**Department of Mathematics & Statistics**

| Course | Number | Section(s) |
|---|---|---|
| Mathematics | 209 | All Except EC |
| **Examination** | **Date** | **Pages** |
| Alternate | April 2013 | 3 |
| **Instructors** | | **Course Examiner** |
| ALL | | R. Raphael |

**Special Instructions**

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators are allowed.

*MARKS*

[6] **1.** Find the following limits:

(a) $\lim_{x \to t} (x^2 - \frac{7}{x})$ where t is a non-zero real number.

(b) $\lim_{x \to 5} \dfrac{x^2 - 3x - 10}{x - 5}$

(c) $\lim_{x \to 0} \dfrac{x^2}{(x^2 - 7)}$

[4] **2.** Find the derivative $f'(x)$ of the following (you do not need to simplify):

(a) $f(x) = 3x^6 - 7x^3 - 3$

(b) $f(x) = x^{\frac{-3}{2}} - x^{\frac{3}{4}}$

[10] **3.** Find $\dfrac{dy}{dx}$ (you do not need to simplify):

(a) $y = (x^3 + 2)^3(x^2 - 5)^2$

(b) $y = ln(x^3 + 2)$

(c) $y = \dfrac{e^x}{x^2 - 2}$

(d) $y = [5 - x\ ln(x)]^3$

[10] **4.** Given $f(x) = x^3 - x^2 + 5$

(a) Calculate the slope of the tangent line to the curve when $x = -3$.

(b) Find the equation of the tangent line to the curve when $x = -3$.

[10] **5.** For the function $f(x) = x^4 - 2x^3$ find:

*[Please list the following neatly]*

(a) the critical and inflection points;

(b) the intervals where $f(x)$ is increasing and decreasing;

(c) the intervals on which $f(x)$ is concave up and concave down;

(d) Use the above to sketch the graph of $f(x)$.

[10] **6.** Given the price-demand equation

$$0.02x + p = 60$$

(A) Express the demand $x$ as a function of the price $p$.

(B) Express the revenue $R$ as a function of the price $p$.

(C) Find the elasticity.

[6] **7.** Find the absolute extrema of the function $f(x) = x^3 + 3x^2 - 9x - 7$ on the interval $[-2, 2]$.

[4] **8.** If interest is compounded continuously, and the interest rate is 4.2%, how long will it take for the money invested to double?

[10] **9.** (a) Find $\dfrac{dy}{dx}$ from the implicit relationship $x^2 + y^2 = 25$.

(b) Find the equation of the tangent line to the graph of the above implicit function at the point (-3,-4).

(c) Sketch the curve and the tangent.

[10] **10.** Compute these antiderivatives:

(a) $\int (3x^4 - 3x^2 + 7)\,dx$

(b) $\int \left(\frac{1}{\sqrt{x}} - \frac{1}{x^3}\right)\,dx$

(c) $\int \frac{e^{3x}}{5 + e^{3x}}\,dx$

[10] **11.** Evaluate the integrals:

(a) $\int_0^1 \sqrt{4 + 4x}\;dx$

(b) $\int_3^5 \frac{5}{x + 5}\;dx$

(c) $\int_0^1 e^{5x}\;dx$

[10] **12.** Find the area between the graph of $f(x) = x^2 - 4x$ and the $x$-axis over the interval [-2, 3].