SYSC 3600 – Assignment #1 Solution

1/. $\dot{p} + kp = \emptyset$

fits general form $\dot{y} + \frac{1}{\tau} y = 0$

$\therefore\ p(t) = p_0 e^{-kt} = 100 e^{-kt}$ grams

taking t in years (for convenience)

$$100 e^{-k(24100)} = 50 \quad \text{half of original 100 grams}$$

$$e^{-k(24100)} = 0.5$$

$$k = -\ln(0.5)/24100 = 2.876 \times 10^{-5}\ \text{1/years}$$

time constant $= \tau = 1/k = 34{,}769$ years

It makes sense that the time constant is longer than the half life, as at one time constant only about 37% of the original amount will remain.

2/.

area of can = $2\left(\frac{\pi d^2}{4}\right) + \pi dh$ volume of can = $\frac{\pi d^2 h}{4}$

mass of beer = volume times density = $\frac{\pi d^2 h\rho}{4}$

heat transfer rate = $\dot{Q} = (T_R - T_B) * U * area$

rate of temperature change = $\dot{T}_B = \dot{Q}/(mass * c_p)$

putting everything together:

$$\dot{T}_B = \left[(T_R - T_B)U\left(\frac{\pi d^2}{2} + \pi dh\right)\right] / \left(\frac{\pi d^2 h\rho c_P}{4}\right)$$

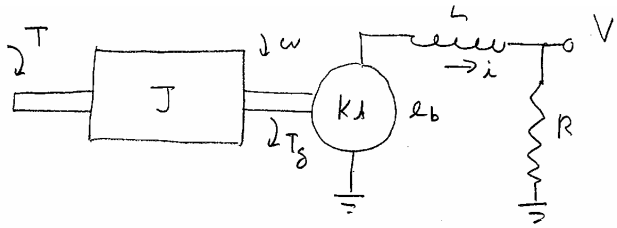
$$\dot{T}_B + \frac{4U\left(\frac{d}{2} + h\right)}{dh\rho c_P} T_B = \frac{4U\left(\frac{d}{2} + h\right)}{dh\rho c_P} T_R$$

The system is analogous to the charging capacitor example.

Note that the units work out (as they must).

$\frac{4U\left(\frac{d}{2} + h\right)}{dh\rho c_P}$ has units of 1/sec

3/.



mechanical: $\dot{\omega} = \dfrac{T - T_g}{J}$ (1)

electrical: $-e_b + L\dot{i} + Ri = 0$ (2)

generator: $e_b = k_t \omega$ (3)

$T_g = k_t i$ (4)

output: $V = Ri$, $i = V/R$ (5)

$\dot{i} = \dot{V}/R$ (6)

start with (2)

use (5), (6) $\quad -e_b + L\left(\dot{V}/R\right) + R\left(V/R\right) = 0$

use (3) $\quad -k_t\omega + \frac{L}{R}\dot{V} + V = 0$

differentiate $\quad -k_t\dot{\omega} + \frac{L}{R}\ddot{V} + \dot{V} = 0$

use (1) $\quad -k_t\left(\frac{T - T_g}{J}\right) + \frac{L}{R}\ddot{V} + \dot{V} = 0$

use (4) $\quad -k_t\left(\frac{T - k_t i}{J}\right) + \frac{L}{R}\ddot{V} + \dot{V} = 0$

use (5) $\quad -k_t\left(\frac{T - k_t\left(\frac{V}{R}\right)}{J}\right) + \frac{L}{R}\ddot{V} + \dot{V} = 0$

simplify: $\quad \frac{L}{R}\ddot{V} + \dot{V} + \frac{k_t^2}{RJ} = \frac{k_t}{J}T$

check units: all terms are in $\dfrac{kg\, m^3}{coulomb\ s^2}$ ✓

in the steady state $\ddot{V}$ and $\dot{V}$ are zero

$$\therefore \frac{k_t^2}{RJ} = \frac{k_t}{J} T \quad \text{from our equation.}$$

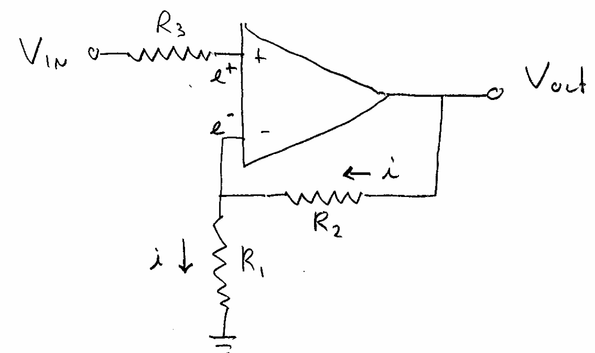
$$\frac{k_t}{R} V = T$$

we also know that in the steady state T_g must equal T and e_b must equal V

$$\therefore \quad T = T_g = k_t i = k_t\left(\frac{V}{R}\right) = \frac{k_t}{R} V \quad \text{from eqn (4)}$$

this agrees with our derived equation.. ✓

4/.



no current through R_3, $\therefore e^+ = V_{IN}$
inputs tend to same voltage $\therefore e^- = e^+ = V_{IN}$

$$i = \frac{V_{out}}{R_1 + R_2} = \frac{e^-}{R_1} = \frac{V_{IN}}{R_1}$$

$$\therefore \quad V_{out} = V_{IN}\left(\frac{R_1 + R_2}{R_1}\right)$$

non-inverting amplifier