EECE 359: Signals and Systems
Midterm #1

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This midterm is closed note, closed book, and calculators are not allowed. A *single-sided, one-page* cheat sheet is allowed and must be handed in with your midterm. (It will be returned to you with your graded midterm.) For partial credit, be sure to show all your work.

SOLUTIONS

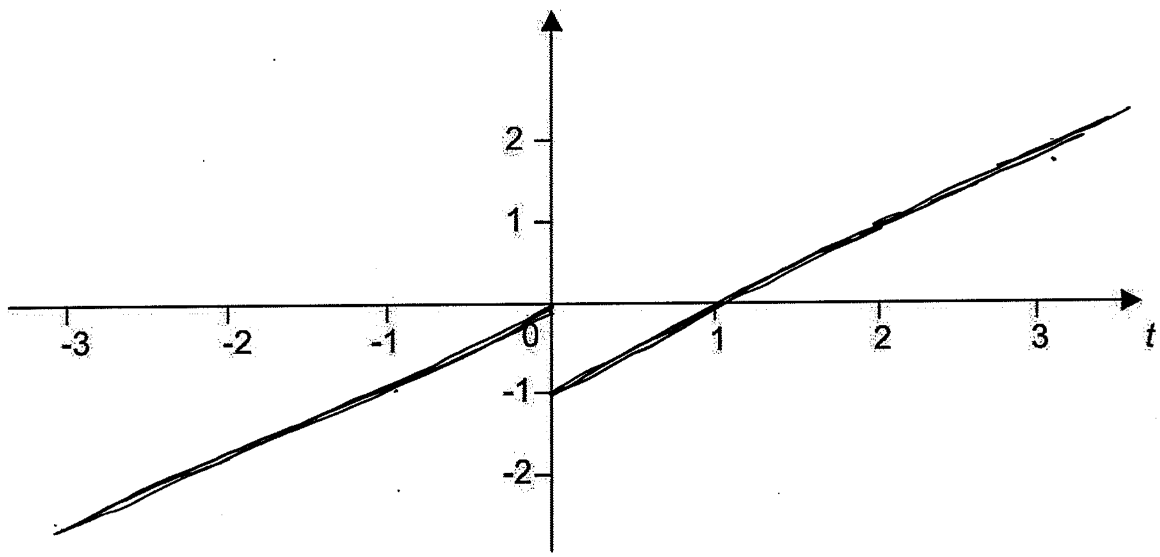
Name　　Student ID #　　Signature

Problem #	Actual points	Possible points
1		30
2		25
3		40
	5	5
Total:		100

Problem 1 (30 points)

Consider a system $y(t) = -x(t) + t$ with input $x(t)$ and output $y(t)$.)

1. Compute and sketch $y(t)$ when $x(t) = u(t)$ on the graph below.



$$y(t) = -u(t) + t = \begin{cases} t & t \leq 0 \\ t-1 & t > 0 \end{cases}$$

Problem 1 (continued)

Recall that $y(t) = -x(t) + t$.

3. Determine and mathematically justify whether the system is:
   (a) linear
   (b) time-invariant

a) Given $x_1(t) \rightarrow y_1(t) = -x_1(t) + t$
$x_2(t) \rightarrow y_2(t) = -x_2(t) + t$

Then $ax_1(t) + bx_2(t) \rightarrow -(ax_1(t) + bx_2(t)) + t$ for $a, b \in \mathbb{R}$

$= -ax_1(t) + t - bx_2(t) + t - t$

$= y_1(t) + y_2(t) - t$

$\neq y_1(t) + y_2(t)$ $\therefore$ nonlinear

b) Given $x_1(t) \rightarrow y_1(t) = -x_1(t) + t$

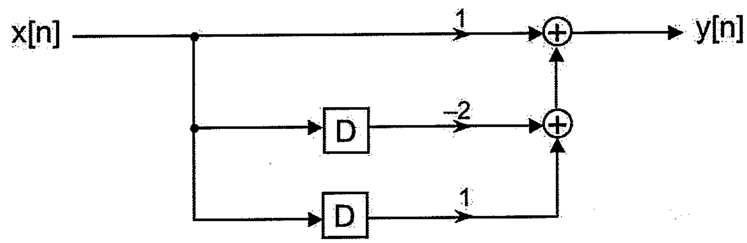
$x_1(t - t_0) \rightarrow -x_1(t - t_0) + t \neq y_1(t - t_0)$,

since $y_1(t - t_0) = -x_1(t - t_0) + (t - t_0)$

$\therefore$ time-varying

Problem 2 (25 points)

Consider a causal linear time-invariant (LTI) system whose input $x[n]$ and output $y[n]$ are related as shown below.



1. Find the impulse response $h[n]$ for the system.
   If you cannot find the impulse response $h[n]$, then assume for the remainder of the problem that $h[n] = \left(\frac{1}{2}\right)^n (u[n] - u[n-2])$. Note that this is not necessarily the correct answer.

2. Determine and mathematically justify whether the system is:

   (a) stable

   (b) memoryless

1. $y[n] = x[n] - 2x[n-1] + x[n-2]$ from block diagram

$\therefore h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$

2. a) $\sum_{k=-\infty}^{\infty} |h[k]| = 1 + 2 + 1 = 4 < \infty$

$\Rightarrow$ stable.

b) $h[n] \neq k\,\delta[n] \Rightarrow$ not memoryless

Problem 3 (40 points)

Consider a periodic signal $x(t)$ with fundamental frequency $\omega_0 = \pi/4$ and Fourier series coefficients

$$\begin{aligned} a_0 &= 1 \\ a_1 &= a_{-1} = 2 \\ a_2 &= a_{-2} = -1 \\ a_k &= 0,\ |k| \geq 3 \end{aligned}$$

1. What are the Fourier series coefficients b_k of $y(t) = \frac{1}{2}x(-t)$? (*Note that you do not need to solve for $x(t)$ in order to solve this problem.*)
2. Solve for $x(t)$ and for full credit, simplify using Euler's relation.

1. $b_0 = \frac{1}{2}a_0 = 1/2$

$b_1 = \frac{1}{2}a_{-1} = 1$ $\qquad b_{-1} = \frac{1}{2}a_1 = 1$

$b_2 = 1/2\, a_{-2} = -1/2$ $\qquad b_{-2} = \frac{1}{2}a_2 = -1/2$

2. $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$= a_0 + a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_2 e^{2j\omega_0 t} + a_{-2} e^{-2j\omega_0 t}$

$= 1 + 2e^{j\pi/4 t} + 2e^{-j\pi/4 t} - 1 \cdot e^{j\pi/2 t} - 1 e^{-j\pi/2 t}$

$= 1 + 4\cos(\pi/4\, t) - 2\cos(\pi/2\, t)$

Problem 3 (continued)

Now consider the periodic continuous-time signal $z(t) = 3 - \cos\left(\frac{2\pi}{3}t\right) + 2\sin\left(\frac{\pi}{2}t\right)$.

3. What are the fundamental period and the fundamental frequency of $z(t)$?

4. What are the Fourier series coefficients c_k of $z(t)$?
(If you did not solve part 3, assume that $\omega_0 = \pi/6$. Note that this is not necessarily the correct answer.)

3. For $\cos\left(\frac{2\pi}{3}t\right)$, $T = \frac{2\pi}{2\pi/3} = 3$
For $\sin\left(\pi/2\, t\right)$, $T = \frac{2\pi}{\pi/2} = 4$
and $\operatorname{lcm}(3,4) = 12 = T_0$
$\therefore\ \omega_0 = \frac{2\pi}{12} = \pi/6$.

4. $z(t) = 3\cdot e^{0\cdot j\omega_0 t} + 2\sin\left(\frac{\pi}{6}\cdot 3\cdot t\right) - \cos\left(\frac{\pi}{6}\cdot 4t\right)$

$= 3\cdot e^{0\cdot j\omega_0 t} + 2\left(\frac{1}{2j}\right)\left(e^{j\omega_0\cdot 3t} - e^{-j\omega_0 3t}\right) - \left(\frac{1}{2}\right)\left(e^{j\omega_0 4t} + e^{-j\omega_0 4t}\right)$

$= 3\cdot e^{0\cdot j\omega_0 t} - j e^{3\cdot j\omega_0 t} + j e^{-3j\omega_0 t} - \frac{1}{2}e^{4\cdot j\omega_0 t} - \frac{1}{2}e^{-4\cdot j\omega_0 t}$

$c_0 = 3$ $\qquad c_1 = c_{-1} = 0$

$c_3 = -j$ $\qquad c_2 = c_{-2} = 0$

$c_{-3} = +j$ $\qquad c_k = 0,\ |k| \geq 5,\ k \in \mathbb{Z}$.

$c_4 = -1/2$

$c_{-4} = -1/2$