STAT 2509 A — Assignment #1.

SOLUTION

//36

Q.1: [2] a) (ii) a population (1), b) (iii) a statistic (1)

Q.2: [1] b) mean & median (1)

Q.3: [7]

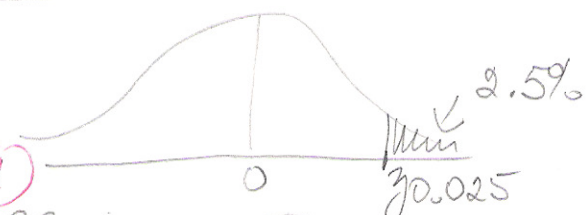
a) quantitative (1/2) & continuous (1/2)
b) purely categorical (1) (or qualitative)
c) quantitative (1/2) & discrete (1/2)
d) categorical (1/2) & ranked (1/2)
e) categorical (1/2) & ranked (1/2)
f) quantitative (1/2) & discrete (1/2)
g) quantitative (1/2) & continuous (1/2)

Q.4: [6]

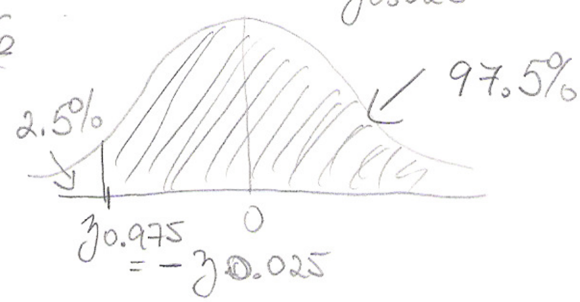
(i) $\bar{x}$ – statistic (1), (ii) σ^2 – parameter (1)
(iii) μ – parameter (1), (iv) s^2 – statistic (1)
(v) β_1 – parameter (1), (vi) $\hat{\beta}_1$ – statistic (1)

Q.5: [6]

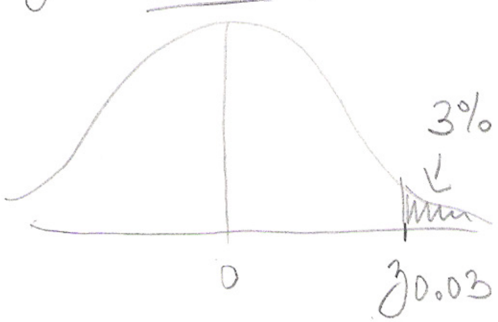
a) $z_{0.025} = \underline{1.96}$ (1)



b) $z_{0.975} = -z_{0.025} = \underline{-1.96}$ (1)

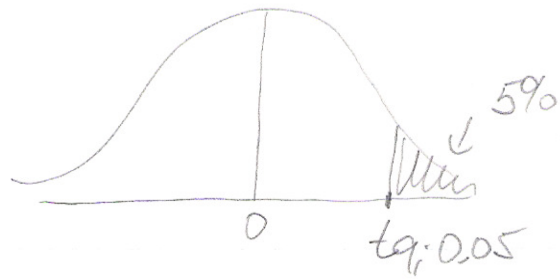


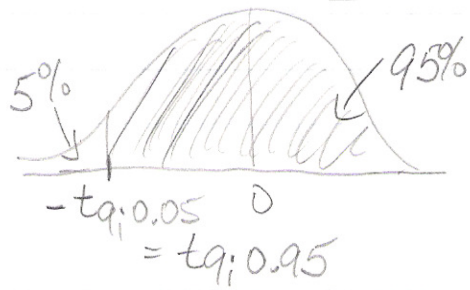
c) $z_{0.03} = \underline{1.88}$ (1)



d) $t_{9;0.05} = \underline{1.833}$ 1

e) $-t_{9;0.05} = \underline{-1,833}$ 1





f) $t_{9;0.95} = -t_{9;0.05} = \underline{-1,833}$ 1

Q.6. $N(\mu,\sigma^2)$, σ^2 known

[6]

a) (i) $\bar{x} \pm 1,96\,\sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 1.96 \Rightarrow \alpha/2 = 0.025$

$\alpha = 0,05$

$\therefore$ 1 $\underline{95\%\ \text{C.I.}}$ for μ $\quad 1-\alpha = 0.95$

(ii) $\bar{x} \pm 2.24\,\sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 2.24 \Rightarrow \alpha/2 = 0.0125$

$\alpha = 0.025$

$\therefore$ 1 $\underline{97.5\%\ \text{C.I.}}$ for μ $\quad 1-\alpha = 0.975$

(iii) $\bar{x} \pm 3,09\,\sigma/\sqrt{n} \Rightarrow z_{\alpha/2} = 3.09 \Rightarrow \alpha/2 = 0.0010$

$\alpha = 0,002$

$\therefore$ $\underline{99.8\%\ \text{C.I.}}$ for μ 1 $\quad 1-\alpha = 0.998$

b) $\left(\bar{x} \pm z_{\alpha/2}\,\sigma/\sqrt{n}\right)$ (i) 89.68% C.I. $\Rightarrow 1-\alpha = 0.8968$

$\alpha = 0.1032$

$\underline{z_{\alpha/2} = 1.63}$ 1 $\Leftarrow \alpha/2 = 0.0516$

(ii) 99.20% C.I, $\Rightarrow 1-\alpha = 0.9920$

$\alpha = 0.0080$

$\alpha/2 = 0,0040 \Rightarrow z_{\alpha/2} = \underline{2.65}$ 1

(iii) 75.40% C.I. $\Rightarrow 1-\alpha = 0.7540$

$\alpha = 0.2460$ (1)

$\alpha/2 = 0.1230 \Rightarrow z_{\alpha/2} = 1.16$

Q.7: [4]

a) Ha (alternative hypothesis) (1) is the one we want to show (or prove) i.e. we are trying to find sufficient evidence for

Ho (null hypothesis) - is negating the (1) statement in Ha. It is called a "fall-back" hypothesis. It tests against Ha.

b) Type I error = error we make when we reject Ho when it is true (1)

P[Type I error] = α

Type II error = error we make when we do not reject Ho when it is false (1)

P[Type II error] = β.

Q.8: [4]

$$\sum_{i=1}^{N}(x_i-\mu)^2 \overset{(1)}{=} \sum_{i=1}^{N}(x_i^2+\mu^2-2\mu x_i) = \sum_{i=1}^{N} x_i^2 + \sum_{i=1}^{N}\mu^2 - 2\mu\sum_{i=1}^{N}x_i =$$

$$= \sum_{i=1}^{N} x_i^2 + N\mu^2 - 2\mu\sum_{i=1}^{N} x_i \quad \left(\text{sub in } \mu = \frac{\sum_{i=1}^{N} x_i}{N}\right) \Rightarrow (2)$$

$$= \sum_{i=1}^{N} x_i^2 + \cancel{N}\frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N^{\cancel{2}}} - 2\frac{\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} x_i\right)}{N} =$$

$$= \sum_{i=1}^{N} x_i^2 + \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N} - \frac{2\left(\sum_{i=1}^{N} x_i\right)^2}{N} = \sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N} \quad (1)$$

$$\therefore \frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2 = \frac{1}{N}\left[\sum_{i=1}^{N} x_i^2 - \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N}\right]$$

$\square$