

THE UNIVERSITY OF BRITISH COLUMBIA
Department of Electrical and Computer Engineering
EECE 359 – Signals and Communications
Final Examination – 29. July 2005

This exam consists of 7 pages. Please check that you have a complete copy.

Time: 2 hrs and 30 min

		Problem	Points	max.
		1		17
		2		22
		3		15
		4		20
		5		15
		6		20
		Total:		109

		SURNAME	First Name
		Signature	Student ID

INSTRUCTIONS

1. All writings must be on the exam's booklet. The blank sides on the reverse of each page may also be used.
2. Each candidate should be prepared to produce, upon request, his/her Library/AMS card.
3. Read and observe the following rules:
No candidate shall be permitted to enter the examination room after the expiration of one-half hour, or to leave during the first half-hour of the examination.
Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
Caution – Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
 - Making use of any books, papers or memoranda, calculators, audio or visual cassette players or other memory aid devices, other than as authorized by the examiners.
 - Speaking or communicating with other candidates.
 - Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.

4. Show all your work. Justify your answers. Partial credit is possible for an answer, but only if you show the intermediate steps in obtaining the answer.

Problem 1 (17 points)

(Important: Justify all your answers!)

Assume you measured two continuous-time signals $x_1(t)$ and $x_2(t)$, which can be represented as follows:

$$\begin{aligned}x_1(t) &= 2\cos(t + \pi/4)u(t) \\x_2(t) &= \cos(10\pi t) + \sin(5\pi t + 1)\end{aligned}$$

- a) Determine whether or not the two signals $x_1(t)$ and $x_2(t)$ are periodic.
- b) If $x_1(t)$ and/or $x_2(t)$ are found periodic, determine the fundamental period.

The two signals are now sampled such that the two discrete-time signals

$$\begin{aligned}x_1[n] &= 2\cos(n + \pi/4)u[n] \\x_2[n] &= \cos(10\pi n) + \sin(5\pi n + 1)\end{aligned}$$

are obtained.

- c) Determine whether or not the two signals $x_1[n]$ and $x_2[n]$ are periodic.
- d) If $x_1[n]$ and/or $x_2[n]$ are found periodic, determine the fundamental period.

Finally, the signal $x_1[n]$ is applied to a system, and at the output of the system the signal

$$y_1[n] = \cos(n + \pi/4)$$

is observed.

- e) Is this system a causal system?

Problem 2 (22 points)

Consider a two-path discrete-time communication channel, which can be represented as a discrete-time linear time-invariant (LTI) system with input $x[n]$, output $y[n]$, and the relation

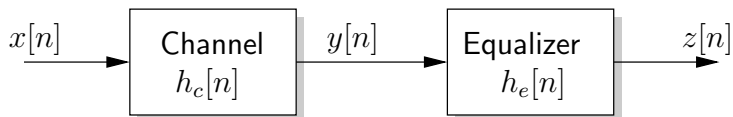
$$y[n] = x[n] + 0.5 \cdot x[n - 1] .$$

- Draw the block diagram of this system.
- Calculate the impulse response $h_c[n]$ of this system.
- Calculate the frequency response $H_c(e^{j\omega})$ of this system.
- Calculate the output $y[n]$ when signal

$$x[n] = u[n] - u[n - 5]$$

is transmitted over the channel (i.e., $x[n]$ is the input signal to the LTI system).

We now wish to recover the transmitted signal $x[n]$ from the received signal $y[n]$ by means of a second LTI system with output $z[n]$, which we refer to as “Equalizer”. The cascade of the channel and the equalizer systems is depicted in the figure:

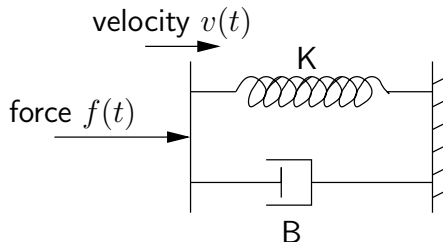


The impulse response of the equalizer system will be denoted by $h_e[n]$, and by $h_o[n]$ we denote the impulse response of the overall system between $x[n]$ and $z[n]$. Correspondingly, $H_e(e^{j\omega})$ and $H_o(e^{j\omega})$ denote the frequency responses of the equalizer and the overall system.

- Write $h_o[n]$ as function of $h_c[n]$ and $h_e[n]$.
- Write $H_o(e^{j\omega})$ as function of $H_c(e^{j\omega})$ and $H_e(e^{j\omega})$.
- What form should $h_o[n]$ and $H_o(e^{j\omega})$ have to achieve perfect recovery of $x[n]$, i.e., $z[n] = x[n]$?
- Determine $h_e[n]$ and $H_e(e^{j\omega})$ to achieve $z[n] = x[n]$.

Problem 3 (15 points)

Consider the mechanical system depicted in the figure, where the input force $f(t)$ acts on a spring and a dashpot. We are interested in the resulting velocity $v(t)$.



The differential equation relating the force $f(t)$ with the velocity $v(t)$ is

$$B \frac{dv(t)}{dt} + Kv(t) = \frac{df(t)}{dt},$$

where the real-valued variables B and K are the damping and the spring constant.

- Determine the frequency response $H(j\omega)$ of this mechanical system with input $f(t)$ and output $v(t)$.
[If you cannot find $H(j\omega)$, use $H(j\omega) = \frac{j\omega \cdot K}{K \cdot B + j\omega \cdot B^2}$ in the following. Note that this is not necessarily the correct answer.]
- Calculate the impulse response $h(t)$ of this system.

For the following, we assume the input force

$$f(t) = \cos(2t + \pi/2)$$

being applied to the system.

- Find the Fourier series coefficients a_k of the input $f(t)$.
[If you cannot solve Problem c), assume fundamental frequency $\omega_0 = 2\pi$ and Fourier series coefficients $a_0 = a_{-2} = a_2 = 1$ and $a_k = 0$ for all other k . Note that this is not necessarily the correct answer.]
- Find the Fourier series representation of the output signal $v(t)$, and give an expression for $v(t)$ when $B = K = 1$.

Problem 4 (20 points)

(Important: Label your sketches properly!)

Calculate and sketch the Fourier transforms of each of the following signals:

a) $x(t) = \frac{\sin(t)}{t}$

b) $x(t) = \frac{1}{\pi} \left(\frac{\sin(t)}{t} \right)^2$

c) $x(t) = u(-t + 1) - u(-t - 1)$

Consider the two discrete-time signals

$$x_1[n] = \sum_{k=0}^3 \delta[n - k]$$

$$x_2[n] = \sum_{k=-\infty}^{\infty} \delta[n - 6k]$$

d) Sketch $x_1[n]$ and $x_2[n]$.

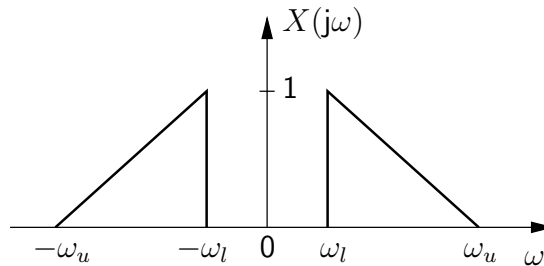
e) Determine and sketch

$$y[n] = x_1[n] * x_2[n] .$$

Problem 5 (15 points)

(Important: Label your sketches properly!)

A continuous-time real-valued audio signal $x(t)$ has the real-valued spectrum $X(j\omega)$ as depicted in the following figure.



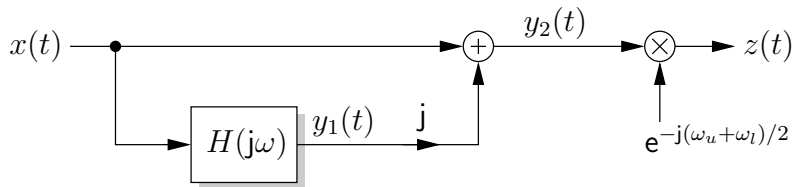
The spectrum $X(j\omega)$ is nonzero for a minimum (lower) frequency $\omega_l = 2\pi \cdot 300$ rad/sec and for a maximum (upper) frequency $\omega_u = 2\pi \cdot 3400$ rad/sec.

We will consider two methods of sampling of $x(t)$ in the following.

First, $x(t)$ is sampled directly.

- a) What is the minimum sampling rate ω_x if aliasing is to be avoided?

Second, we process $x(t)$ before sampling as shown in the following figure.



The intermediate signal $y_1(t)$ is the output of a linear time-invariant (LTI) system with frequency response

$$H(j\omega) = \begin{cases} j, & \text{for } \omega \leq 0 \\ -j, & \text{for } \omega > 0 \end{cases}$$

The intermediate signal $y_2(t)$ and the output $z(t)$ are obtained as:

$$\begin{aligned} y_2(t) &= x(t) + jy_1(t) \\ z(t) &= y_2(t) \cdot e^{-j(\omega_u + \omega_l)/2} \end{aligned}$$

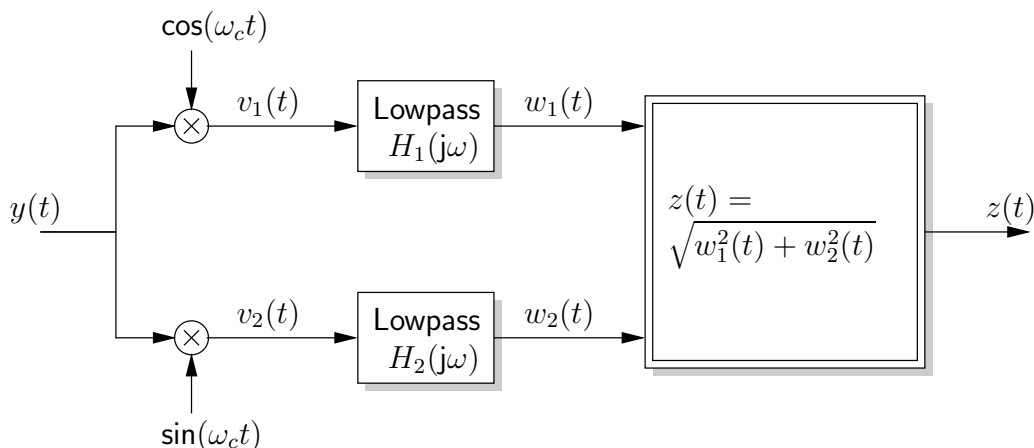
- b) Sketch the spectra $Y_1(j\omega)$, $Y_2(j\omega)$, and $Z(j\omega)$ of $y_1(t)$, $y_2(t)$, and $z(t)$, respectively.
- c) Is $z(t)$ real-valued or complex-valued? Justify your answer!

The signal $z(t)$ is now sampled.

- d) What is the minimum sampling rate ω_z if aliasing is to be avoided?

Problem 6 (20 points)

We want to analyze the following demodulator (receiver) for an amplitude modulation (AM) signal with carrier.



First, we consider the modulated signal, which is given by

$$y(t) = (x(t) + A)\cos(\omega_c t + \phi_c) ,$$

where A is a positive constant, the carrier frequency $\omega_c = 2\pi \cdot 10^5$ rad/sec, and ϕ_c is an arbitrary phase offset. The message signal $x(t)$ has a spectrum $X(j\omega)$ with maximum bandwidth $\omega_M = 2\pi \cdot 4000$ rad/sec.

- What condition must $(x(t) + A)$ satisfy for all t to allow the use of an envelope detector?
- Express the spectrum $Y(j\omega)$ of $y(t)$ as function of $X(j\omega)$ and the parameters A , ω_c , and ϕ_c .

Now, we consider the demodulator shown in the figure. We assume that the carrier frequency ω_c is known, but not the phase offset ϕ_c .

- Express the spectra $V_1(j\omega)$ of $v_1(t)$ and $V_2(j\omega)$ of $v_2(t)$ as function of $Y(j\omega)$.
- Express the spectra $W_1(j\omega)$ of $w_1(t)$ and $W_2(j\omega)$ of $w_2(t)$ as function of $X(j\omega)$.

Using your result from Problem d), solve the following design problem.

- Design the two lowpass filters shown in the figure (specify their frequency responses $H_1(j\omega)$ and $H_2(j\omega)$) such that

$$w_1(t) = (x(t) + A)\cos(\phi_c) \quad \text{and} \quad w_2(t) = (x(t) + A)\sin(\phi_c) \quad \text{hold.}$$

[Hint: It might be useful to first determine the spectrum of the desired signals $w_1(t)$ and $w_2(t)$, respectively.]

- Given $w_1(t)$ and $w_2(t)$ as in Problem e), determine $z(t)$.