

Sample Variance: $s^2 = \frac{\sum(x_i - \bar{x})^2}{n - 1}$. Equivalent alternative formula: $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n - 1}$

Sample z -score: $z = \frac{x - \bar{x}}{s}$

If we transform the data using the linear transformation $x^* = a + bx$:

$$\bar{x}^* = a + b\bar{x}, s_{x^*} = |b|s_x, s_{x^*}^2 = b^2 s_x^2$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B).$$

The conditional probability of A, given B has occurred is $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Two events A and B are independent if and only if:

$$P(A|B) = P(A), P(B|A) = P(B), P(A \cap B) = P(A)P(B).$$

Expected Value and Variance of a Discrete Random Variable

$$E(X) = \mu = \sum xp(x), \sigma^2 = E[(X - \mu)^2] = \sum (x - \mu)^2 p(x).$$

$$\text{Alternative method: } E[(X - \mu)^2] = E(X^2) - [E(X)]^2.$$

Properties of Expectation and Variance

$$E(a + bX) = a + bE(X), \sigma_{a+bX}^2 = b^2 \sigma_X^2, \sigma_{a+bX} = |b| \sigma_X$$

If X and Y are two random variables then $E(X + Y) = E(X) + E(Y)$.

If X and Y are independent: $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$ and $\sigma_{X-Y}^2 = \sigma_X^2 + \sigma_Y^2$

Binomial and Poisson Distributions

If X is a binomial random variable then:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \text{ for } x = 0, 1, \dots, n. \binom{n}{x} = \frac{n!}{x!(n-x)!}. \mu = np, \sigma^2 = np(1-p).$$

Poisson distribution: $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \lambda = \mu = \sigma^2$.