Midterm 2 markers:

Q1

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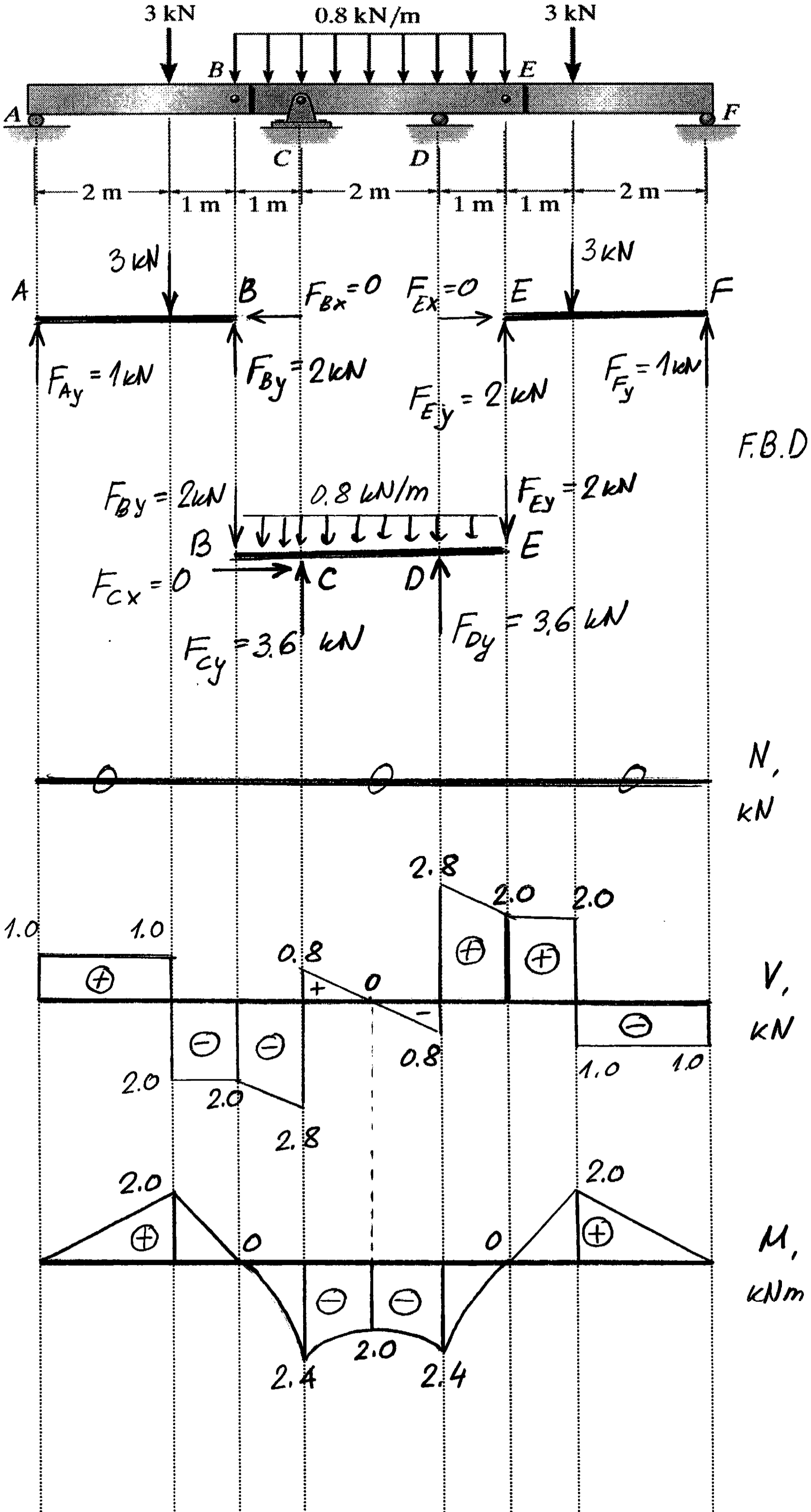
Question One (20 marks)

Draw the normal, shear and moment diagrams for the compound beam.
The three segments are connected by pins at B and E.



Parts AB and EF

$\Sigma M_B = 0: \; F_{Ay}(2.0+1.0) - 3 \times 1.0 = 0$

$F_{Ay} = 1$ kN

$\Sigma M_A = 0 \; F_{By}(2.0+1.0) - 3 \times 2.0 = 0$

$F_{By} = 2$ kN

Due to symmetry: $F_{Ey} = 2$ kN

$F_{Fy} = 1$ kN

No Ext. Horizontal loading: $F_{Bx} = 0$

$F_{Ex} = 0$

Part BCDE

$\Sigma M_D = 0: \; F_{Cy} 2.0 - 2(1.0+2.0) + 2 \times 1.0 - 0.8(1.0+2.0+1.0)(2.0/2) = 0$

$F_{Cy} = 3.6$ kN

Due to symmetry: $F_{Dy} = 3.6$ kN

No Ext. horizontal loading: $F_{Cx} = 0$

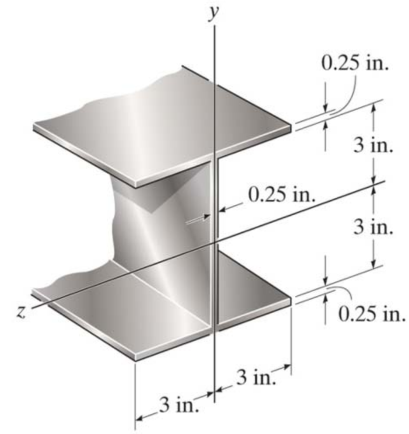
Check: $\Sigma F_y \stackrel{?}{=} 0$ O.K.

$3.6 + 3.6 - 2 - 2 - 0.8(1.0+2.0+1.0) = 0$

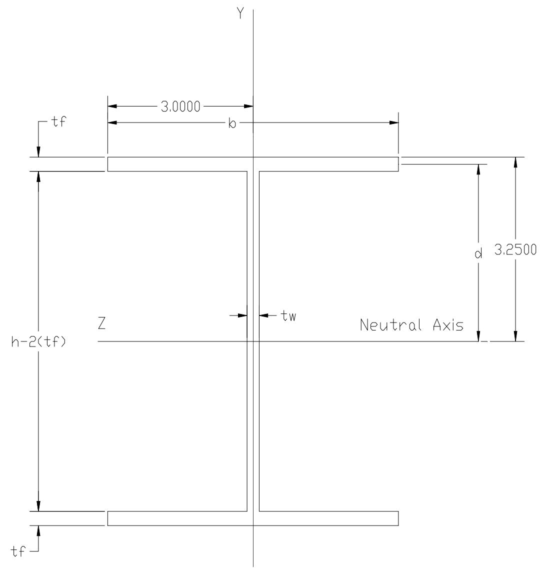
- Moment is equal to zero at pin supports A and F
- Moment is equal to zero at internal pins B and E
- Shear is equal to zero at midspan of CD, Moment is max at this point.

Question Two(20 marks)

A beam has the cross section shown. If it is made of steel that has an allowable stress of σ_{allow} = 24 ksi, determine the largest internal moment the beam can resist if the moment is applied
(a) about the z axis, (b) about the y axis.



Answer:



$$I = I(upper.flange) + I(lower.flange) + I(web)$$

[4 marks]

$$I_y = \left[\frac{t_f(b)^3}{12}\right]x2 + \frac{b(t_w)^3}{12} = \left[\frac{0.25(6)^3}{12}\right]x2 + \frac{6(0.25)^3}{12} = 9.0in^4$$

[4 marks]

$$I_z = \left[\frac{b(t_f)^3}{12} + A.d^2\right]x2 + \frac{b(t_w)^3}{12} = \left[\frac{6(0.25)^3}{12} + 6x0.25x(3.125)^2\right]x2 + \frac{0.25(6)^3}{12} = 33.8in^4$$

[6 marks]

$\sigma_y = \frac{M_y.z}{I_y}$ .... By reorganizing the equation we will have: $M_y = \frac{\sigma_y.I_y}{z} = \frac{24x9}{3} = 72kip.in = 6.0kip.ft$

[6 marks]

$\sigma_z = \frac{M_z.y}{I_z}$ ....By reorganizing the equation we will have: $M_z = \frac{\sigma_z.I_z}{y} = \frac{24x33.8}{3.25} = 249.6kip.in = 20.8kip.ft$

Where (z) is the farthest point in the section from the Neutral axis along the Y-axis
And (y) is the farthest point in the section from the Neutral axis along the Z-axis

Question Three (20 marks)

The resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 200$ N $\cdot$ m and is directed as shown. Determine (1) the maximum bending stress in the strut (2) The location y of the centroid C of the strut's cross-sectional area (3) specify the orientation of the neutral axis.

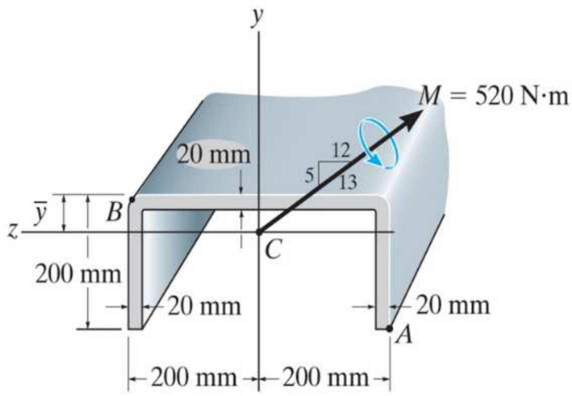


Figure: 06-37-04P06.106/107

Determine the location $\bar{y}$ of the centroid C of the strut's cross-sectional area (4 marks):

Method 1:

$$\begin{aligned}\bar{y} &= \frac{\sum \bar{y}A}{\sum A} \\ &= \frac{10(400)(20) + 2(110)(180)(20)}{400(20) + 2(180)(20)} \\ &= 57.37 \text{ mm}\end{aligned}$$

Method 2:

$$\begin{aligned}\bar{y} &= \frac{\sum \bar{y}A}{\sum A} \\ &= \frac{10(360)(20) + 2(100)(200)(20)}{360(20) + 2(200)(20)} \\ &= 57.37 \text{ mm}\end{aligned}$$

Determine the resultant internal moment components directed along the principle y and z axis (2 marks):

$$\begin{aligned}M_y &= \frac{5}{13}520 = 200 \text{ N} \cdot \text{m} \\ M_z &= \frac{-12}{13}520 = -480 \text{ N} \cdot \text{m}\end{aligned}$$

Determine the principle moment of inertia about the principle y and z axis (4 marks):

Method 1:

$$
\begin{aligned}
I_z &= \frac{1}{12}(400)(20)^3 + 400(20)(57.37-10)^2 + 2\frac{1}{12}(20)(180)^3 + 2(20)(180)(57.37-110)^2 \\
&= 57.6 \times 10^6 \text{ mm}^2 \\
&= 5.76 \times 10^{-5} \text{ m}^2
\end{aligned}
$$

$$
\begin{aligned}
I_y &= \frac{1}{12}(20)(400)^3 + 2\frac{1}{12}(180)(20)^3 + 2(20)(180)(190)^2 \\
&= 367 \times 10^6 \text{ mm}^2 \\
&= 3.67 \times 10^{-4} \text{ m}^2
\end{aligned}
$$

Method 2:

$$
\begin{aligned}
I_z &= \frac{1}{12}(360)(20)^3 + 360(20)(57.37-10)^2 + 2\frac{1}{12}(20)(200)^3 + 2(20)(200)(57.37-100)^2 \\
&= 57.6 \times 10^6 \text{ mm}^2 \\
&= 5.76 \times 10^{-5} \text{ m}^2
\end{aligned}
$$

$$
\begin{aligned}
I_y &= \frac{1}{12}(20)(360)^3 + 2\frac{1}{12}(200)(20)^3 + 2(20)(200)(190)^2 \\
&= 367 \times 10^6 \text{ mm}^2 \\
&= 3.67 \times 10^{-4} \text{ m}^2
\end{aligned}
$$

Determine the maximum bending stress in the strut (8 marks):

Determine the bending stress at point A: At point A, $y = \bar{y} - 200 = -142.6$ mm $= -0.1426$ m and $z = -200$ mm $= -0.2$ m, therefore

$$
\begin{aligned}
\sigma_A &= -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \\
&= -\frac{(-480)(-0.1426)}{5.76 \times 10^{-5}} + \frac{200(-0.2)}{3.67 \times 10^{-4}} \\
&= -1.30 \times 10^6 \text{ Pa} \\
&= -1.30 \text{ MPa}
\end{aligned}
$$

Determine the bending stress at point B: At point A, $y = \bar{y} = 57.37$ mm $= 0.05737$ m and $z = 200$ mm $= 0.2$ m, therefore

$$
\begin{aligned}
\sigma_A &= -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \\
&= -\frac{(-480)(0.05737)}{5.76 \times 10^{-5}} + \frac{200(0.2)}{3.67 \times 10^{-4}} \\
&= 5.87 \times 10^5 \text{ Pa} \\
&= 0.587 \text{ MPa}
\end{aligned}
$$

Therefore, point A undergoes the maximum bending stress at 1.30 MPa.

Specify the orientation of the neutral axis (2 marks):

The angle θ that the internal makes with the principal z axis is

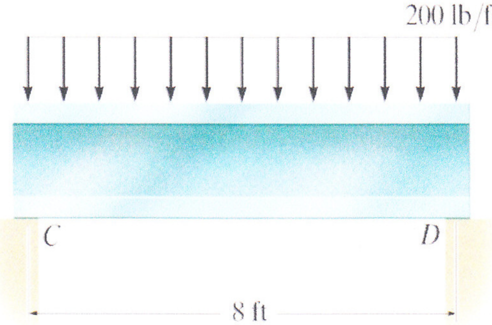
$$
\begin{aligned}
\theta &= \tan^{-1}(\frac{5}{-12}) \\
&= -22.6^\circ
\end{aligned}
$$

The angle α that the neutral axis makes with the principal z axis is

$$
\begin{aligned}
\alpha &= \tan^{-1}\left(\frac{I_z}{I_y}\tan\theta\right) \\
&= -3.74^\circ
\end{aligned}
$$

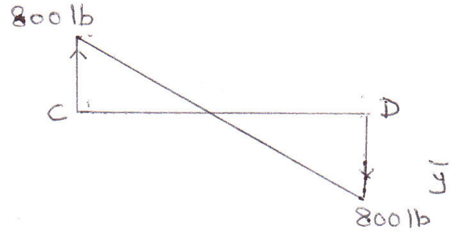
Question Four (20 marks)

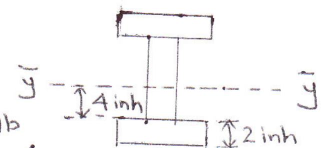
The beam is made from three plastic pieces glued together at the seams A and B.
If it is subjected to the loading shown, determine the shear stress developed in the glued joints at the critical section.
Note: The supports at C and D exert only vertical reactions on the beam.

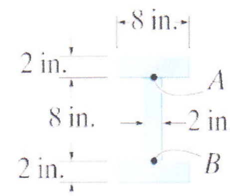


Solution:-

a) Shear force V at Supports C and D:-







∴ Max shear force V = 800 lb —— (5 Marks)

b) I about the neutral axis:-

$$I = \Sigma(I + Ad^2)$$
$$= \frac{1}{12} * 2 * (8)^3 + 2\left[\frac{1}{12} * 8 * (2)^3 + 2 * 8 * (5)^2\right]$$
$$= 896 \text{ inh}^4$$ —— (5 Marks)

c) Shear Stress at the Seams A and B:- —— (10 Marks)

$$\tau = \frac{VQ}{It}$$

$$Q = Q_A = Q_B = \Sigma \bar{y} A'$$
$$= 5 * 2 * 8$$
$$= 80 \text{ inh}^3$$

Therefore $\tau = \frac{800 * 80}{896 * 2} = 35.7 \text{ lb/inh}^2 =$

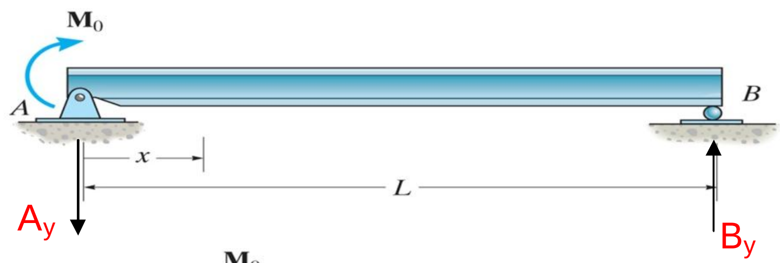
∴ Shear Stress at the Seams A and B = 35.7 Psi

Question Five (20 marks)

Determine the deflection at the center of the beam and the slope at B. EI is constant.

Solution No.1

"Slope & displacement by integration"

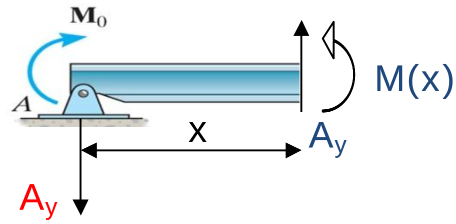


- **Support Reactions:**

$\sum M_B = 0$

3 $A_y . L - M_o = 0.0 \quad \rightarrow \quad A_y = \frac{M_o}{L}$

At distance (x)



$M(x) + \frac{M_o}{L} . x - M_o = 0.0$

$M(x) = M_o(1 - \frac{x}{L})$ *(Moment function)*

- **Moment Equation:**

$EI . \frac{d^2 v}{dx^2} = M(x) = M_0(1 - \frac{x}{L}) \qquad \rightarrow (1)$

5 *By Integration twice;*

$EI . \frac{dv}{dx} = \int_0^x \left(M_0 \left(1 - \frac{x}{L}\right)\right) . dx = M_0 \left(x - \frac{x^2}{2L}\right) + c_1 \qquad \rightarrow (2)$

$EI . v = \int_0^x \left(M_0 \left(x - \frac{x^2}{2L}\right) + c_1\right) . dx = M_0 \left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + c_1 x + c_2 \qquad \rightarrow (3)$

- **Applying Boundary Conditions:**

At $x = 0.0 \rightarrow v = 0.0$ (I)

At $x = L \rightarrow v = 0.0$ (II)

From (I) into eqn. (3)

$EI . (0.0) = M_0 \left(\frac{(0.0)^2}{2} - \frac{(0.0)^3}{6L}\right) + c_1(0.0) + c_2 \qquad \rightarrow c_2 = 0.0$

From (II) into eqn. (3)

4 $EI . (0.0) = M_0 \left(\frac{(L)^2}{2} - \frac{(L)^3}{6L}\right) + c_1(L) + 0.0$

$c_1(L) = M_0 \left(\frac{(L)^3}{6L} - \frac{(L)^2}{2}\right) \qquad \rightarrow c_1 = M_0 \left(\frac{L}{6} - \frac{L}{2}\right) = -\frac{M_0 . L}{3}$

Substitute with c_1 & c_2 into eqn. (3)

$EI . v = M_0 \left(\frac{x^2}{2} - \frac{x^3}{6L}\right) - M_0 \left(\frac{x . L}{3}\right) = M_0 \left(\frac{x^2}{2} - \frac{x^3}{6L} - \frac{x . L}{3}\right) \qquad \rightarrow (III)$

- **Deflection at the center of the beam at (C)**

At (C), x =L/2 into eqn. (III)

4 $EI . v = M_0 \left(\frac{\left(\frac{L}{2}\right)^2}{2} - \frac{\left(\frac{L}{2}\right)^3}{6L} - \frac{\left(\frac{L}{2}\right)L}{3}\right) = M_0 \left(\frac{L^2}{8} - \frac{L^2}{48} - \frac{L^2}{6}\right) = M_0 \left(\frac{6 L^2 - L^2 - 8L^2}{48}\right)$

$EI . v = M_0 \left(\frac{-3L^2}{48}\right) = \frac{-M_0 . L^2}{16} \rightarrow$

$$v = \frac{-M_0 . L^2}{16 \, EI}$$

- **Slope at (B), where (x=L)**

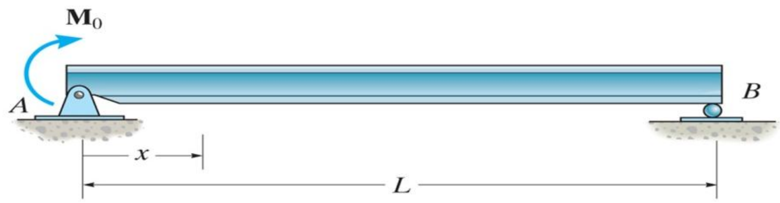
Substitute with c_1 into eqn. (2)

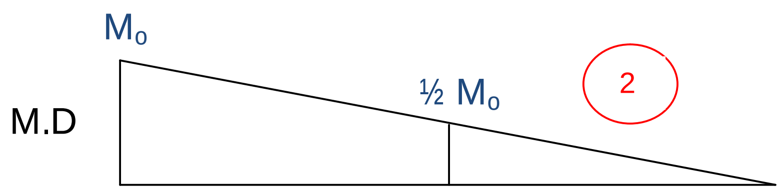
4 $EI . \frac{dv}{dx} = M_0 \left(x - \frac{x^2}{2L}\right) - \frac{M_0 . L}{3} = M_0 \left(x - \frac{x^2}{2L} - \frac{L}{3}\right) = M_0 \left(\frac{L}{6}\right)$

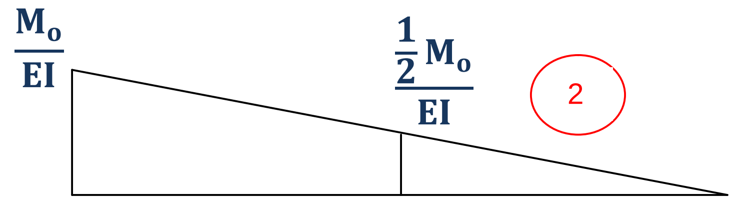
$$\theta_B = \frac{dv}{dx} = \frac{M_0 L}{6EI}$$

Solution No.2

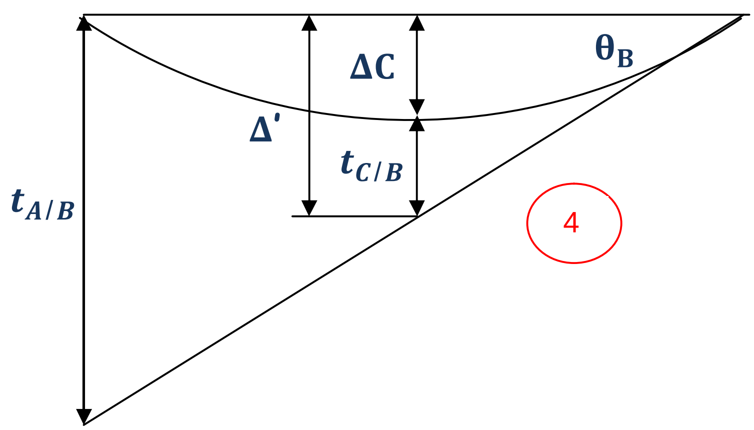
"Slope & displacement by the Moment-Area Method"







$\frac{M}{EI}$ **Diagrame**



2

- **From the Elastic Curve:**

$$\Delta c = \Delta' - t_{C/B}$$

$$\frac{\Delta'}{\left(\frac{L}{2}\right)} = \frac{t_{A/B}}{(L)} \rightarrow \Delta' = \frac{t_{A/B}}{2}$$

$$\Delta c = \frac{t_{A/B}}{2} - t_{C/B} \qquad \rightarrow (1)$$

5

- **From the Moment-Area Theorem**

$$t_{A/B} = \left(\frac{1}{3}L\right)\left[\frac{1}{2}.L.\left(\frac{M_o}{EI}\right)\right] = \frac{M_o.L^2}{6EI}$$

$$t_{C/B} = \left(\frac{1}{3}\left(\frac{L}{2}\right)\right)\left[\frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_o}{2EI}\right)\right] = \frac{M_o.L^2}{48EI}$$

Into eqn. (1)

$$\Delta c = \frac{1}{2}\left(\frac{M_o.L^2}{6EI}\right) - \left(\frac{M_o.L^2}{48EI}\right)$$

$$\boxed{\Delta c = \frac{M_o.L^2}{16EI}}$$

5

- **From the Elastic Curve**

The slope at (B)

$$\theta_B = \frac{t_{A/B}}{L} = \frac{\frac{M_o.L^2}{6EI}}{L}$$

$$\boxed{\theta_B = \frac{M_o.L}{6EI}}$$

By:

Osama Salem