1.

a) $y(t) = p(t)\, x(t)$

$x(t) = a\, x_1(t) + b\, x_2(t)$

$\leadsto y(t) = a \underbrace{p(t)\, x_1(t)}_{y_1(t)} + b \underbrace{p(t)\, x_2(t)}_{y_2(t)}$

$\leadsto$ is linear   2

b) $y(t-t_0) = p(t-t_0)\, x(t-t_0)$

$\neq p(t)\, x(t-t_0)$ $\leadsto$ not time-inv   2

c) $y(t)$ only depends on $x(t)$ $\leadsto$ is memoryless   2

d) not memoryless since $h_2(t) \neq k\, \delta(t)$ 2

e) causal since $h_2(t) = 0$ for $t<0$ 2

f) stable since $\int_{-\infty}^{\infty} |h_2(t)|\, dt = T < \infty$ 2

g) not an LTI syst since S_1 not LTI   2

h) yes, since S_1 not LTI   2

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2 a)

$$h[n] = \frac{\sin\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{2}n\right)}{\pi n}$$ 3

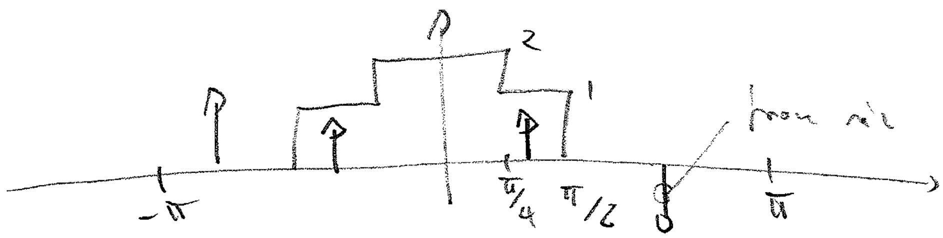
b) not causal since $h[n] \neq 0$ for $n < 0$ 2

c) $y[n] = x[n] * h[n]$

$= h[n] - h[n-1]$

$$= \frac{\sin\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{2}n\right)}{\pi n} + \frac{\sin\left(\frac{\pi}{4}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-1)\right)}{\pi (n-1)}$$ 2

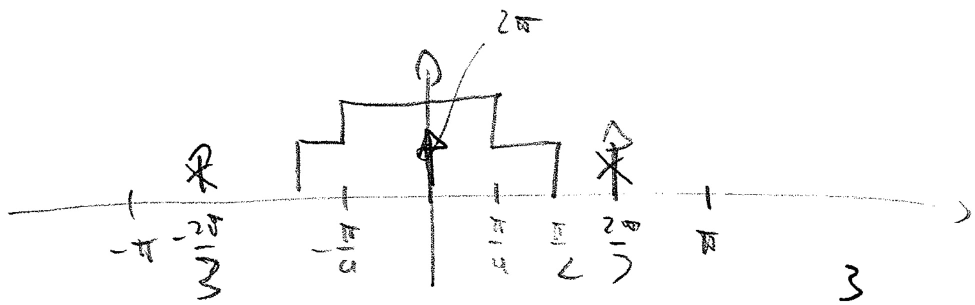
d)



3

$\rightsquigarrow y[n] = \cos\left(\frac{\pi}{3}n\right)$

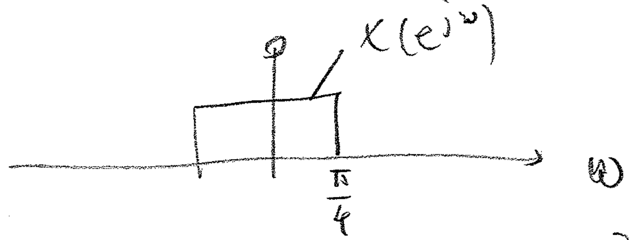
e)



3

$\rightsquigarrow y[n] = 1$

f)

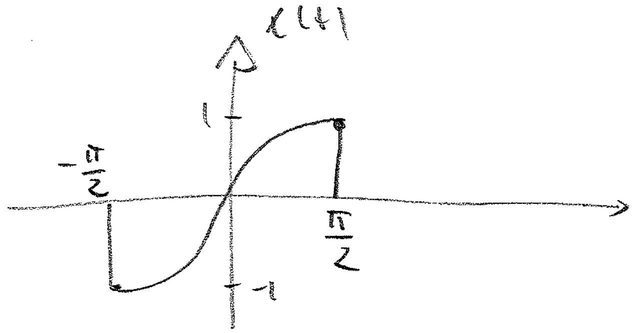


$$\rightsquigarrow y[n] = 2x[n] = 2\,\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$$

3

/16

3a)

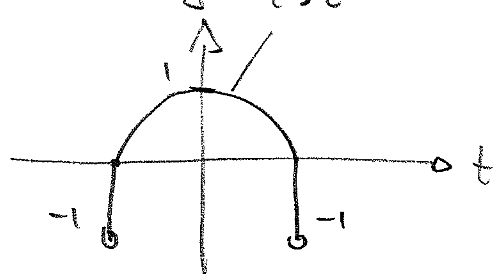


2

b) $X(j0) = \int_{-\infty}^{\infty} x(t)\, dt = 0$ 2

c) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi\, x(0) = 0$ 2

d) $j\omega\, X(j\omega) = \frac{dx(t)}{dt} = y(t)$



$\int_{-\infty}^{\infty} \omega\, X(j\omega)\, d\omega = \frac{1}{j}\, 2\pi\, y(0) = \frac{2\pi}{j}$ // 2

e) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$ 2

$= 2\pi \cdot 2 \int_0^{\pi/2} \sin^2 t\, dt$

$= 4\pi\, \frac{1}{2} \int_0^{\pi/2} (1 - \cos 2t)\, dt$

$= 2\pi \cdot \frac{\pi}{2} - \underbrace{2\pi \tfrac{1}{2} \sin 2t \Big|_0^{\pi/2}}_{0} = \pi^2$ //

f) x(t) real & odd $\Rightarrow$ $Re\{X(j\omega)\} = 0$ 2

g) no, since x(t) not a sum of δ-functions 2

/ 14

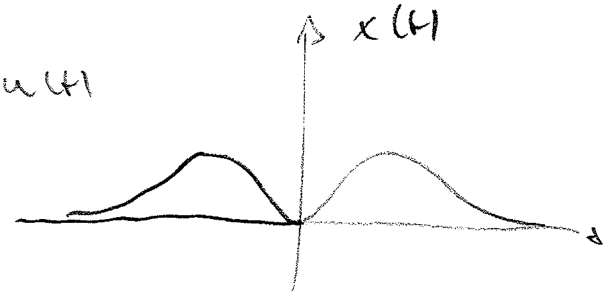
4a

$$x(t) = z(t) + z(-t)$$

$$z(t) = t\,e^{-t}\,u(t)$$

$$\updownarrow$$

$$\frac{1}{(1+j\omega)^2}$$

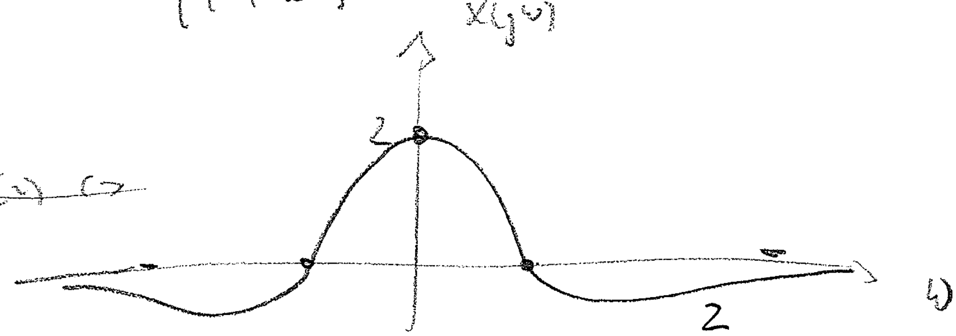


2

$$\Rightarrow X(j\omega) = Z(j\omega) + Z(-j\omega)$$

$$= \frac{1}{(1+j\omega)^2} + \frac{1}{(1-j\omega)^2}$$

$$= 2\,\frac{1-\omega^2}{(1+\omega^2)^2} \quad 3$$



b) ~~x[n] = z[n] c~~

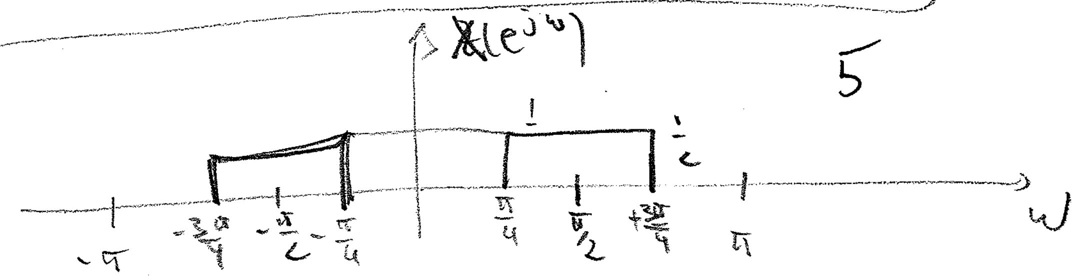
2

b) $x[n] = z[n]\cos\left(\frac{\pi}{2}n\right)$

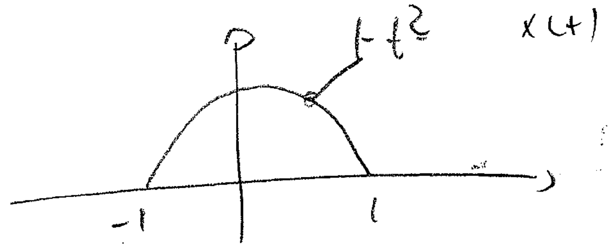
$$z[n] \leftrightarrow Z(e^{j\omega}) = \begin{cases} 1 & |\omega| < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\omega| < \pi \end{cases}$$

$$\boxed{X(e^{j\omega}) = \frac{1}{2}\left[Z\left(e^{j(\omega+\frac{\pi}{2})}\right) + Z\left(e^{j(\omega-\frac{\pi}{2})}\right)\right]}$$

5



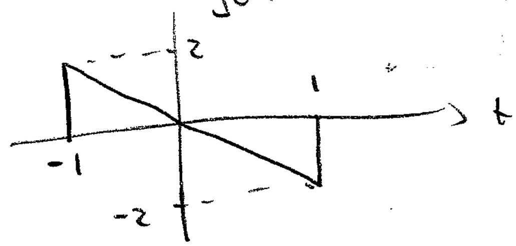
4b,



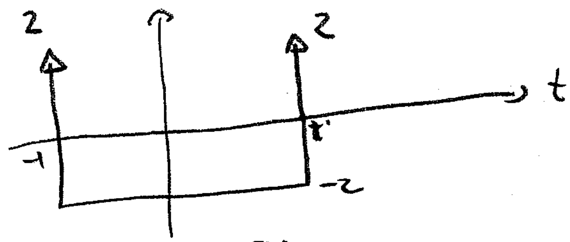
2

$$y(t) = \frac{d}{dt} x(t) = -2t$$

y(t)



$$z(t) = \frac{d}{dt} y(t)$$



$$Z(j\omega) = 2e^{j2\omega} + 2e^{-j2\omega} - 2 \quad \frac{2\sin\omega}{\omega}$$

$$= 4\left[\cos 2\omega - \frac{\sin\omega}{\omega} \right]$$

$$\leadsto Y(j\omega) = \frac{Z(j\omega)}{j\omega} + \pi Z(0)\,\delta(\omega) \; ; \; Z(0) \equiv \lim_{\omega\to 0} 1 - 1 = 0$$

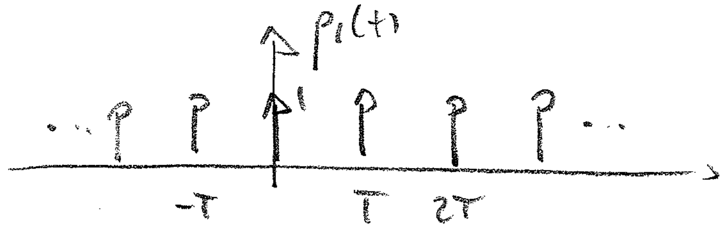
$$X(j\omega) = -\frac{Z(j\omega)}{\omega^2} + \pi Y(j\omega)\,\delta(\omega) \; ; \; Y(0) = \lim_{\omega \to 0} \frac{1}{j\omega} - \frac{\omega}{j\omega^2} = 0$$

$$\leadsto \boxed{X(j\omega) = 4\left(\frac{\sin\omega}{\omega^3} - \frac{\cos 2\omega}{\omega^2} \right)}$$

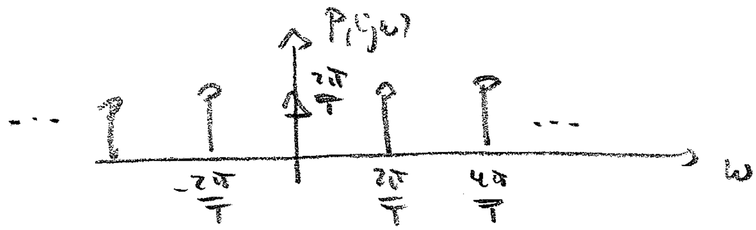
6

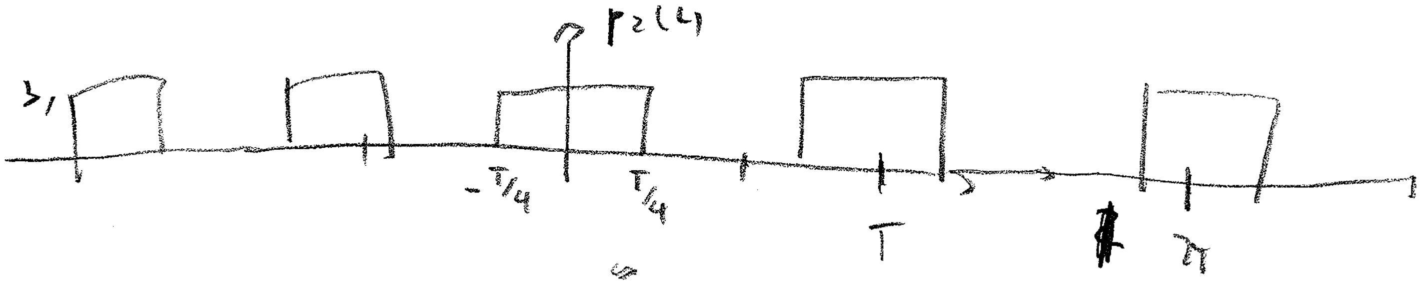
/20

5 a)



$$P_1(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T}k\right)$$ 2

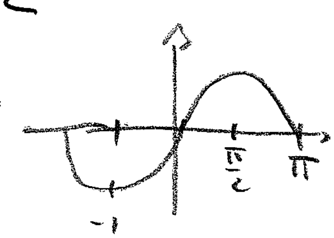


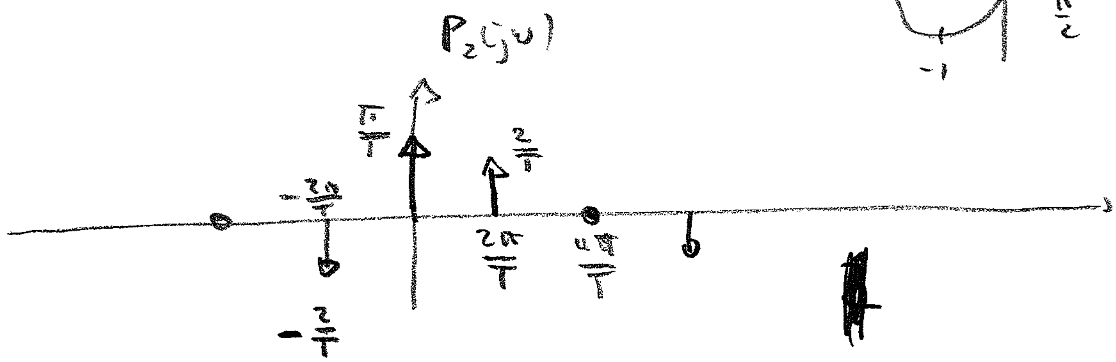


$$P_2(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} a_k\, \delta\left(\omega - \frac{2\pi}{T}k\right)$$

2

$$a_k = \frac{\sin\left(k\, 2\pi \frac{T/4}{T}\right)}{k\pi} = \frac{\sin\left(\frac{\pi}{2}k\right)}{k\pi}$$





c)

$p_1(t)$: $X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c[j(\omega - \frac{2\pi}{T}k)]$

$p_2(t)$: $X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} a_k X_c[j(\omega - \frac{2\pi}{T}k)]$

d)

$\frac{2\pi}{T} > 2\omega_M$

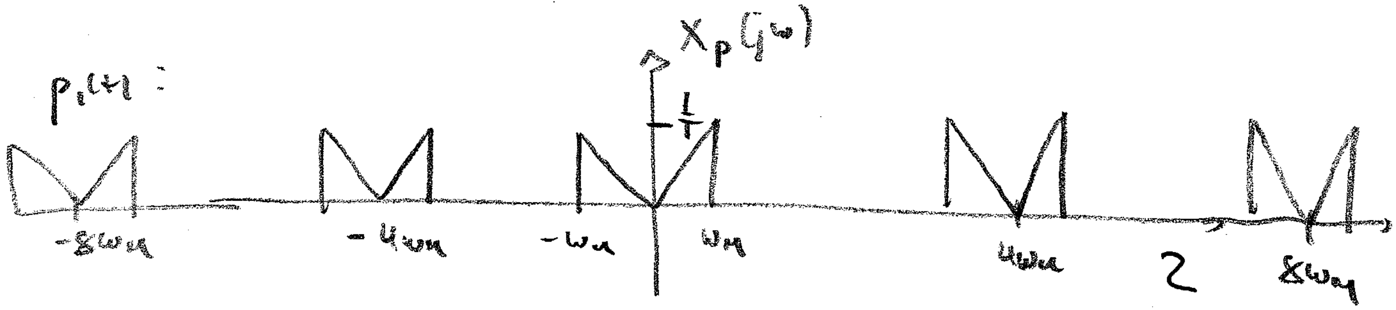
$T < \frac{\pi}{\omega_M}$ in both cases

2

e)

$\frac{2\pi}{T} = \frac{2\pi}{\pi} 2\omega_M = 4\omega_M$

$p_1(t)$:



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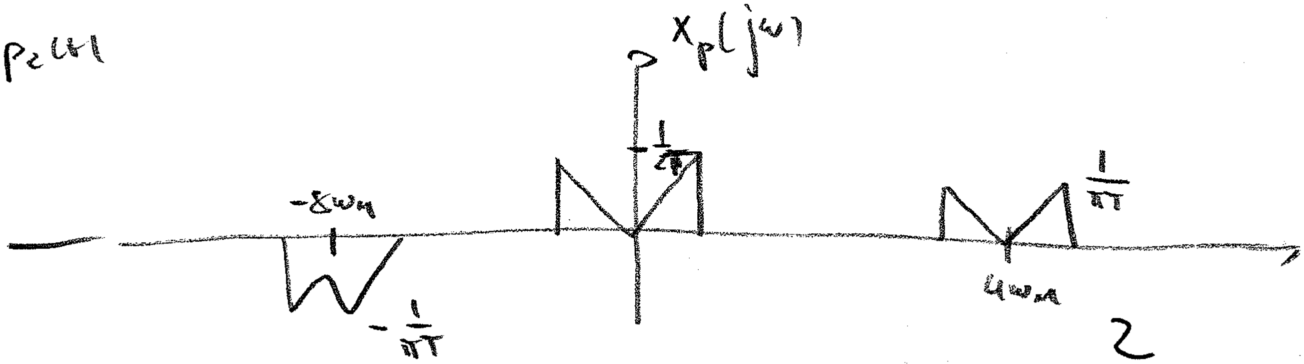
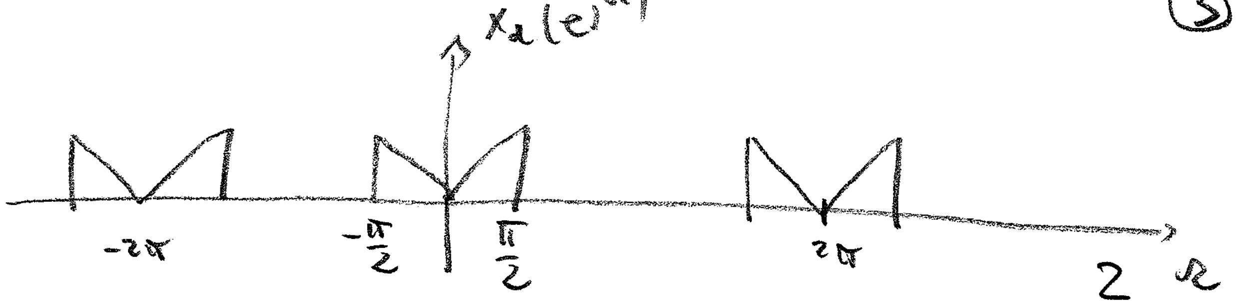
$p_2(t)$



2

③

f)



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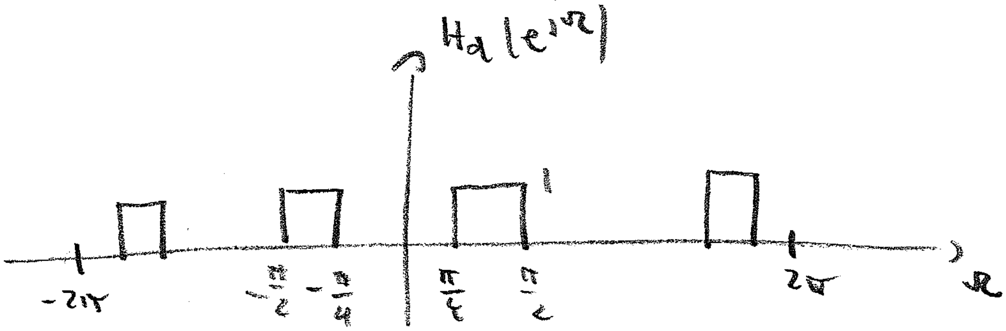
g)



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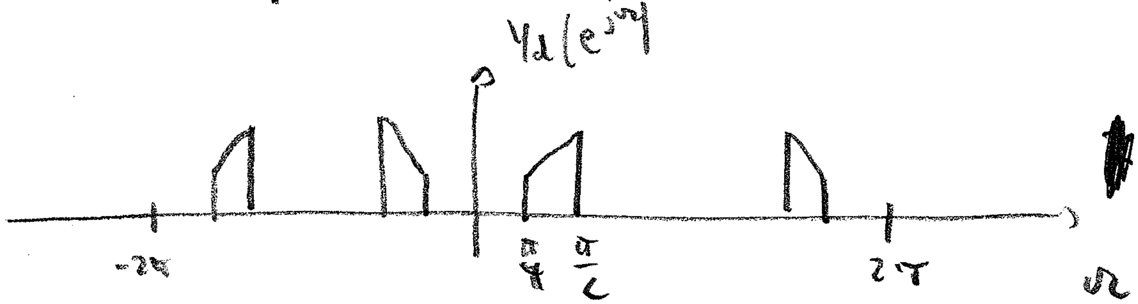
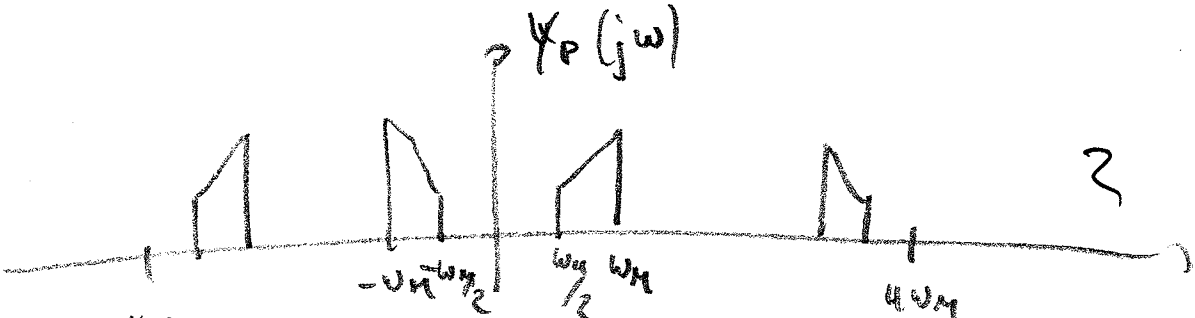
band pass filter

h)



i)



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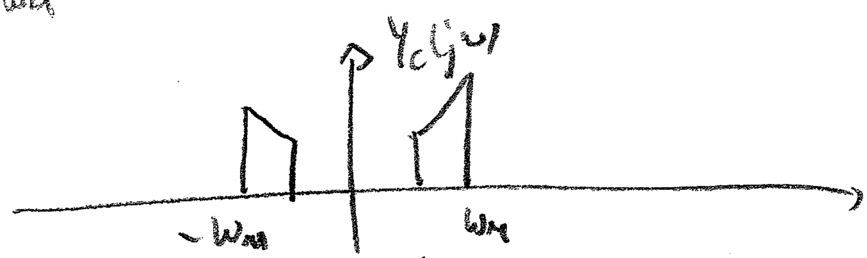
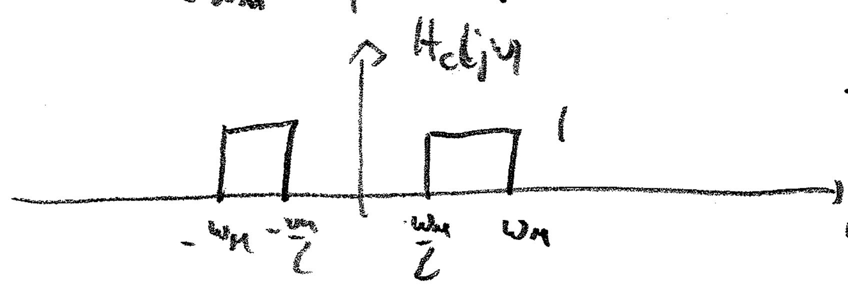
j)



k)



2

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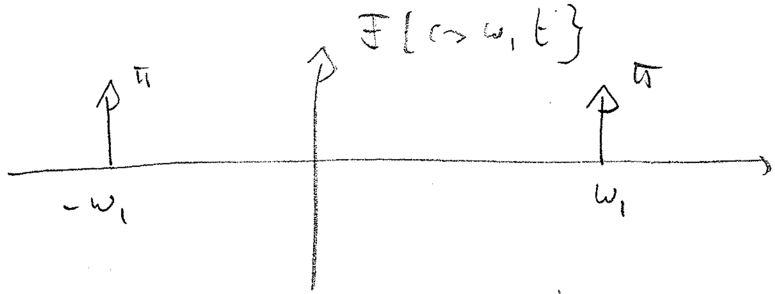
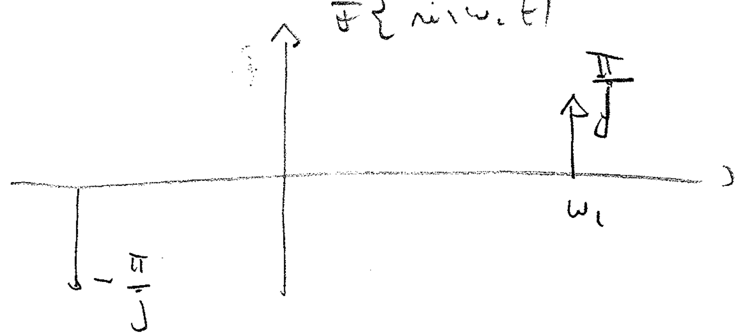
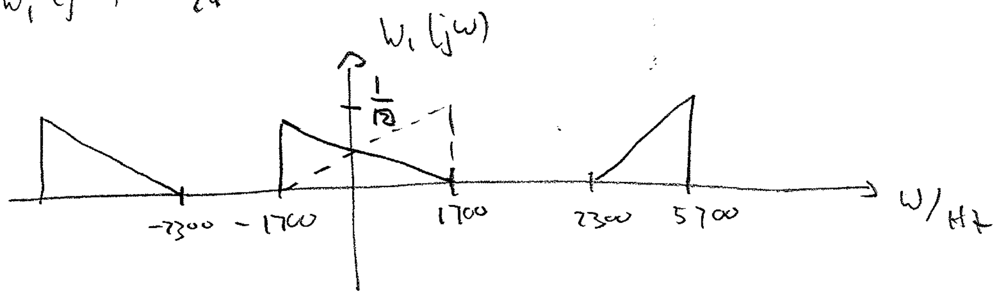
6a)

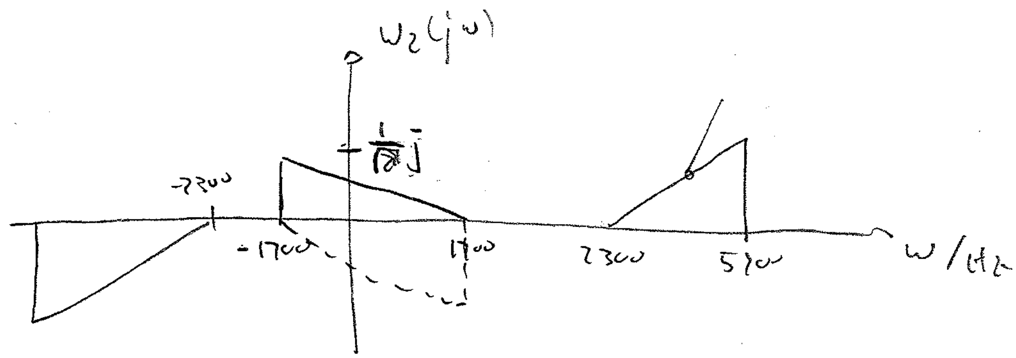
①





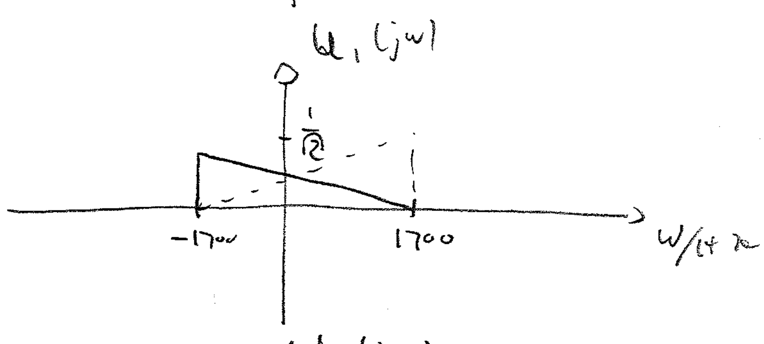
$W_1(j\omega) = \frac{1}{2\pi}\,\mathcal{F}\{\cos\omega_1 t\} * X(j\omega)$; $\omega_1 = 2000$ Hz

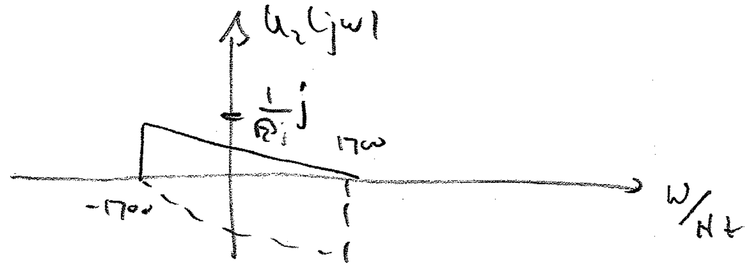




6

b)

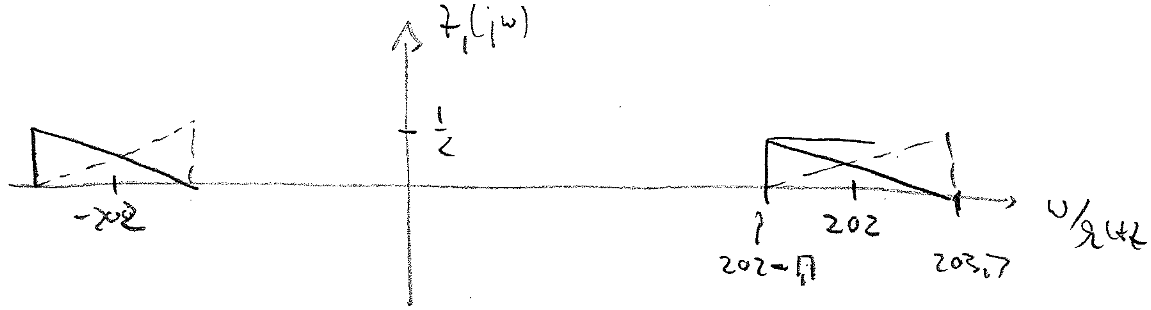


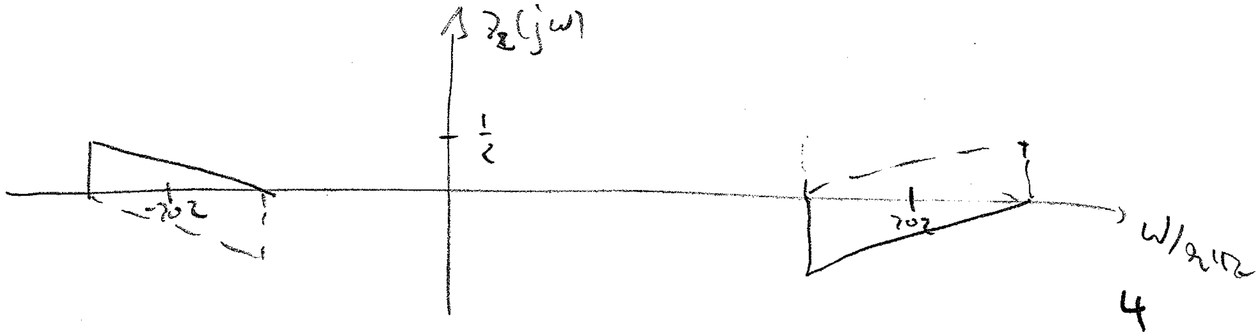


2

2

c)





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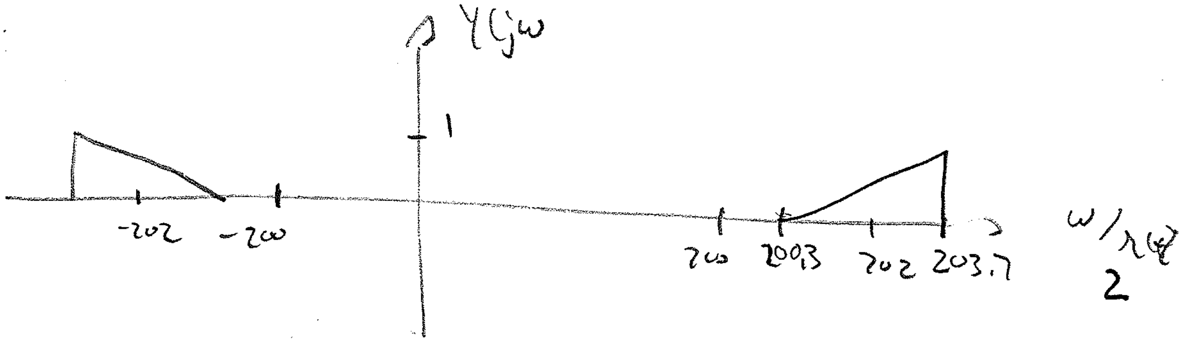
d)



2

e) single-sideband modulation 1

f) $\underline{f_c = 200 \text{ kHz}}$ 1

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