

# CARLETON UNIVERSITY

FINAL  
EXAMINATION  
December 2008

## DURATION: 3 HOURS

**Department Name and Course Number:** School of Mathematics and Statistics, MATH1104.

**Course Instructors:** J. Abdulrahman, P. Chafe, X. Hua and M. Sadeghi

AUTHORIZED MEMORANDA

NON-PROGRAMMABLE CALCULATOR PERMITTED

This examination paper **MAY NOT** be taken from the examination room.  
The scratch paper is not permitted, you can use the backs of the pages if needed.

**First (Given) Name :**

**Last Name:**

**Student No :**

Circle your section : A: Chafe, B: Abdulrahman C: Hua D Sadeghi

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PLEASE READ THE FOLLOWING INSTRUCTIONS BEFORE PROCEEDING

1. Please count your pages now. **This examination has 9 pages numbered from 1–9**, including this page. The page numbered 9 is blank and is to be used for rough work only. If you feel there is a page missing please report this to your Proctor.
2. **The examination is out of a total of 50 marks** and consists of 25 Multiple choice questions, 2 points each. Select the best possible answers.

**Place your answers in the following table.**

|     |     |     |     |     |     |    |    |    |     |      |      |     |      |
|-----|-----|-----|-----|-----|-----|----|----|----|-----|------|------|-----|------|
| # 1 | # 2 | # 3 | # 4 | # 5 | # 6 | #7 | #8 | #9 | #10 | # 11 | # 12 | #13 | # 14 |
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|------|------|------|------|------|------|------|------|------|------|------|
| # 15 | # 16 | # 17 | # 18 | # 19 | # 20 | # 21 | # 22 | # 23 | # 24 | # 25 |
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1. Given  $A^{-1} = \begin{bmatrix} -1 & 3 \\ 4 & 1 \end{bmatrix}$  and  $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ .

What is the solution of the equation  $A\mathbf{x} = \mathbf{b}$ ?

(a)  $\begin{bmatrix} 5 \\ 9 \end{bmatrix}$     (b)  $\begin{bmatrix} 1 \\ 9 \end{bmatrix}$     (c)  $\begin{bmatrix} 5 \\ -7 \end{bmatrix}$     (d)  $\begin{bmatrix} -5 \\ 7 \end{bmatrix}$     (e)  $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

2. Suppose that a system of equations has the augmented matrix  $\left[ \begin{array}{cc|c} 1 & h & 3 \\ 5 & -10 & k \end{array} \right]$ . If the system is inconsistent, then the values of  $h$  and  $k$  must be:

(a)  $h = -2, k = 15$

(b)  $h \neq -2, k \neq 15$

(c)  $h \neq -2, k = 15$

(d)  $h = -2, k \neq 15$

(e)  $h = 2, k = 15$

3. Find all the values of  $h$  such that the vectors  $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ -2 \\ 0 \\ h \end{bmatrix}$

are linearly dependent:

(a)  $h = 2$     (b)  $h \neq 2$     (c)  $h = -2$     (d)  $h \neq -2$     (e)  $h \neq \pm 2$

4. Let  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation defined by

$$T \left( \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x + y - z \\ x + y - w \\ y + z \end{bmatrix}. \text{ Which of the following is true?}$$

- (a)  $T$  is onto and one-to-one
- (b)  $T$  is onto, but not one-to-one
- (c)  $T$  is one-to-one, but not onto
- (d)  $T$  is neither onto nor one-to-one
- (e) none of the above statements is true

5. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation such that

$$T \left( \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ and } T \left( \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \text{ Find } T \left( \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \right).$$

- (a)  $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$
- (b)  $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$
- (c)  $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$
- (d)  $\begin{bmatrix} -11 \\ 2 \end{bmatrix}$
- (e)  $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$

6. Let  $A = [v_1 \ v_2 \ v_3 \ v_4] = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ .

Which of the following can not be a basis of  $\text{Col}A$ ?

- (a)  $\{v_1, v_3\}$
- (b)  $\{v_2, v_4\}$
- (c)  $\{v_3, v_4\}$
- (d)  $\{v_1, v_2\}$
- (e)  $\{v_2, v_3\}$

7. Let  $A$ ,  $B$  and  $C$  be  $n \times n$  invertible matrices. What is  $(A^{-1}B^T(C^{-1})^T)^{-1}$ ?

- (a)  $AB^TC^T$     (b)  $CBA$     (c)  $A(B^T)^{-1}C^T$     (d)  $C^T(B^{-1})^TA$     (e)  $C^T(B^T)^{-1}A^{-1}$

8. Let  $A$  be a  $5 \times 7$  matrix such that  $\dim \text{Col}A = 4$ . What is  $\dim \text{Nul}A$ ?

- (a) 0    (b) 1    (c) 2    (d) 3    (e) 4

9. Let  $\mathcal{B} = \{v_1, v_2\}$  be a basis for a subspace  $H$ , and let  $x = 2v_2 - v_1$ . Find  $[x]_{\mathcal{B}}$ , i.e., the coordinate vector of  $x$  relative to  $\mathcal{B}$ .

- (a)  $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$     (b)  $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$     (c)  $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$     (d)  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$     (e)  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

10. Let  $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 4 & 4 \\ 5 & 5 & 5 & 5 & 6 \end{bmatrix}$ . What is  $\det A$ ?

- (a) 0    (b) 6    (c) 12    (d) 24    (e) 48

11. Let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$  and  $B = \begin{bmatrix} g & h & i \\ d - a & e - b & f - c \\ -2a & -2b & -2c \end{bmatrix}$

If  $\det(A) = 2$ , what is  $\det(B)$ ?

- (a) 2      (b) 4      (c) -4      (d) -2      (e) 0

12. Let  $A$  be an  $2 \times 2$  matrix and  $\det A = -3$ . What is  $\det(2A^2A^T A^{-1})$ ?

- (a) -36      (b) -18      (c) 18      (d) 36      (e) 9

13. Consider the matrix equation  $A\mathbf{x} = \mathbf{b}$ , where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

What is the value of  $x_1$ ? (Hint: Use Cramer's Rule)

- (a)  $\frac{2}{17}$       (b)  $\frac{4}{17}$       (c) 17      (d) 2      (e) 0

14. Let  $z = \cos(\pi/16) + i \sin(\pi/16)$  and  $w = \cos(\pi/8) + i \sin(\pi/8)$ . What is  $z^2 w$  ?

- (a)  $\frac{1}{\sqrt{2}}(1+i)$       (b)  $\frac{1}{\sqrt{2}}(1-i)$       (c)  $1+i$       (d)  $1-i$       (e)  $1$

15. What is the standard form of the complex number  $\frac{2+4i}{1+i}$  ?

- (a)  $1-3i$       (b)  $1-i$       (c)  $1+2i$       (d)  $3+i$       (e)  $3-i$

16. Let  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix}$ . What are the eigenvalues of  $A$ ?

- (a)  $1, -2, 2$       (b)  $1, 2, -4$       (c)  $2, 2, 4$       (d)  $0, 1, 2$       (e)  $-2, 1, 6$

17. Let  $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$ . You are given that  $\lambda = 1$  is an eigenvalue of  $A$ .

What is the dimension of the corresponding eigenspace?

- (a)  $0$       (b)  $1$       (c)  $2$       (d)  $3$       (e)  $4$

18. Which of the following sets are subspaces of  $\mathbb{R}^3$ .

$$U = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \mid a + b = 1 \right\}, \quad V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a - b = c \right\},$$

$$W = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \quad H = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(a)  $U$  and  $V$     (b)  $V$  and  $W$     (c)  $V$  and  $H$     (d)  $U$  and  $W$     (e)  $V$  only

19. Let  $A = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$ . What are the eigenvalues of  $A$ ?

(a)  $1 \pm 3i$     (b)  $-1 \pm 3i$     (c)  $\pm 3i$     (d)  $\pm 1$     (e) 1 and  $3i$

20. Let  $A$  be a  $2 \times 2$  matrix with eigenvectors  $v_1 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$  and  $v_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

corresponding to eigenvalues  $\lambda_1 = 1$  and  $\lambda_2 = -1$ , respectively. What is the matrix  $A$ ?

(a)  $\begin{bmatrix} 7 & -2 \\ -7 & 24 \end{bmatrix}$     (b)  $\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$     (c)  $\begin{bmatrix} 7 & -2 \\ 24 & -7 \end{bmatrix}$     (d)  $\begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$     (e)  $\begin{bmatrix} 7 & 24 \\ -2 & -7 \end{bmatrix}$

21. Let  $x = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$ ,  $u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ ,  $v = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$  and  $W = \text{Span}\{u, v\}$ .

What is the orthogonal projection of the vector  $x$  onto the subspace  $W$ ?

- (a)  $\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$       (b)  $\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$       (c)  $\begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$       (d)  $\begin{bmatrix} 4 \\ -4 \\ -7 \end{bmatrix}$       (e)  $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

22. Let  $u_1 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ ,  $u_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ , and  $u_3 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ .

You are given that  $S = \{u_1, u_2, u_3\}$  is an orthogonal basis for  $R^3$ .

Let  $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ . What is the value of  $c_3$  in the equation  $x = c_1u_1 + c_2u_2 + c_3u_3$ ?

- (a) -1      (b) 1      (c) -2      (d) 2      (e) 4

23. Let  $x = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$  and  $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ .

What is the distance from  $x$  to the line through  $u$  and the origin?

- (a)  $\sqrt{20}$       (b)  $\sqrt{45}$       (c)  $\sqrt{50}$       (d)  $\sqrt{90}$       (e)  $\sqrt{125}$

24. Let  $u = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$  and  $v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ . What is the angle between the vectors  $u$  and  $v$ ?

- (a)  $\pi/3$       (b)  $-\pi/3$       (c)  $2\pi/3$       (d)  $-2\pi/3$       (e)  $\pi/2$

25. Let  $A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ . Which of the following vectors is in  $(\text{Col}A)^\perp$ ?

- (a)  $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$       (b)  $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$       (c)  $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$       (d)  $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$       (e)  $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

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