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Last name: Solutions

First name:

Student number:

MAT 1348 — First Homework Assignment — Due Jan. 22, 2014 at 11:20am

Instructions: Show all relevant work to receive full credit. Submit a finished product, not a draft. You may write on both sides of the paper or insert additional pages if necessary. Please staple the pages. Submit the assignment to your TA in the DGD or in the appropriate submission box in the Department of Mathematics and Statistics. Late assignments will not be accepted.

[5]

1. Write each of the following statements as a compound proposition using correct logical connectives. You must clearly **define the propositional variables** you used in your compound propositions.

(1 pt each part)

(a) Sidewalks being sanded is a necessary condition for students to go to school.
Compound proposition: $g \to s$

(b) Sidewalks are sanded only if there has been freezing rain.
Compound proposition: $s \to f$

(c) Sidewalks are not sanded when roads have not been plowed or students do not go to school.
Compound proposition: $(\neg r \vee \neg g) \to \neg s$

(d) Students go to school if and only if the roads have been plowed or there has been no freezing rain.
Compound proposition: $g \leftrightarrow (r \vee \neg f)$

(e) Students do not go to school unless the roads are plowed and the sidewalks are sanded.
Compound proposition: $\neg(r \wedge s) \to \neg g$ or $\neg g \vee (r \wedge s)$

Your propositional variables:

s: "Sidewalks are sanded."
g: "students go to school."
f: "There is freezing rain."
r: "Roads are plowed."

[4]

2. Use a **truth table** to determine whether the given set of three propositions is **consistent**. Clearly explain what feature of the truth table supports your answer.

$$\{(\neg b \leftrightarrow a) \vee c, b \rightarrow \neg(c \wedge a), (b \rightarrow \neg c) \wedge \neg(b \vee a)\}$$

a b c	$(\neg b \Leftrightarrow a)$	$\vee c$	$b \Rightarrow$	$\neg$	$(c \wedge a)$	$(b \Rightarrow \neg c)$	$\wedge$	$\neg$	$(b \vee a)$
T T T	F	T	F	F	T	F	F	F	T
T T F	F	F	T	T	F	T	F	F	T
T F T	T	T	T	F	T	T	F	F	T
T F F	T	T	T	T	F	T	F	F	T
F T T	T	T	T	T	F	F	F	F	T
F T F	T	T	T	T	F	T	F	F	T
F F T	F	T	T	T	F	T	T	T	F
F F F	F	F	T	T	F	T	T	T	F

there is a truth assignment (a=F , b = F, c = T)
that makes all 3 compound propositions T.
Hence the set is consistent.

(3pts table, 1pt answer)

[5]

3. The table below is a truth table for two mystery compound propositions P and Q. Each of these consists of atomic propositions x and y, and logical connectives.

x	y	P	Q
T	T	F	T
T	F	T	F
F	T	F	T
F	F	F	T

(a) Write a **DNF formula** (Disjunctive Normal Form) for each of the compound propositions $\cancel{A}$ P and $\cancel{B}$. Q

(b) Write a formula for each of the compound propositions $\cancel{A}$ P and $\cancel{B}$ Q **using only logical connectives** $\neg$ **and** $\vee$.

(c) Write a formula for each of the compound propositions $\cancel{A}$ P and $\cancel{B}$ Q **using only logical connectives** $\neg$ **and** $\rightarrow$.

(a) $P = x \wedge \neg y$

$Q = (x \wedge y) \vee (\neg x \wedge y) \vee (\neg x \wedge \neg y)$

(b) $P \equiv \neg(\neg x \vee y)$

$Q \equiv (x \wedge y) \vee \neg x \equiv \neg(\neg x \vee \neg y) \vee \neg x$

(c) $P \equiv \neg(x \rightarrow y)$

$Q \equiv (\neg x \vee \neg y) \rightarrow \neg x \equiv (x \rightarrow \neg y) \rightarrow \neg x$

(1+2+2 pts)

[5]

4. *Fully justify your answers below.* Consider the following compound propositions in propositional variables a, b, c:

P: $(a \to c) \vee (b \vee c)$
Q: $\neg(a \to b) \to c$
R: $((a \vee b) \wedge (b \to c) \wedge (a \to c)) \to c$

(a) Determine for each of the compound propositions above whether it is a tautology, a contradiction, or a contingency.

(b) For each contingency, give all truth assignments of the propositional variables for which the compound proposition is **false**.

(c) Which pairs of compound propositions (if any) in the list above are **logically equivalent**?

* $P = (a \to c) \vee (b \vee c) \equiv \neg a \vee c \vee b \vee c \equiv \neg a \vee b \vee c$
* $Q = \neg(a \to b) \to c \equiv (a \to b) \vee c \equiv \neg a \vee b \vee c \equiv P$
* R is true when c is true; when c is false we have

a	b	c	$a \vee b$	$b \to c$	$a \to c$	$((a \vee b) \wedge (b \to c) \wedge (a \to c))$	$\to c$
T	T	F	T	F	F	F	T
T	F	F	T	T	F	F	T
F	T	F	T	F	T	F	T
F	F	F	F	T	T	F	T

Thus R is always true.

(a) P & Q are contingencies
R is a tautology

(b) both P & Q are F if and only if $a = T$, $b = F$, $c = F$

(c) $P \equiv Q$

(3pts answers, 2pt work)

[6]

5. On the Island of Knights and Knaves you meet two natives, A and B. Given their statements below, what can you determine about their identities? Fully explain your reasoning.

   (a) B says: "I am a knight only if A is a knave". What do you conclude?

   (b) A says: "Either I am a knight or B is a knight". What do you conclude?

   (c) In Case (b) above, what do you conclude if A later adds: "I am a knave or B is a knave"?

Define propositions:

p: "A is a knight."

q: "B is a knight."

(a) B says: $q \to \neg p$

p	q	$q \to \neg p$
T	T	F
T	F	T
F	T	T ←
F	F	T

compare

Since the truth values of q and $q \to \neg p$ must be equal, we conclude that A is a knave and q is a knight.

(b) A says: $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T ←
F	T	T
F	F	F ←

compare

Conclude: $\boxed{\text{B is a knave. A can be either}}$

(2 pts each part)

(c) A says: $p \oplus q$ and also $\neg p \vee \neg q$

p	q	$p \oplus q$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	T

Conclude:

A is a knight

B is a knave