THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
EECE 359 – Signals and Communications
Final Examination – 12. April 2005

This exam consists of 11 pages. Please check that you have a complete copy.

Time: 2 hrs and 30 min

SURNAME	First Name
Signature	Student ID

Problem	Points	max.
1		18
2		18
3		22
4		14
5		22
6		26
Total:		120

INSTRUCTIONS

1. All writings must be on the booklet provided.
2. Each candidate should be prepared to produce, upon request, his/her Library/AMS card.
3. Read and observe the following rules:
   No candidate shall be permitted to enter the examination room after the expiration of one–half hour, or to leave during the first half–hour of the examination.
   Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
   Caution – Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
   - Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.
   - Speaking or communicating with other candidates.
   - Purposely exposing written papers to the view of other candidates.

   The plea of accident or forgetfulness shall not be received.
4. Show all your work. Justify your answers. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.

Supporting Material

Notation

- $\delta(t)$ denotes the continuous-time unit impulse function.
- $u(t)$ denotes the continuous-time unit step function.
- $\delta[n]$ denotes the discrete-time unit impulse function.
- $u[n]$ denotes the discrete-time unit step function.

Formulas

- Continuous–time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{\mathrm{j}k\omega_0 t} \quad \overset{\mathcal{FS}}{\longleftrightarrow} \quad a_k = \frac{1}{T}\int_T x(t)\mathrm{e}^{-\mathrm{j}k\omega_0 t}\,dt$$

with $\omega_0 = 2\pi/T$ (T: Period).

- Discrete–time Fourier series

$$x[n] = \sum_{k=<N>} a_k \mathrm{e}^{\mathrm{j}k\omega_0 n} \quad \overset{\mathcal{FS}}{\longleftrightarrow} \quad a_k = \frac{1}{N}\sum_{n=<N>} x[n]\mathrm{e}^{-\mathrm{j}k\omega_0 n}$$

with $\omega_0 = 2\pi/N$ (N: Period).

- Continuous–time Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mathrm{j}\omega)\mathrm{e}^{\mathrm{j}\omega t}\,d\omega \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(\mathrm{j}\omega) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}\omega t}\,dt$$

- Discrete–time Fourier transform

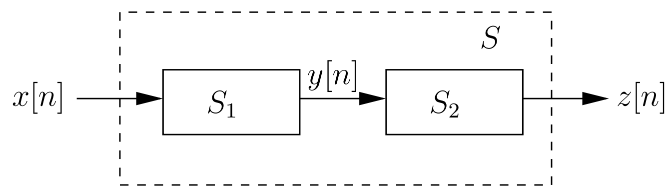
$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\mathrm{e}^{\mathrm{j}\omega})\mathrm{e}^{\mathrm{j}\omega n}\,d\omega \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad X(\mathrm{e}^{\mathrm{j}\omega}) = \sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-\mathrm{j}\omega n}$$

- Useful formulas

$$\begin{aligned}
\mathrm{e}^{\mathrm{j}t} &= \cos(t) + \mathrm{j}\sin(t) \\
\cos(t) &= \frac{1}{2}\left(\mathrm{e}^{\mathrm{j}t} + \mathrm{e}^{-\mathrm{j}t}\right) \\
\sin(t) &= \frac{1}{2\mathrm{j}}\left(\mathrm{e}^{\mathrm{j}t} - \mathrm{e}^{-\mathrm{j}t}\right) \\
\cos^2(t) &= \frac{1}{2}(1 + \cos(2t)) \\
\sin^2(t) &= \frac{1}{2}(1 - \cos(2t))
\end{aligned}$$

Problem 1 (18 marks)

In the following, we consider an "equalizer" with real–valued input signal $x[n]$. The equalizer S shown below is the cascade of two discrete–time systems S_1 and S_2.



The first system S_1 is *fully characterized* by its impulse response

$$h_1[n] = \sum_{k=K_1}^{K_2} a[k]\delta[n-k],$$

where K_1 and K_2 are constants with $K_1 < 0$ and $K_2 > 0$, and $a[k]$ are the real–valued impulse response coefficients.

The second system S_2 is characterized by the input–output relation

$$z[n] = \mathrm{sign}(y[n]),$$

where the signum function is defined as

$$\mathrm{sign}(m) = \begin{cases} +1 & m \geq 0 \\ -1 & m < 0 \end{cases}.$$

Consider first system S_1.

a) Sketch $h_1[n]$ for the special case $K_1 = -1$, $K_2 = 1$, and $a[k] = 1$ for $-1 \leq k \leq 1$.

Determine whether or not S_1 has the following properties. Justify all your answers.

b) linear

c) time invariant

d) causal

Now, consider the second system S_2. Determine whether or not S_2 has the following properties. Justify all your answers.

e) linear

f) memoryless

Consider now the overall system S. Determine whether or not S has the following properties. Justify all your answers.

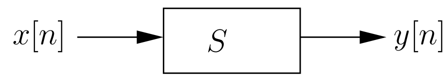
g) linear

h) stable

i) Is the output of S affected if the order of S_1 and S_2 is changed? Justify your answer.

Problem 2 (18 marks)

Consider the *linear time–invariant* (LTI) system S shown below.



A measurement campaign has shown that an input signal of the form

$$x[n] = \delta[n] + a\,\delta[n-1] + \frac{1}{4}\,\delta[n-2]$$

results in an output signal of the form

$$y[n] = \delta[n] - b\,\delta[n-1],$$

where a and b are constants.

a) Calculate the frequency response $H(e^{j\omega})$ of S.
[If you cannot solve this problem, use $H(e^{j\omega}) = (e^{j\omega} + b)/(\mathrm{e}^{j\omega} + a + \frac{1}{4}e^{-j\omega})$ in the following. Note that this is not necessarily the correct answer.]

b) Calculate the impulse response $h[n]$ of S for the special case $a = -1$ and $b = 1$.

c) Find the difference equation describing S for general a and b.

d) Draw a block diagram representation of S for general a and b.

e) For $a = 0$ and input signal $x[n] = \cos(\pi n/2)$ it can be shown that the output signal has the form

$$y[n] = C_1 \cos(\pi n/2) + C_2 \sin(\pi n/2).$$

Determine the constants C_1 and C_2 as functions of b.

f) For which values of a and b has the impulse response $h[n]$ of S the properties

$$\sum_{n=-\infty}^{\infty} h[n] = 1$$

and

$$\sum_{n=-\infty}^{\infty} (-1)^n\, h[n] = 0.$$

Note that it is not necessary to calculate $h[n]$ itself to solve this problem.

Problem 3 (22 marks)

This problem consists of three *independent* subproblems.

Subproblem 3.1

Consider the continuous–time signal

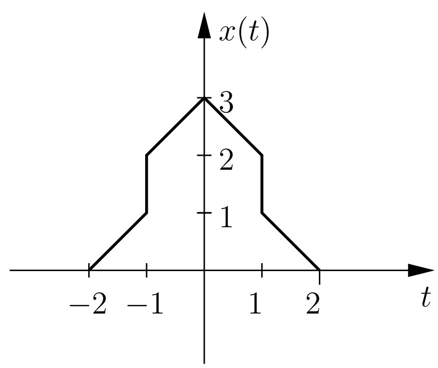
$$x(t) = \frac{\sin(2(t-1))\cos^2(3\pi t)}{\pi(t-1)}.$$

a) Calculate the Fourier transform $X(j\omega)$ of $x(t)$.

b) Sketch $|X(j\omega)|$.
[Note: $|X(j\omega)|$ denotes the absolute value of $X(j\omega)$.]

Subproblem 3.2

Calculate the Fourier transform $X(j\omega)$ of the continuous–time signal $x(t)$ shown below.



Subproblem 3.3

From a continuous–time signal $x(t)$ with Fourier transform $X(j\omega)$ only its energy density spectrum

$$|X(j\omega)|^2 = \begin{cases} 1-\omega^4 & |\omega| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

is known.

a) Calculate the energy of $x(t)$ given by

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt.$$

b) Calculate the magnitude of

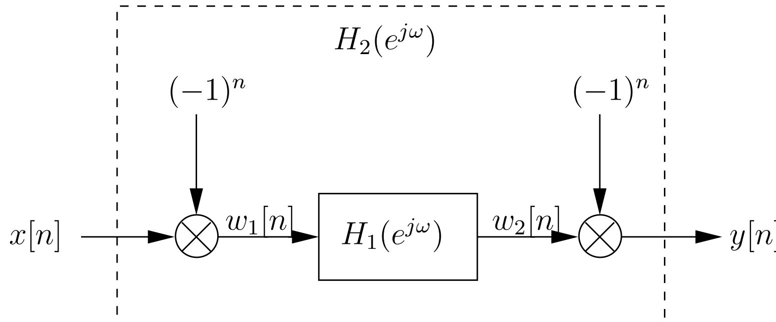
$$m = \int_{-\infty}^{\infty} x(t) \, dt.$$

c) The derivative of $x(t)$ is denoted by $y(t) = \frac{dx(t)}{dt}$. Calculate the energy of $y(t-1)$ given by

$$E_y = \int_{-\infty}^{\infty} |y(t-1)|^2 \, dt.$$

Problem 4 (14 marks)

Consider the following block diagram.



The spectrum $X(e^{j\omega})$ of $x[n]$ is defined by

$$X(e^{j\omega}) = \pi - |\omega|$$

for $|\omega| \leq \pi$ and the frequency response $H_1(e^{j\omega})$ is defined by

$$H_1(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \pi/4 \\ 0 & \pi/4 \leq |\omega| \leq \pi \end{cases}$$

a) Sketch $X(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

b) Sketch $H_1(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

c) What type of filter (low–pass, high–pass, or band–pass filter, etc.) corresponds to $H_1(e^{j\omega})$?

d) Determine the spectrum $W_1(e^{j\omega})$ of $w_1[n]$ as function of $X(e^{j\omega})$ and sketch $W_1(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

e) Determine the spectrum $W_2(e^{j\omega})$ of $w_2[n]$ as function of $X(e^{j\omega})$ and $H_1(e^{j\omega})$ and sketch $W_2(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

f) Determine the spectrum $Y(e^{j\omega})$ of $y[n]$ as function of $X(e^{j\omega})$ and $H_1(e^{j\omega})$ and sketch $Y(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

g) The resulting overall frequency response $H_2(e^{j\omega})$ is defined as $H_2(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$. Determine $H_2(e^{j\omega})$ as function of $H_1(e^{j\omega})$ and sketch $H_2(e^{j\omega})$ in the interval $-2\pi \leq \omega \leq 2\pi$.

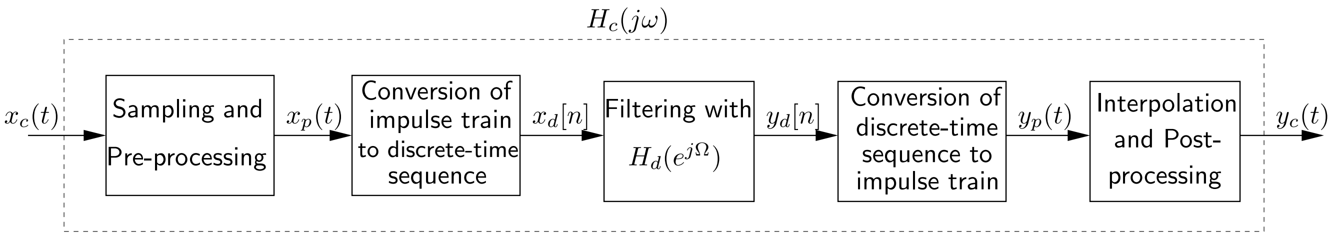
h) What type of filter (low–pass, high–pass, or band–pass filter, etc.) corresponds to $H_2(e^{j\omega})$?

i) How are the impulse responses $h_1[n] = \mathcal{F}^{-1}\{H_1(e^{j\omega})\}$ and $h_2[n] = \mathcal{F}^{-1}\{H_2(e^{j\omega})\}$ related? *[$\mathcal{F}^{-1}\{\cdot\}$ denotes inverse Fourier transform.]*

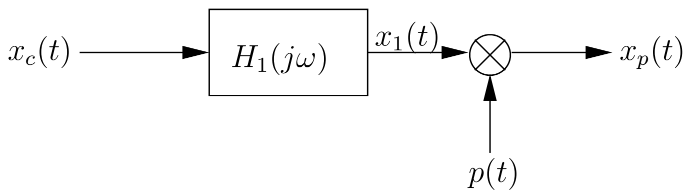
Problem 5 (22 marks)

Note that subproblems a) and j)–k) are completely independent from subproblems b)–i)!

For implementation of a continuous–time filter with frequency response $H_c(j\omega)$ the structure shown below is proposed.



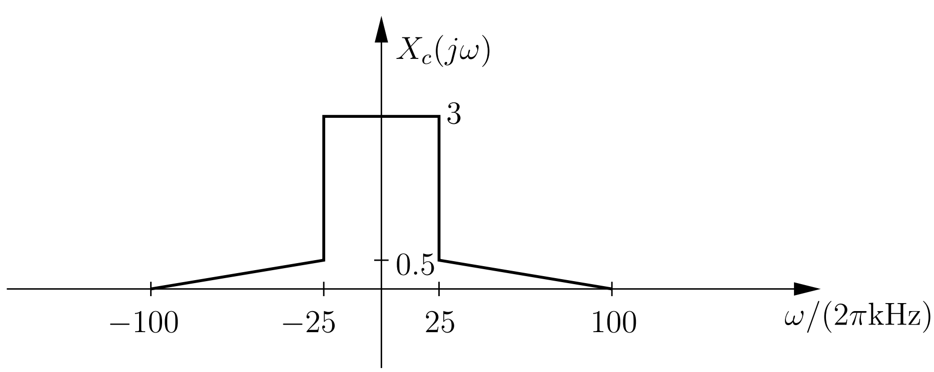
First, we consider the "Sampling and Pre–processing" unit which is shown more in detail below.



Since the bandwidth of $x_c(t)$ is not known, an ideal low–pass filter with frequency response $H_1(j\omega)$ is used to avoid aliasing and to simplify any subsequent processing. The output $x_1(t)$ of the low–pass filter is sampled by an impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ with a sampling period of $T = 10^{-5}$ seconds.

a) Determine the maximum cut–off frequency ω_1 of $H_1(j\omega)$ if aliasing is to be avoided.

In the following, we assume that $H_1(j\omega)$ has a cut–off frequency of $\omega_1 = 2\pi \cdot 25$ kHz and $H_1(0) = 1$. Furthermore, we assume that $x_c(t)$ has the spectrum shown below.



b) Sketch the spectrum $X_1(j\omega)$ of $x_1(t)$.

c) Sketch the spectrum $X_p(j\omega)$ of the impulse train sampled signal $x_p(t)$ in the interval $-2\pi \cdot 150$ kHz $\leq \omega \leq 2\pi \cdot 150$ kHz.

d) Sketch the spectrum $X_d(e^{j\Omega})$ of the discrete–time sequence $x_d[n]$ in the interval $-3\pi \le \Omega \le 3\pi$.

We first assume that the "Interpolation and Post–processing" unit is an ideal low–pass filter with frequency response $H_2(j\omega)$, cut–off frequency $\omega_2 = 2\pi \cdot 50$ kHz, and $H_2(0) = T$.

The spectrum $Y_c(j\omega)$ of the output signal $y_c(t)$ is given by

$$Y_c(j\omega) = \begin{cases} j3\omega & |\omega| < 2\pi \cdot 25\,\text{kHz} \\ 0 & \text{otherwise} \end{cases}$$

e) Sketch the frequency response $H_c(j\omega)$ of the continuous–time system and mark the frequency interval(s) where $H_c(j\omega)$ can be chosen arbitrarily.

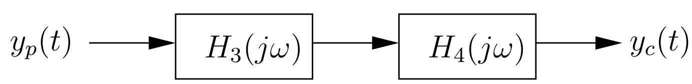
f) How are $x_1(t)$ and $y_c(t)$ related? Which time domain operation is implemented by the considered continuous–time filter?

g) Sketch the spectrum $Y_p(j\omega)$ of $y_p(t)$ in the interval $-2\pi \cdot 150$ kHz $\le \omega \le 2\pi \cdot 150$ kHz.

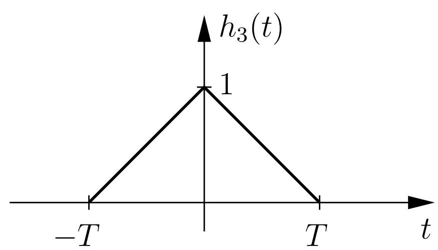
h) Sketch the spectrum $Y_d(e^{j\Omega})$ of the discrete–time output signal $y_d[n]$ in the interval $-3\pi \le \Omega \le 3\pi$.

i) Sketch the frequency response $H_d(e^{j\Omega})$ of the discrete–time system in the interval $-3\pi \le \Omega \le 3\pi$. Mark the frequency interval(s) where $H_d(e^{j\Omega})$ can be chosen arbitrarily.

In the following, we consider the "Interpolation and Post–processing" unit shown below.



The first filter is a first–order hold whose impulse response $h_3(t)$ is shown below.



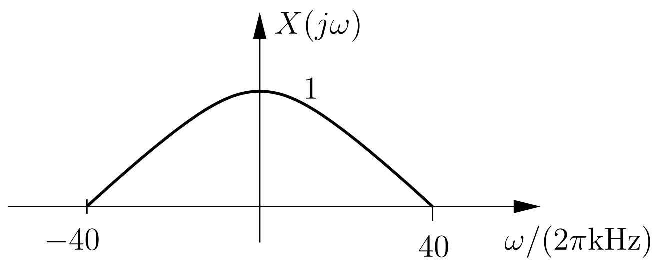
Since interpolation with a first–order hold results in a distortion of the output signal, an anti–imaging filter with frequency response $H_4(j\omega)$ is used.

j) Calculate the frequency response $H_3(j\omega)$ of the first–order hold.

k) Find the frequency response $H_4(j\omega)$ of the anti–imaging filter which results in ideal reconstruction. Note that in this case the cascade of first–order hold and anti–imaging filter is equivalent to an ideal low–pass filter. In which frequency interval(s) can $H_4(j\omega)$ be chosen arbitrarily?

Problem 6 (26 marks)

A second year UBC student in the PIP program has to design a wireless communications system for his project and seeks your advice. The spectrum $X(j\omega)$ of the message signal $x(t)$ is shown below.



Which of the following analog modulation schemes would you recommend for modulation of $x(t)$? Give special consideration to the practical feasibility of the demodulator considering the shape of $X(j\omega)$ and justify all your answers.

a) Double–sideband amplitude modulation (AM) with suppressed carrier

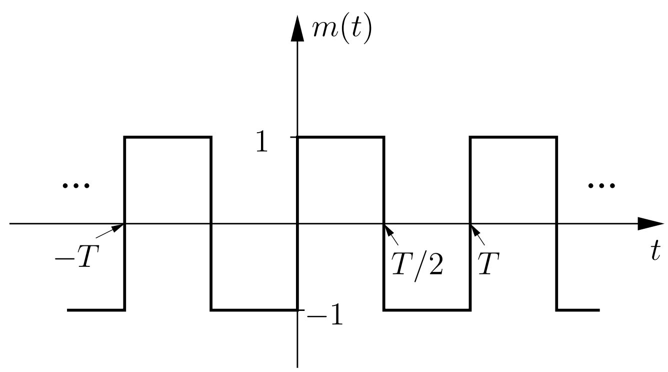
b) Double–sideband AM with carrier

c) Single–sideband AM

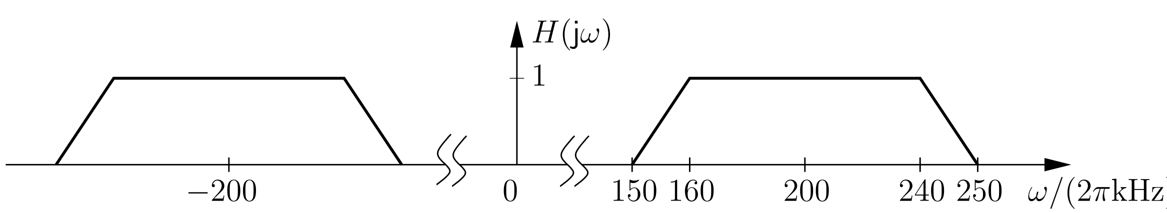
d) Vestigial–sideband AM

e) Frequency modulation

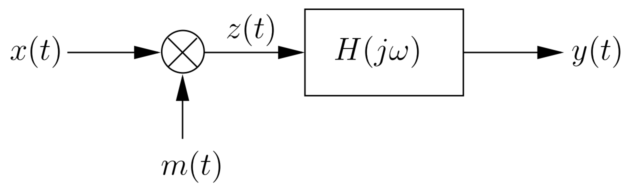
The desired carrier frequency in the PIP project is $\omega_c = 2\pi \cdot 200$ kHz. The PIP student has only a function generator which generates the waveform $m(t)$ shown below



and a non–ideal band–pass filter with frequency response $H(j\omega)$ at his disposal.



The PIP student builds a modulator using the structure shown below but is not sure if his design works and how he should choose the period T of $m(t)$.



f) Determine the Fourier series coefficients a_k of $m(t)$.

g) Give the spectrum $M(j\omega)$ of $m(t)$.

h) Determine the spectrum $Z(j\omega)$ of $z(t)$ as a function of $X(j\omega)$.

i) Sketch $|Z(j\omega)|$ and the magnitude $|Y(j\omega)|$ of the spectrum of the modulated signal $y(t)$ in the interval $-2\pi \cdot 700$ kHz $\leq \omega \leq 2\pi \cdot 700$ kHz for the special case $T = 5 \cdot 10^{-6}$ seconds.
[Note: $|Z(j\omega)|$ is the absolute value of $Z(j\omega)$.]

j) What type of modulation is realized by the chosen structure?

k) Determine all values of T which can be used for modulation of $x(t)$. Justify your answer.