40, x_4 \leq 40$

$i_1 = 10 + x_1 + y_1 - 40,$

$i_2 = i_1 + x_2 + y_2 - 60,$

$i_3 = i_2 + x_3 + y_3 - 75,$

$i_4 = i_3 + x_4 + y_4 - 25$

$i_t \geq 0, y_t \geq 0,$  and  $x_t \geq 0$  ( $t = 1, 2, 3, 4$ )

**3. (6 points)**

The Sweet smell Fertilizer Company markets bags of manure labeled “not less than 30 kilograms dry weight.” The packaged manure is a combination of compost and sewage. To provide good quality fertilizer, each bag should contain at least 10 kilograms of compost and not more than 20 kilograms of sewage. Each kilogram of compost costs Sweet Smell \$1.30 and each kilogram of sewage costs \$0.95.

- (a) Formulate algebraically this problem as a Linear Program to determine the least cost blend of compost and sewage in each bag (1 point).

Let  $X$  = number of kilograms of compost, in each bag,  $Y$  = number of kilograms of sewage in each bag.

Objective: Minimize cost =  $\$1.30X + \$0.95Y$

Subject to:

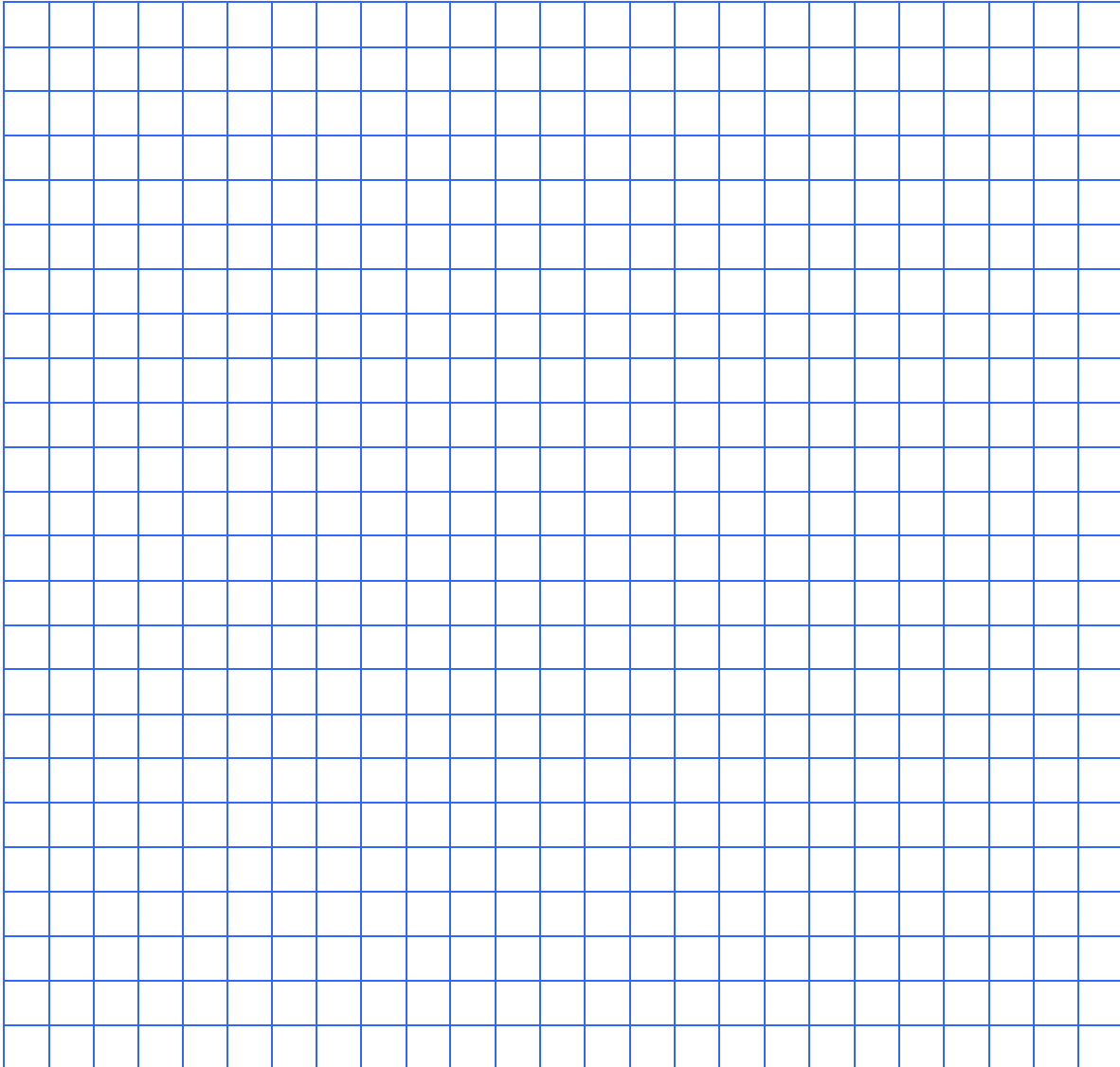
$X + Y \geq 30$  Kilograms per bag

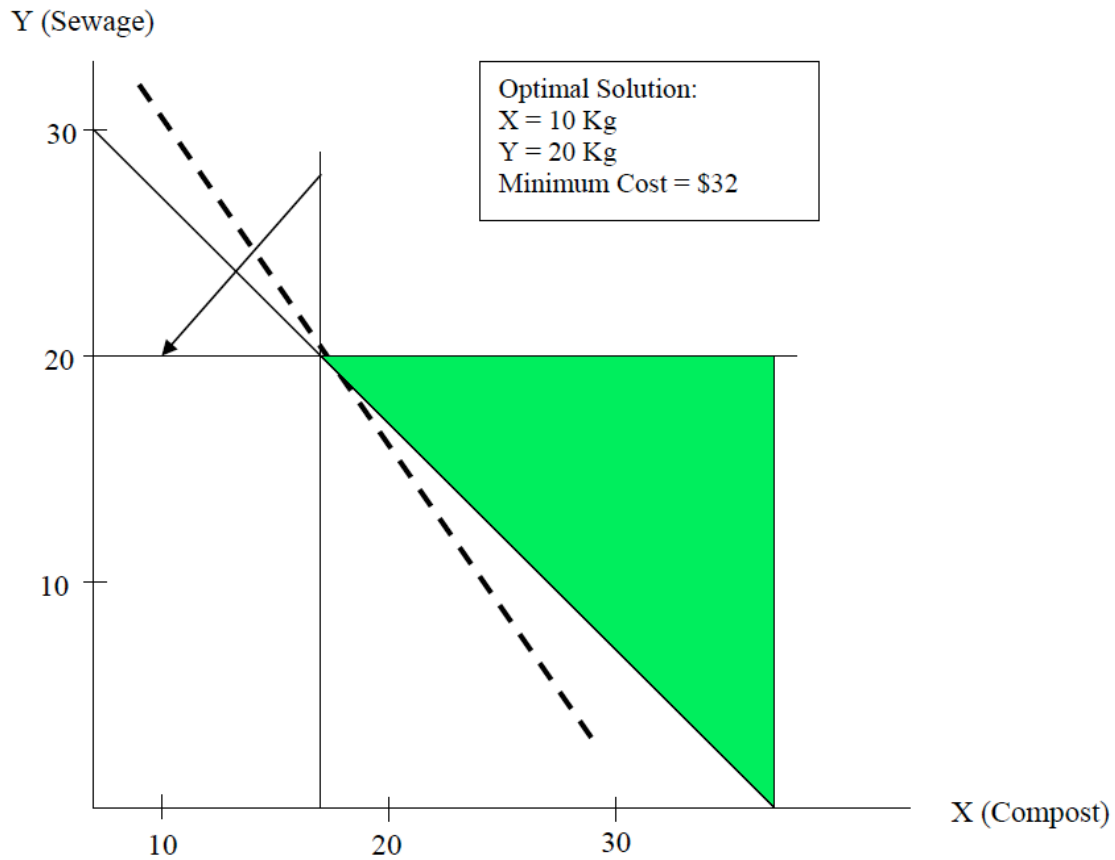
$X \geq 10$  Min compost, kg

$Y \leq 20$  Max sewage, kg

$X, Y \geq 0$  Non-negativity

- (b) Using the grid below, graph the constraint lines and mark them clearly with the numbers to indicate which line corresponds to which constraint. Darken the feasible region. Establish optimal values of the decision variables and objective function. (5 points)





**4. (8 points)**

Canning Transport is to move goods from three factories to three distribution centers. Information about the move is given below. Give the **network model** and the linear programming model for this problem. **DO NOT SOLVE.**

Source	Supply	Destination	Demand
A	200	X	50
B	100	Y	125
C	150	Z	125

Shipping costs are:

Source	Destination		
	X	Y	Z
A	3	2	5
B	9	10	--
C	5	6	4

(Source B cannot ship to destination Z)

$$\text{Min } 3X_{AX} + 2X_{AY} + 5X_{AZ} + 9X_{BX} + 10X_{BY} + 5X_{CX} + 6X_{CY} + 4X_{CZ}$$

$$\text{s.t. } X_{AX} + X_{AY} + X_{AZ} \leq 200$$

$$X_{BX} + X_{BY} \leq 100$$

$$X_{CX} + X_{CY} + X_{CZ} \leq 150$$

$$X_{AX} + X_{BX} + X_{CX} = 50$$

$$X_{AY} + X_{BY} + X_{CY} = 125$$

$$X_{AZ} + X_{CZ} = 125$$

$$X_{ij} \geq 0$$

