7. **[5 +5 marks]** a) Calculate $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ given that $f(x, y) = e^x \cos y$.

b) Find the point where the function f defined by $f(x, y) = e^{-x^2-y^2}$, for $-\infty < x, y < +\infty$ attains its absolute maximum, and determine its value.

a) $\frac{\partial f}{\partial x} = e^x \cos y$ (2), $\frac{\partial^2 f}{\partial x^2} = e^x \cos y$ (2), $\frac{\partial f}{\partial y} = -e^x \sin y$ (2) and

$\frac{\partial^2 f}{\partial y^2} = -e^x \cos y$. (2) $\therefore$ $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. (2)

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b). $f(x,y) = e^{-x^2-y^2} \Rightarrow \frac{\partial f}{\partial x} = -2x e^{-x^2-y^2}$ (1), $\frac{\partial f}{\partial y} = -2y e^{-x^2-y^2}$ (1)

and so there is only ONE critical point, namely, $(0,0)$. (2)

It is a relative max. by the second derivative test,

since $f_{11}(0,0) f_{22}(0,0) - f_{12}(0,0) > 0$ and $f_{11}(0,0) < 0$.

It is an absolute maximum because $f(x,y) \to 0$ as (x,y) grow without bound, (or because $e^{-x^2-y^2} \leq 1$ always.)

The value of the maximum is "1". (1)