Final Exams

ADM 3351 A

Fixed Income Securities
2 hours and 50 minutes
Fall 2007
Dr. C. Guo

Student's Name: (Print legibly) _____

Student Number: _____
Closed-book exam. A calculator is required. A formula sheet is attached at the end.

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Question	Credit	Mark
1	20	
2	10	
3	10	
4	10	
5	15	
6	20	
7	15	
Total	100	

Question 1 (20 points)

A 6.5% (pass-through rate) 30-year mortgage-pass-through security is already seasoned with a weighted average maturity (WAM) of 332 months left. The current outstanding balance of $200,000,000, and the weighted average coupon (WAC) rate is 8%. The interest rates are quoted as annual percentage rate (APR) based on monthly compounding assumption. Assume that the prepayment rate is 150 PSA.

a)Calculate the total cash flow of the security to the investors for the next 2 months. Fill out the table on the next page. (15 points)

b)Home owners do not want to be locked in high mortgage rates. Briefly explain why financial institutions are unwilling to offer fixed rate mortgage for long term (e.g., 30 years), and how this problem has been solved by the federally sponsored agencies such as Fannie Mae and Freddie Mac. (5 points)

Answer to Question 1:

(a)

Input Information:		
Beginning Balance:	200,000,000	
Pass-through rate:	0.065	(Monthly compounding)
WAC	0.08	(Weighted-average-coupon of the asset pool)
WAM (No. of months)	332	(Weighted-average-maturity of the asset pool)
CPR of 150PSA	0.09	
Converted values below:		
Monthly rate	0.005416667	(pass-through-rate/12)
Monthly WAC:	0.006666667	(WAC/12)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Month	Outstanding Balance	SMM	Mortgage Payment	Interest to Investors	Scheduled Principal	Prepayment	Total Principal	Cash Flow to Investors
1	200,000,000	0.00755626	1,498,366	1,083,333	165,032	1,510,004	1,675,037	2,758,370
2	198,324,963	0.00782842	1,487,044	1,074,260	164,877	1,551,280	1,716,158	2,790,418

(b) Financial institutions are unwilling to offer long term fixed rate mortgages because their sources of funds are usually short-term deposits. Because their businesses are based on spread between borrowing and lending, they are going to suffer huge losses if the cost of funds during the life of a mortgage loan exceeds the fixed mortgage rate. However, there are fixed income investors who are willing to accept lower interest rate if the general level of the term structure declines. The federal agencies provide funds to financial institutions by buying their mortgage contracts, then package and sell the contracts to the end investors. As a result, the home owners get the prepayment options because the risks are assumed by the end investors, while the financial institutions and the federal agencies receive fees from providing financial services.

Question 2 (10 points)

In a Collateralized Mortgage Obligations (CMO), a 6.5% fixed-rate tranche with $60,000,000 principal balance is used to create a floating-rate tranche (floater) with $40,000,000 balance and an inverse-floating-rate tranche (inverse floater) with $20,000,000 balance. If the floater receives one-month LIBOR + 1%, what interest rate should the inverse floater receive? If the floor rate of the inverse floater is 1.5%, what is the cap rate of the floater?

Answer to Question 2

The floater counts for 2/3 of the total balance and the inverse floater counts for 1/3. The interest rate of the inverse floater should be

$$K - L \times LIBOR = K - 4 \times LIBOR,$$

where K is such that

$$(2/3)(LIBOR + 1\%) + (1/3)(K - 2 \times LIBOR) = (2/3) \times 1\% + (1/3)K = 6.5\%.$$

Hence, $K = 17.5\%$.

The inverse floater hits floor when

$$17.5\% - 2 \times LIBOR^{Max} = 1.5\%.$$

Solve for $LIBOR^{Max} = 8.0\%$

Hence, the cap for the floater is 8% + 1% = 9%.

Question 3 (10 points)

Assume that a CMO has just one Planned Amortization Class (PAC) bond, and one supporting bond. The PAC bond is created between PSA 90 and PSA 300. Part of the principal payments are calculated and shown in the following table. Assume that there is a sufficient supporting bond at the beginning, but it will be exhausted by Month 210. Given the actual principal payments in the 4th column, fill out the respective principal payments to this PAC bondholders in the last column, then, explain briefly on your results.

Month	At 90% PSA	At 300% PSA	Actual Principal Payment	*Principal Payment to PAC Bondholder*
1	508,169	1,075,931	600,000	*508,169*
2	569,843	1,279,412	800,000	*569,843*
…				
(The supporting bond has gone on Month 210)				
211	949,282	213,309	200,000	*200,000*
212	946033	209,409	300,000	*300,000*
…				
349	613,875	12,314		
350	*612,292*	*12,008*		

Explanation:

For Months 1 and 2, because the actual principal payments are between the PAC window, and the supporting bond is available, the PAC bondholders will receive the minimum of the PAC window principal payments, which are under PSA 90.

After the supporting bond is gone, there is no more protection on the PAC bond. Hence, the PAC bondholders will receive the actual principal payments, i.e., $200,000 on Month 211 and $300,000 on Month 212, respectively.

Question 4 (10 points)

Assume that the yield curve is always flat. The current interest rate level is at 7% (BEY). The U.S. Federal Reserve has indicated that it will cut interest rate gradually over the next two years. Based on this information, you have estimated that the interest rate will fall 50 basis points every 6 months for the next 2 years. You have a two-year investment horizon. You are considering investing in an 8% U.S. Treasury bond which has 20 years to maturity. What annualized return (in BEY) can you realize over two years if you invest in this bond?

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Solution:

If investing in the 10% 10-year note,

Year	R(BEY)	r(semiannual)	# of years	# of periods	Coupon(% p.a.)	Price	Interest	Cum Int
0.0	0.070	0.035	20.0	40	8	110.678	0	0
0.5	0.065	0.033	19.5	39	8	116.448	4.000	4.000
1.0	0.060	0.030	19.0	38	8	122.492	4.000	8.130
1.5	0.055	0.028	18.5	37	8	128.796	4.000	12.374
2.0	0.050	0.025	18.0	36	8	135.334	4.000	16.714
Horizon R	0.0826	BEY=	0.1653					

Horizon Return $= [(135.334+16.714)/110.678]^{1/4} - 1 = 0.0826$ per half-year,
and in BEY, 2*0.0826 = 0.1653.

Question 5 (15 points)

The current term structure of interest rate is given as par-yield, i.e., Bond equivalent yield on par valued coupon bonds with maturities greater than 1 year. Some of the zero-coupon spot rates have been identified (also quoted as BEY).

a)Calculate the missing spot rates (10 points)
b)Find the price of a 10% coupon bond with 5 years to maturity (5 points).

Maturity (Year)	Par-yield (BEY)	Spot Rate
0.5	5.250	
1.0	5.500	
1.5	5.750	
2.0	6.000	
2.5	6.250	6.2822
3.0	6.500	6.5494
3.5	6.750	6.8213
4.0	6.800	6.8694
4.5	7.000	7.0947
5.0	7.100	7.2045

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Answer:

			100.0001	100.0001	100.0001	100.0001	100.0001	100.0000	99.9999	100.0002
			5.750	6.000	6.250	6.500	6.750	6.800	7.000	7.100
			1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000
Year	YTM	Spot Rate	2.875	3.000	3.125	3.250	3.375	3.400	3.500	3.550
0.5	5.250	5.2500	2.801	2.923	3.045	3.167	3.289	3.313	3.410	3.459
1.0	5.500	5.5000	2.723	2.842	2.960	3.078	3.197	3.220	3.315	3.363
1.5	5.750	5.7597	94.475	2.755	2.870	2.985	3.099	3.122	3.214	3.260
2.0	6.000	6.0191	0.000	91.480	2.775	2.887	2.998	3.020	3.109	3.153
2.5	6.250	6.2822	0.000	0.000	88.350	2.784	2.891	2.913	2.999	3.041
3.0	6.500	6.5494	0.000	0.000	0.000	85.099	2.782	2.802	2.885	2.926
3.5	6.750	6.8213	0.000	0.000	0.000	0.000	81.745	2.689	2.768	2.807
4.0	6.800	6.8694	0.000	0.000	0.000	0.000	0.000	78.921	2.671	2.710
4.5	7.000	7.0947	0.000	0.000	0.000	0.000	0.000	0.000	75.629	2.594
5.0	7.100	7.2045	0.000	0.000	0.000	0.000	0.000	0.000	0.000	72.687

Par-yield	Spot Rate	Discount Factor	10% 5Y Cashflow	PV
5.25	5.25	0.974421437	5	4.872107186
5.50	5.50	0.947188331	5	4.735941653
5.75	5.76	0.918351697	5	4.591758485
6.00	6.02	0.888157608	5	4.440788039
6.25	6.28	0.856724538	5	4.283622688
6.50	6.55	0.824207093	5	4.121035464
6.75	6.82	0.790757144	5	3.953785722
6.80	6.87	0.763255472	5	3.816277361
7.00	7.09	0.730716814	5	3.653584068
7.10	7.20	0.701953148	105	73.70508057
			Price	112.1739812

Question 6 (20 points)

Assume all the coupon bonds have just paid out their respective semi-annual interest payments today. Assume further that all the bonds have a par value of $100 and are perfectly divisible (i.e., buying or selling any fraction of a bond unit is possible).

You have a fixed liability of $1,000,000 at the end of the **fifth** year. Assume that you can only invest in two bonds: Bond 1 carries 10% coupon with 10 years to maturity and Bond 2 carries 13% coupon with 5 years to maturity.

a)(10 points) The term structure is flat and the yield (expressed as bond-equivalent yield) is 8 percent. Construct an immunization portfolio today.
b)(10 points) Assume the term structure shifts to 9% flat in six months, check if you are able to meet your obligation, then rebalance your portfolio.

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Answer:

Part (a)	Answer to				
Input Information					
Liability value at the end	$ 1,000,000				
Bond	1	2	**Liability**		
Maturity (year)	10	5	5		
Coupon	0.100	0.130	0.000		
Yield	0.080	0.080	0.080		
Converted Values					
Maturity (half-year)	20.000	10.000	10.000		
Semiannual coupon	0.050	0.065	0.000		
Semiannual yield	0.040	0.040	0.040		
Price (par $100)	113.59	120.28	675,564.17		
Macaulay Duration	13.54	7.89	10		
Weight	0.373575966	0.626424034			
Unit	2,221.80	3,518.45	1.00		
Market Value	252,374.54	423,189.63	675,564.17	0.00	(A-L)

(b)

Answer:

Part (b): Six Months Later	0.5				
Liability value at the end	$ 1,000,000				
Bond	1	2	**Liability**		
Maturity (year)	9.5	4.5	4.5		
Coupon	0.100	0.130	0.000		
Yield	0.090	0.090	0.090		
Converted Values					
Maturity (half-year)	19.000	9.000	9.000		
Semiannual coupon	0.050	0.065	0.000		
Semiannual yield	0.045	0.045	0.045		
Price (par $100)	106.30	114.54	672,904.43		
Macaulay Duration	12.90	7.23	9		
Interest received	11,108.98	22,869.93	0.00		
Bond Value	236,169.47	402,994.92	672,904.43		
Mkt Value before rebalancing	247,278.45	425,864.86	672,904.43	238.88	(A-L)
New weight	0.313	0.687			
New unit	1,978.31	4,038.99			
Net purchase(sales)	(243.48)	520.54	Interests available	Net Cash Flow	
Cash flow	(25,881.29)	59,621.32	33,978.91	238.88	
Mkt value after rebalancing	210,288.18	462,616.24	672,904.43	0.00	(A-L)

Question 7 (15 points)

Briefly answer the follwing short questions (3 points each) :

1. What is meant by on-the-run Treasuries? List all of the on-the-run Treasury securities.

Answer:
The on-the-run Treasury issues are the most recently auctioned issue of a given maturity. These issues include the 3-month, 6-month, and 1-year Treasury bills; the 2-year, 5-year, and 10-year Treasury notes; and the 30-year Treasury bond. Treasury bills are zero-coupon instruments; the notes and the bond are coupon securities.

2 What is TIPS? Suppose that the coupon rate for a TIPS is 3% and an investor has purchased $10,000 of par value (initial principal) of this issue today and that the anual inflation rate is 2%. What is the dollar coupon interest that will be paid in cash at the end of the first six months?

Answer:
The coupon rate for a TIPS is 3%, the annual inflation rate is 2%, and an investor purchases today $10,000 par value (principal) of this issue. The semiannual inflation rate is 1% (2% divided by 2). The inflation-adjusted principal at the end of the first six-month period is found by multiplying the original par value by one plus the semiannual inflation rate: (1.01)$10,000 = $1,100. The coupon payment is then 1.5% (one-half the real rate of 3%) The coupon payment is therefore 0.015($10,100) = $151.50.

3. What is a sinking fund requirement in a bond issue? How does it work?

Answer:
Corporate bond indentures may require the issuer to retire a specified portion of an issue each year. This is referred to as a sinking fund requirement. Generally, the issuer may satisfy the sinking fund requirement by either (i) making a cash payment of the face amount of the bonds to be retired to the corporate trustee, who then calls the bonds for redemption using a lottery, or (ii) delivering to the trustee bonds purchased in the open market that have a total face value equal to the amount that must be retired.

4. What does the yield spread between commercial paper and Treasury bills of the same maturity reflect?

Answer:
In brief, the yield spread between commercial paper and Treasury bills of the same maturity reflects differences in credit risk, taxability, and liquidity. The commercial paper rate is higher than that on Treasury bills for the same maturity for three reasons. First, the investor in commercial paper is exposed to credit risk. Second, interest earned from investing in Treasury bills is exempt from state and local income taxes. Finally, commercial paper is less liquid than Treasury bills..

5. The following excerpt is taken from an article titled “MERUS to Boost Corporates,” which appeared in the January 27, 1992, issue of *BondWeek*, p.6:

> MERUS Capital Management will increase the allocation to corporates in its $790 million long investment-grade fixed-income portfolio by $39.5 million over the next six months to a year, according to George Wood, managing director. MERUS will add corporates rated single A or higher in the expectation that spreads will tighten as the economy recovers and that some credits may be upgraded.

What types of active portfolio strategies is MERUS Capital Management pursuing?

Answer:

MERUS is employing a **credit spread strategy**. Credit or quality spreads change because of expected changes in economic prospects. Credit spreads between Treasury and non-Treasury issues widen in a declining or contracting economy and narrow during economic expansion (which is MERUS's case).

Formulas:

Standard Coupon bond pricing:

$$P = c \times PVIFA(y,m) + F/(1+y)^m, \quad PVIFA(r,m) = \frac{1-1/(1+r)^m}{r}.$$

General formula of Present Value of Fixed Income Security Discounted by Spot Rates:

$$P = \sum_{t=1}^{n} \frac{c_t}{(1+r_t)^t}.$$

Macaulay Duration for coupon bond:

$$D = \frac{1+y}{y} - \frac{1+y+m(i-y)}{i[(1+y)^m - 1] + y}.$$

In the above formulas, $c = iF$ with i being the semiannual coupon interest rate (i.e., one-half of the quoted annual coupon interest rate), y is the semiannual discount rate, and m is the number of half-years. The result is in unit of half-years. Divide this result by 2 to convert it to number of years.

Modified Duration = Macaulay Duration / (1+y)

Duration Approximation:

$$\text{Modified D} \approx \frac{P_+ - P_-}{2P_0(\Delta y)} = \frac{P_+ - P_-}{P_0((y_+ - y_-)/2)}$$

Convexity measure approximation

$$\approx \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2}$$

Approximating the percentage change of bond price:

$$\Delta P / P = -(\text{Modefied D}) \times \Delta y + \tfrac{1}{2}(\text{Convexity}) \times (\Delta y)^2$$

Duration of bond portfolio:

$$D_P = \sum_i w_i D_i \text{ with } \sum_i w_i = 1.$$

In the following mortgage calculations, where r is the monthly interest rate and m is the number of months.
r = Quoted Mortgage rate / 12.

Monthly Mortgage Payment = Beginning Outstanding Principal/PVIFA(r,m).
Mortgage Balance before refinancing = Old Monthly payment×PVIFA(r,m'),
m' is the remaining amortization periods (months).

The 100 PSA Conditional prepayment rate (CPR) is calculated as

$$CPR = \begin{cases} 6\%(t/30) & \text{if } t \le 30 \\ 6\% & \text{if } t > 30 \end{cases}$$

where t is the number of months elapsed since the origination of the pass-through security.

Single-month-mortality rate $SMM = 1 - (1 - CPR)^{1/12}$.

The floater receives *LIBOR* + a%, and the inverse floater receives $K - L \times LIBOR$,
where the principal of the floater is L times of the inverse floater, i.e., the total principal is 1+L, and the floater counts for L/(1+L).