

THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
EECE 359 – Signals and Communications
Final Examination – 22. April 2006

This exam consists of 11 pages. Please check that you have a complete copy.

Time: 2 hrs and 30 min

SURNAME	First Name
Signature	Student ID

Problem	Points	max.
1		15
2		21
3		19
4		10
5		18
6		23
7		19
Total:		125

INSTRUCTIONS

1. All writings must be on the booklet provided.
2. Each candidate should be prepared to produce, upon request, his/her Library/AMS card.
3. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half-hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

Caution – Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

 - Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.
 - Speaking or communicating with other candidates.
 - Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.
4. Show all your work. Justify your answers. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.

Supporting Material

Notation

- $\delta(t)$ denotes the continuous-time unit impulse function.
- $u(t)$ denotes the continuous-time unit step function.
- $\delta[n]$ denotes the discrete-time unit impulse function.
- $u[n]$ denotes the discrete-time unit step function.

Formulas

- Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \xleftrightarrow{\mathcal{FS}} a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

with $\omega_0 = 2\pi/T$ (T : Period).

- Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \xleftrightarrow{\mathcal{FS}} a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

with $\omega_0 = 2\pi/N$ (N : Period).

- Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \xleftrightarrow{\mathcal{F}} X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

- Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \xleftrightarrow{\mathcal{F}} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

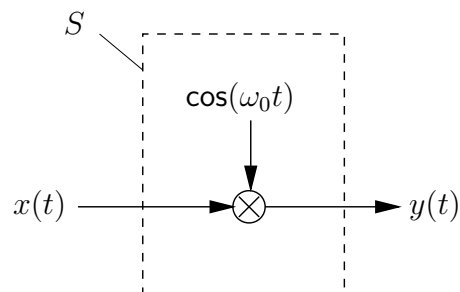
- Useful formulas

$$\begin{aligned} e^{jt} &= \cos(t) + j\sin(t) \\ \cos(t) &= \frac{1}{2} (e^{jt} + e^{-jt}) \\ \sin(t) &= \frac{1}{2j} (e^{jt} - e^{-jt}) \\ \cos^2(t) &= \frac{1}{2} (1 + \cos(2t)) \\ \sin^2(t) &= \frac{1}{2} (1 - \cos(2t)) \end{aligned}$$

Problem 1

(15 marks)

In the following, we consider the system S with input signal $x(t)$ and output signal $y(t)$ as shown below.



S is an amplitude modulator. Mathematically, $x(t)$ and $y(t)$ are related by

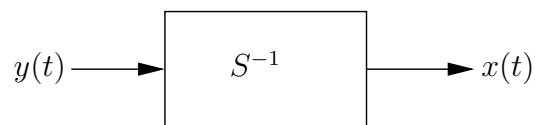
$$y(t) = \cos(\omega_0 t)x(t),$$

where $\omega_0 > 0$.

Determine whether or not the amplitude modulator S has the following properties. Justify all your answers.

- linear
- time-invariant
- memoryless
- causal
- stable

In the following, we assume that $x(t)$ is a speech signal with bandwidth $\omega_M = 2\pi \cdot 4$ kHz. Furthermore, we are interested in the inverse system S^{-1} of S .



- For which values of ω_0 does a stable inverse system for S exist?
- Sketch a block diagram of the stable inverse system S^{-1} for $\omega_0 = 2\omega_M$.

Problem 2

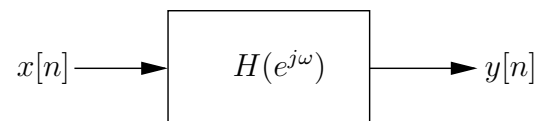
(21 marks)

We consider a linear time-invariant (LTI) discrete-time system S with frequency response

$$H(e^{j\omega}) = \frac{1}{(1 - \frac{1}{2}e^{-j\omega})(1 + \frac{1}{2}e^{-j\omega})}$$

- Calculate the impulse response $h[n]$ of system S .
- Is this system causal or non-causal? Justify your answer.
- Find the impulse response $h_i[n]$ of the inverse system S^{-1} .

In the following, $x[n]$ denotes the input of S and $y[n]$ is the output of S .



Sketch the input $x[n]$ and calculate and sketch the output $y[n]$ for each of the following cases.

- $x[n] = \delta[n - 1] - \frac{1}{2}\delta[n - 2]$
- $x[n] = \cos\left(\frac{\pi}{2}n\right)$
- $x[n] = \sum_{k=-\infty}^{\infty} (-1)^k \delta[n - k]$

Problem 3

(19 marks)

Given is a time-limited continuous-time signal $x(t)$ with $x(t) = 0$ for $|t| > 1$. The periodic signal $\tilde{x}(t)$ is defined as

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t - 2k).$$

- a) Sketch $x(t)$ and $\tilde{x}(t)$ for the special case $x(t) = t^2$ for $|t| \leq 1$.
- b) Show that the spectrum $\tilde{X}(j\omega)$ of $\tilde{x}(t)$ can be expressed as

$$\tilde{X}(j\omega) = \pi \sum_{k=-\infty}^{\infty} X(j\pi k) \delta(\omega - k\pi),$$

where $X(j\omega)$ denotes the spectrum of $x(t)$.

- c) Express the Fourier series coefficients a_k of $\tilde{x}(t)$ as a function of $X(j\omega)$.
- d) Calculate $X(j\omega)$, $\tilde{X}(j\omega)$, and a_k for the special case $x(t) = t^2$ for $|t| \leq 1$.
- e) Sketch $\tilde{X}(j\omega)$ for the special case $x(t) = t^2$ for $|t| \leq 1$.

Problem 4

(10 marks)

Given is the discrete-time signal

$$x[n] = \delta[n - 2] - \delta[n - 1] + 2\delta[n + 2] + \delta[n + 4].$$

a) Sketch $x[n]$.In the following, $X(e^{j\omega})$ denotes the Fourier transform of $x[n]$.b) Calculate $X(e^{j0})$.c) Calculate $\int_{\pi}^{3\pi} X(e^{j\omega}) d\omega$.d) Calculate $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$.e) Calculate $\int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega$.

Problem 5

(18 marks)

In the following, we consider a bandlimited Hilbert transformer S which is often used in single-sideband modulators. The frequency response of S is given by

$$H_c(j\omega) = \begin{cases} j & -\omega_M \leq \omega \leq 0 \\ -j & 0 < \omega \leq \omega_M \\ 0 & |\omega| > \omega_M \end{cases},$$

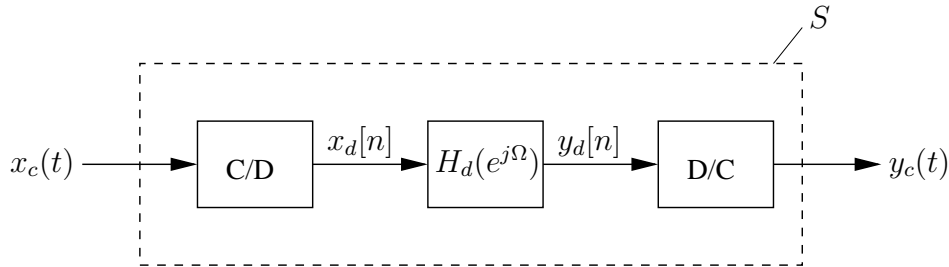
where ω_M is the bandwidth. The spectrum $X_c(j\omega)$ of the input signal $x_c(t)$ is given by

$$X_c(j\omega) = \begin{cases} 1 - \frac{|\omega|}{\omega_M} & |\omega| \leq \omega_M \\ 0 & |\omega| > \omega_M \end{cases}.$$

The output of S is denoted by $y_c(t)$ and its spectrum is $Y_c(j\omega) = \mathcal{F}\{y_c(t)\}$.

- Sketch $H_c(j\omega)$, $X_c(j\omega)$, and $Y_c(j\omega)$.
- Calculate the impulse response $h_c(t)$ of S .
- Calculate $x_c(t)$ and $y_c(t)$.

Since a continuous-time (analog) implementation of S might be too expensive, a discrete-time (digital) implementation is often preferred. A block diagram of such a digital implementation is shown below.



The digital implementation involves a continuous-time to discrete-time converter (C/D), a discrete-time filter having frequency response $H_d(e^{j\Omega})$, and a discrete-time to continuous-time converter (D/C). The input and output signals of the discrete-time filter are given by

$$x_d[n] = x_c(nT)$$

and

$$y_d[n] = y_c(nT),$$

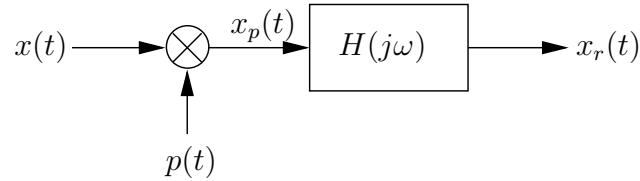
respectively, where T is the sampling rate of the C/D.

- Determine the maximum sampling rate T that allows processing without aliasing.
- Sketch $X_d(e^{j\Omega}) = \mathcal{F}\{x_d[n]\}$, $H_d(e^{j\Omega})$, and $Y_d(e^{j\Omega}) = \mathcal{F}\{y_d[n]\}$ in the interval $-2\pi \leq \Omega \leq 2\pi$ for a sampling frequency of $\omega_s = 2\omega_M$.
- Provide a more detailed block diagram of the D/C block. Explain the purpose of the different components of this block.

Problem 6

(23 marks)

The simplified model of a movie projector is shown below.



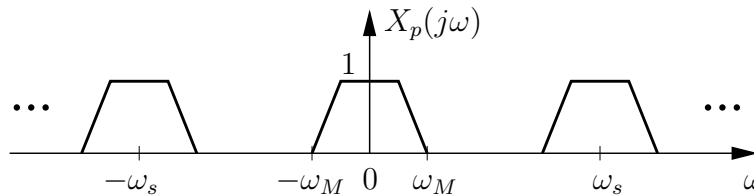
Here, $x(t)$ denotes a bandlimited continuous-time signal (movie), $H(j\omega)$ is the frequency response of the projector interpolation filter, and $x_r(t)$ is the reconstructed signal observed by the audience. $p(t)$ is an impulse train defined as

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT),$$

where T is the sampling rate. The impulse-train sampled signal is denoted by $x_p(t)$. We first consider general interpolation filters and denote their impulse response by $h(t) = \mathcal{F}^{-1}\{H(j\omega)\}$.

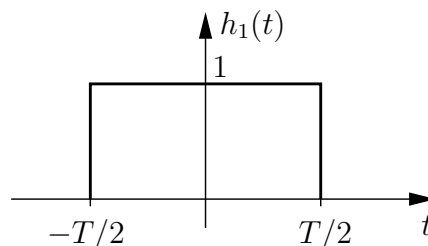
- Express $x_p(t)$ as a function of the samples of $x(t)$.
- Express $x_r(t)$ as a function of the samples of $x(t)$ and $h(t)$.
- What is the impulse response $h(t)$ of the ideal reconstruction filter?

The spectrum $X_p(j\omega)$ of $x_p(t)$ is shown below, where $\omega_s = 2\pi/T$ denotes the sampling frequency.



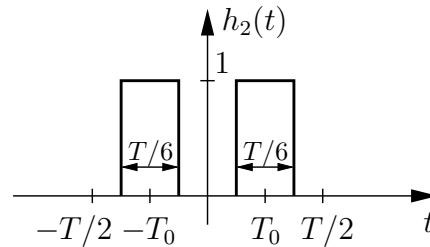
- Sketch the frequency response $H(j\omega)$ of the ideal reconstruction filter. Sketch also the spectrum $X_r(j\omega)$ of $x_r(t)$ for this case.

Next we consider a non-ideal reconstruction filter. The impulse response $h_1(t)$ of the filter is shown below.



- e) Is the reconstruction filter causal or non-causal? Justify your answer.
- f) Sketch the spectrum of $X_r(j\omega)$ in this case and explain the effects of the non-ideal filter in the frequency domain.

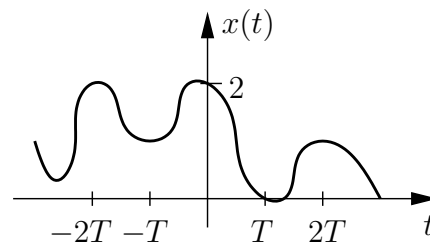
Now consider a second non-ideal reconstruction filter whose impulse response $h_2(t)$ is shown below.



T_0 , $\frac{T}{12} < T_0 < \frac{5T}{12}$, is a parameter which has to be selected properly.

- g) Calculate the frequency response $H_2(j\omega) = \mathcal{F}\{h_2(t)\}$ as a function of T_0 .
- h) It is desirable to suppress the first image of $X(j\omega)$ in $X_r(j\omega)$. This leads to the condition $X_r(\pm j\omega_s) = 0$. For which value of T_0 is this condition met?

Assume for the following that the signal $x(t)$ has the form shown below.



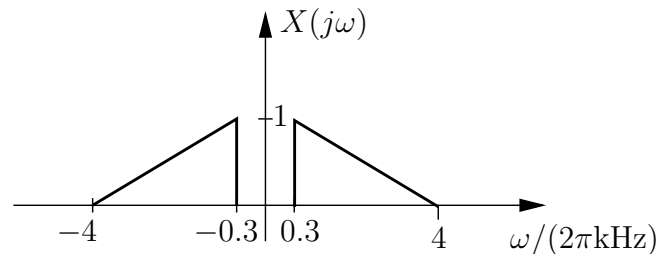
In particular, it is known that $x(-2T) = 2$, $x(-T) = 1$, $x(0) = 2$, $x(T) = 0$, and $x(2T) = 1$.

- i) Sketch the reconstructed signal $x_r(t)$ in the interval $-2T \leq t \leq 2T$ for the following three cases:
- ideal reconstruction,
 - non-ideal reconstruction with $h_1(t)$,
 - non-ideal reconstruction with $h_2(t)$ with $T_0 = T/4$.

Problem 7

(19 marks)

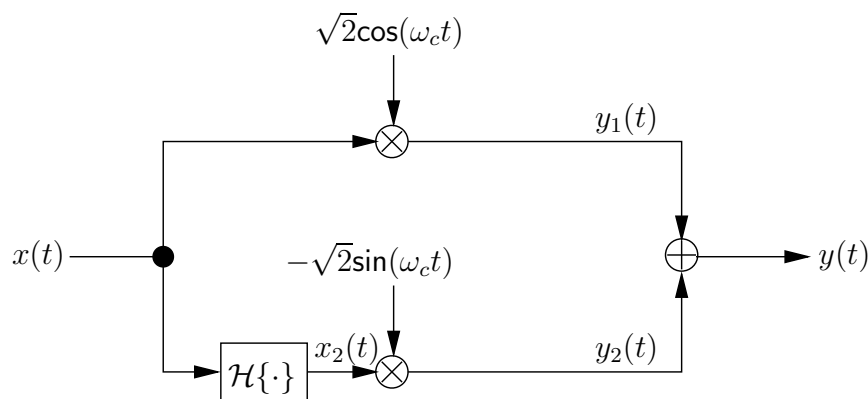
In the following, we consider single-sideband amplitude modulation (AM) for transmission of a speech signal $x(t)$. The spectrum $X(j\omega)$ of $x(t)$ is shown below.



The single-sideband modulated signal is denoted by $y(t)$ and the carrier frequency is $\omega_c = 500$ MHz.

- Sketch the spectrum $Y(j\omega)$ of the single-sideband modulated signal $y(t)$ if the upper sideband is used.
- The conceptually simplest single-sideband modulator (referred to as “Modulator I” in the following) consists of a double-sideband amplitude modulator and a filter. Sketch this simple single-sideband modulator and the frequency response of the filter. How is a filter with such a frequency response called?
- Explain why Modulator I described above is difficult to implement.

An alternative modulator (“Modulator II”) is shown below.



Modulator II involves a Hilbert transformer $\mathcal{H}\{\cdot\}$. The output of the Hilbert transformer can be expressed as

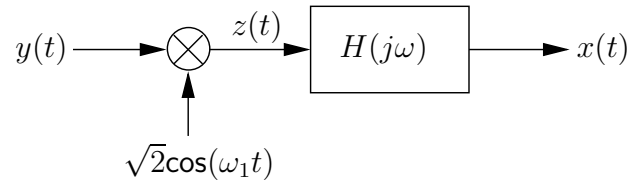
$$x_2(t) = \mathcal{H}\{x(t)\} = \mathcal{F}^{-1}\{-jX(j\omega)\text{sign}(\omega)\}$$

where $\text{sign}(\omega) = 1, \omega \geq 0$ and $\text{sign}(\omega) = -1, \omega < 0$.

- Sketch the spectrum $X_2(j\omega)$ of $x_2(t)$.
- Sketch the spectra $Y_1(j\omega) = \mathcal{F}\{y_1(t)\}$, $Y_2(j\omega) = \mathcal{F}\{y_2(t)\}$, and $Y(j\omega) = \mathcal{F}\{y(t)\}$, respectively.

f) Explain why Modulator II is preferable compared to Modulator I.

At the receiver $y(t)$ can be demodulated using the structure shown below.



g) Sketch the spectrum $Z(j\omega)$ of $z(t)$ and give the appropriate value for frequency ω_1 . Sketch also the frequency response $H(j\omega)$ of the filter. How is a filter with such a frequency response called?