MATH 1339 A-MIDTERM # 1-2013

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NAME and I.D.# Solutions

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

For long answer questions, YOU MUST SHOW YOUR WORK.

ONLY BASIC CALCULATORS ARE ALLOWED.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

A	C	B	A
#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1 *Let $f(x) = x^2 + 1$. Determine the average rate of change of f over the interval* $[0, 2]$.

A) $\boxed{2}$ **B)** $\frac{2}{3}$ **C)** 15 **D)** $\frac{-1}{12}$

$$\frac{f(2)-f(0)}{2-0} = \frac{5-1}{2} = \frac{4}{2} = 2$$

Question 2 *The equation of the tangent line of the function $f(x) = \sqrt[3]{x} + x$ at $x = 1$ is:*

A) $y = \frac{4}{3}x - 1$ **B)** $y = \frac{4}{3}x + 9$ $\boxed{\textbf{C)}}$ $y = \frac{4}{3}x + \frac{2}{3}$ **D)** $y = x + \frac{2}{3}$

$m = f'(1) = \frac{4}{3}$

$f'(x) = (x^{\frac{1}{3}} + x)' = \frac{1}{3}x^{-\frac{2}{3}} + 1 = \frac{1}{3\sqrt[3]{x^2}} + 1$

$f'(1) = \frac{1}{3} + 1 = \frac{4}{3}$

equation of tangent line:

$y - f(1) = \frac{4}{3}(x-1)$

$y - 2 = \frac{4}{3}x - \frac{4}{3}$

$y = \frac{4}{3}x + \frac{2}{3}$

Question 3 *What is the value of the following limit?*

$$\lim_{x \to 3^-} \frac{|x-3|}{x^2-2x-3}.$$

$$|x-3| = \begin{cases} x-3 & x \geq 3 \\ -(x-3) & x<3 \end{cases}$$

(Note: it is **one sided** limit.)

A) $\frac{1}{4}$ $\boxed{\textbf{B)}}$ $\frac{-1}{4}$ **C)** 0 **D)** ∞.

$$\lim_{x \to 3^-} \frac{|x-3|}{x^2-2x-3} = \lim_{x \to 3} \frac{-\cancel{(x-3)}}{\cancel{(x-3)}(x+1)} = \lim_{x \to 3} \frac{-1}{x+1} = \frac{-1}{4}$$

Question 4 *The acceleration of a moving object with the position function* $s(t) = (t^2-2)(t^2+2)$ *at the time* $t=2$ *is:*

$\boxed{\textbf{A)}}$ 48 **B)** 24 **C)** 12 **D)** 85.

$a(t) = s''(t)$

$s(t) = (t^2-2)(t^2+2) = t^4 - 4$

$s'(t) = (t^4-4)' = 4t^3$

$s''(t) = 12t^2$

$s''(2) = 12 \times 4 = 48$

Long Answer Questions (5-7)

Question 5 (8 points) *Evaluate the following limits:*

1. $\lim_{x \to -5} \frac{x^2+4x-5}{25-x^2} = \frac{25-20-5}{25-25} = \frac{0}{0}$ indeterminate

$= \lim_{x \to -5} \frac{\cancel{(x+5)}(x-1)}{\cancel{(x+5)}(x-5)} = \lim_{x \to -5} \frac{x-1}{x-5} = \frac{-5-1}{-5-5} = -\frac{6}{-10} = \frac{6}{10}$

2. $\lim_{x \to 0} \frac{\sqrt{x+16}-4}{x} = \frac{\sqrt{16}-4}{0} = \frac{0}{0}$ indeterminate

$= \lim_{x \to 0} \frac{\sqrt{x+16}-4}{x} \times \frac{\sqrt{x+16}+4}{\sqrt{x+16}+4} = \lim_{x \to 0} \frac{(x+16)-16}{x(\sqrt{x+16}+4)}$

$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+16}+4)} = \lim_{x \to 0} \frac{1}{\sqrt{x+16}+4} = \frac{1}{4+4} = \frac{1}{8}$

3. $\lim_{x \to +\infty} \frac{3x^3+3x^2+1}{2x^3+10} = \lim_{x \to +\infty} \frac{3\cancel{x^3}}{2\cancel{x^3}} = \frac{3}{2}$

Question 6 **(12 points)**

(a) Using only the the First Principle Definition of Derivative, calculate $f'(a)$ where

$$f(x) = \frac{1}{x+5}.$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{1}{a+h+5} - \frac{1}{a+5}}{h} = \lim_{h \to 0} \frac{\cancel{a} + \cancel{5} - (\cancel{a} + h + \cancel{5})}{h(a+5)(a+h+5)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(a+5)(a+h+5)} = \lim_{h \to 0} \frac{-1}{(a+5)(a+5)} = \frac{-1}{(a+5)^2}$$

(b) Determine the set of points that the following function is continuous at them.

$$f(x) = \begin{cases} x^2 + 4 & \text{if } x < 0 \\ \sqrt{x} + 2 & \text{if } 0 \leq x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$$

$\text{Dom}(f) = \mathbb{R}$

$$\begin{cases} \lim_{x \to 0^-} f(x) = \lim_{x \to 0} x^2 + 4 = 4 \\ \qquad\qquad\qquad\qquad\qquad\quad \neq \\ \lim_{x \to 0^+} f(x) = \lim_{x \to 0} \sqrt{x} + 2 = 2 \end{cases} \Rightarrow \lim_{x \to 0} f(x) \text{ does not exist.}$$

$$\begin{cases} \lim_{x \to 1^-} f(x) = \lim_{x \to 1} \sqrt{x} + 2 = 1 + 2 = 3 \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \neq \\ \lim_{x \to 1^+} f(x) = \lim_{x \to 1} x^2 - 1 = 1 - 1 = 0 \end{cases} \Rightarrow \lim_{x \to 1} f(x) \text{ does not exist.}$$

set of continuity points $= \text{Dom}(f) \setminus \{0, 1\}$

$= \mathbb{R} \setminus \{0, 1\}.$

Question 7 (10 points) *Use the derivative Rules to find the derivative of the following functions.*

$$f(x) = \frac{3x^2 - 2}{5x + 1}$$

$$f'(x) = \frac{(3x^2-2)'(5x+1) - (3x^2+2)(5x+1)'}{(5x+1)^2}$$

$$= \frac{6x(5x+1) - (3x^2-2)5}{(5x+1)^2} = \frac{30x^2+6x-15x^2+10}{(5x+1)^2}$$

it is enough.

$$= \frac{15x^2+6x+10}{(5x+1)^2}$$

$$g(x) = (-2x + 1)\sqrt{x^2 + x}$$

$$g'(x) = (-2x+1)'\sqrt{x^2+x} + (-2x+1)\cdot(\sqrt{x^2+x})'$$

$$= -2\sqrt{x^2+x} + (-2x+1)((x^2+x)^{\frac{1}{2}})'$$

$$= -2\sqrt{x^2+x} + (-2x+1)(\frac{1}{2}(2x+1)(x^2+x)^{-\frac{1}{2}})$$

$$= -2\sqrt{x^2+x} + (-2x+1)(\frac{2x+1}{2\sqrt{x^2+x}})$$

it is enough.

$$= \frac{-2(x^2+x) + (2x+1)(-2x+1)}{2\sqrt{x^2+x}}$$

$$= \frac{-2(x^2+x) + (-4x^2+1)}{2\sqrt{x^2+x}} = \frac{-6x^2-2x+1}{2\sqrt{x^2+x}}$$

Space for additional work