EECE 359: Signals and Systems
Midterm #2

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This midterm is closed note, closed book, and calculators are not allowed. A *single-sided, one-page* cheat sheet is allowed and must be handed in with your midterm. (It will be returned to you with your graded midterm.) For partial credit, be sure to show all your work.

SOLUTIONS

Name Student ID # Signature

Problem #	Actual points	Possible points
1		35
2		40
3		35
Total:		100

Formulas

- Continuous-time Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \overset{FS}{\longleftrightarrow} a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

with $\omega_0 = \frac{2\pi}{T}$ (T: Period).

- Discrete-time Fourier series

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} \overset{FS}{\longleftrightarrow} a_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\omega_0 n}$$

with $\omega_0 = \frac{2\pi}{N}$ (N: Period).

- Continuous-time Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \overset{F}{\longleftrightarrow} X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- Discrete-time Fourier Transform

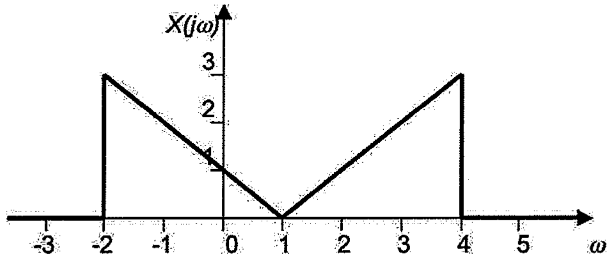
$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega \overset{F}{\longleftrightarrow} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Euler's formula and trigonometric relationships

$$\begin{aligned}
e^{jx} &= \cos(x) + j\sin(x) \\
\cos(x) &= \tfrac{1}{2}\left(e^{jx} + e^{-jx}\right) \\
\sin(x) &= \tfrac{1}{2j}\left(e^{jx} - e^{-jx}\right) \\
\cos^2(x) &= \tfrac{1}{2}(1 + \cos(2x)) \\
\sin^2(x) &= \tfrac{1}{2}(1 - \cos(2x))
\end{aligned}$$

Problem 1 (35 points)

Consider a signal $x(t)$ with Fourier transform $\mathcal{F}\{x(t)\} = X(j\omega)$ as shown below.



Solve the following four questions *without* explicitly calculating the inverse Fourier transform $x(t) = \mathcal{F}^{-1}\{X(j\omega)\}$ for all t.

(10) 1. Calculate $\int_{-\infty}^{\infty} x(t)dt$.

(10) 2. Calculate $x(0)$.

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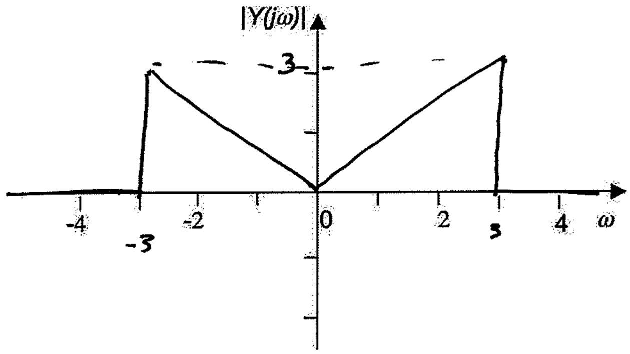
1. $X(j\cdot 0) = \int_{-\infty}^{\infty} x(t)\, e^{-j\cdot 0\cdot t}\, dt = \int_{-\infty}^{\infty} x(t)\, dt = 1.$

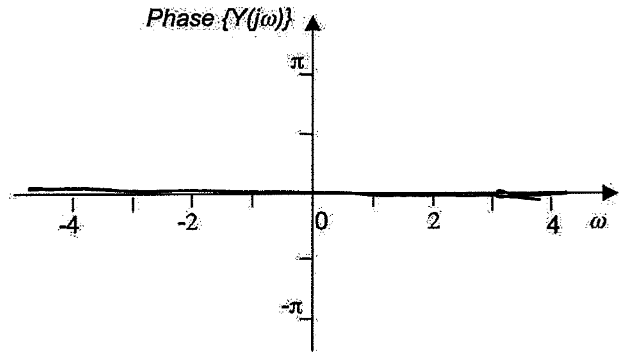
2. $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega\cdot 0}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, d\omega$

$= \frac{1}{2\pi}\cdot 3\cdot 3 = \frac{9}{2\pi}$

(10) 3. Sketch $Y(j\omega) = \mathcal{F}\{y(t)\}$, $y(t) = x(t)e^{-jt}$ on the plots below. Be sure to label the vertical axis scale on the magnitude plot.

(5) 4. True or False: $\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$?





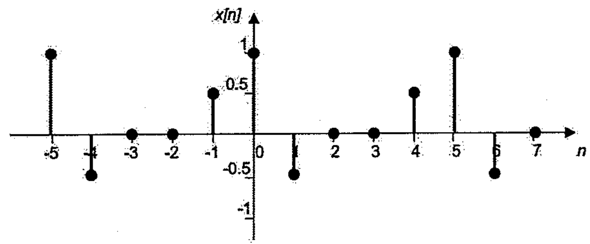
3. $F\{y(t)\} = X(j(\omega+1))$ ← frequency shift of 1 to the left

4. $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$ (Parseval's)

$< \infty$ true.

Problem 2 (30 points)

Consider a periodic signal $x[n]$ with period $N = 5$ as shown below.



(10) 1. Find the Fourier Series coefficients $a_k, k \in \mathbb{Z}$ for $x[n]$. Simplify your result into sine and cosine formulas where possible.

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$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

$$= \frac{1}{5} \sum_{n=-2}^{2} x[n] e^{-jk\frac{2\pi}{5}n}$$

$$= \frac{1}{5} \left(x[-1] e^{-jk\frac{2\pi}{5}(-1)} + x[0] e^{-jk\frac{2\pi}{5}\cdot 0} + x[1] e^{-jk\frac{2\pi}{5}(1)} \right)$$

$$= \frac{1}{5} \left(1 + \frac{1}{2} e^{jk\frac{2\pi}{5}} - \frac{1}{2} e^{-jk2\pi/5} \right)$$

$$= \frac{1}{5} \left(1 + j \left(\frac{1}{2j} e^{jk2\pi/5} - \frac{1}{2j} e^{-jk2\pi/5} \right) \right)$$

$$= \frac{1}{5} \left(1 + j \sin\left(\frac{2\pi}{5} \cdot k \right) \right), \quad k \in \mathbb{Z}$$

(10) 2. Find the Fourier transform $Y(e^{j\omega})$ of $y[n] = \frac{5}{2}x[-n]$.

(10) 3. (a) Are a_k periodic, and if so, with what fundamental period?
(b) Is $Y(e^{j\omega})$ periodic, and if so, with what fundamental period?

2. $Y(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} b_k \, \delta(\omega - \frac{2\pi}{5}k)$, $b_k = \frac{5}{2} a_{-k}$

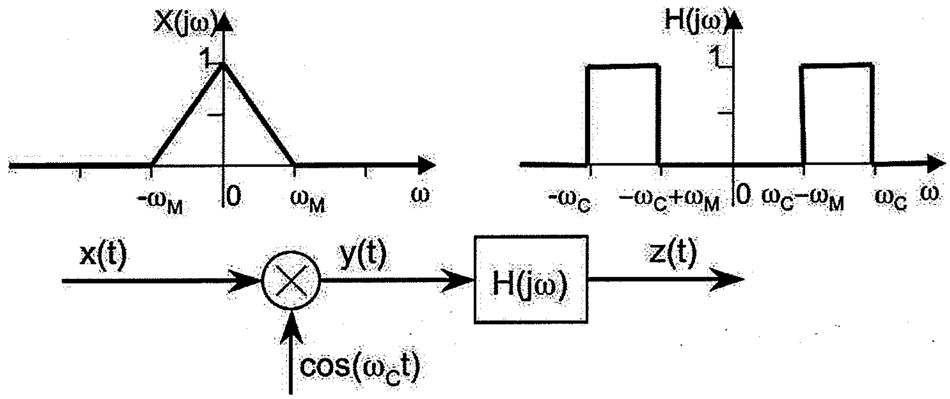
$= 2\pi \sum_{k=-\infty}^{\infty} \frac{5}{2} \cdot \frac{1}{5} (1 + j\sin(\frac{2\pi}{5}k)) \, \delta(\omega - \frac{2\pi}{5}k)$

$= \pi \sum_{k=-\infty}^{\infty} (1 + j\sin(\frac{2\pi}{5}k)) \, \delta(\omega - \frac{2\pi}{5}k)$

3. a) a_k are periodic w/ period $N=5$.
b) $Y(e^{j\omega})$ is periodic w/ period 2π.

Problem 3 (35 points)

Consider the continuous-time signal $x(t)$ with spectrum $X(j\omega)$ as shown below, left. This signal is first multiplied by the signal $s(t) = \cos(\omega_c t)$, then filtered by $H(j\omega)$, shown below, right.



(10) 1. Sketch the magnitude and phase of $Y(j\omega)$. Label all relevant points.

(10) 2. Sketch the magnitude and phase of $Z(j\omega)$. Label all relevant points.

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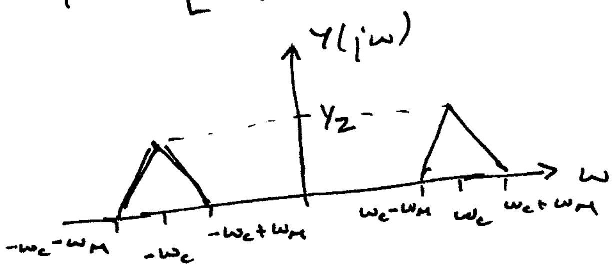
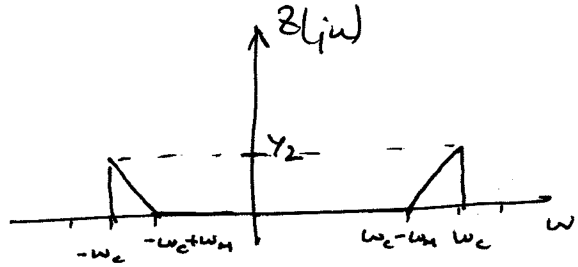
1. $Y(j\omega) = \left[X(j\omega) * \pi\left(\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right)\right] \cdot \frac{1}{2\pi}$



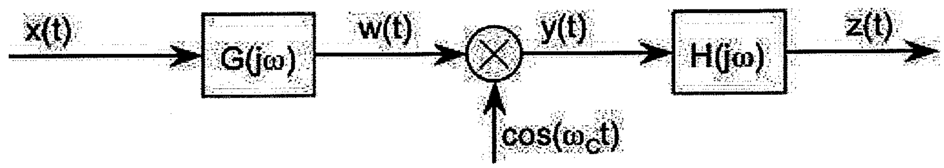
Note $\angle Y(j\omega) = 0$ since $Y(j\omega) \in \mathbb{R}$.

2. $Z(j\omega) = Y(j\omega) \cdot H(j\omega)$



Note $\angle Z(j\omega) = 0$ since $Z(j\omega) \in \mathbb{R}$.

(10) 3. Now consider the case in which $x(t)$ is pre-filtered such that $2\frac{dx(t)}{dt}$ is the input to the multiplier. Find the frequency response for the pre-filtering system $G(j\omega)$ with input $x(t)$ and output $w(t) = 2\frac{dx(t)}{dt}$.



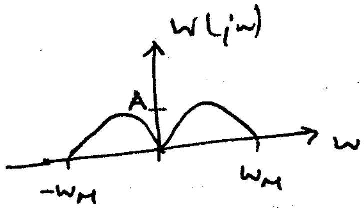
(5) 4. Sketch the magnitude and phase of $Z(j\omega)$ in this case. Label all relevant points.

3. $W(j\omega) = G(j\omega) \cdot X(j\omega)$, $W(j\omega) = 2j\omega X(j\omega)$

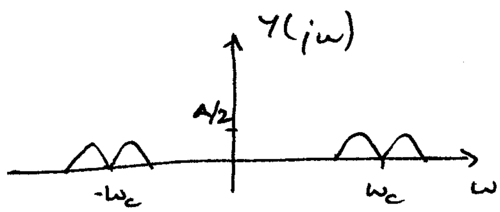
$$\frac{2j\omega X(j\omega)}{X(j\omega)} = G(j\omega)$$

$$\Rightarrow G(j\omega) = 2j\omega$$

4. $W(j\omega)$:



$Y(j\omega)$:



$Z(j\omega)$