Math 1119B: Week 7, Lecture 2

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October 25, 2011

Table of Contents

Recap of the last 2 lectures

- Linear independence of vectors,
- recap of span of vectors,
- spanning $\mathbb{R}^m$, how many vectors? Too many vectors.
- Bases.
- The rank of a matrix,
- (vague) what is a space?
- dimension of a space,
- column space of a matrix,
- row space of a matrix,
- null space of a matrix,
- the Rank Theorem.

Reminders

- Vectors are linearly independent if any only if there is only the trivial solution to the vector equation

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- The Row space of a matrix A is the span of the rows of A.
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$$\dim \mathrm{null}(A) + \mathrm{rank}(A) = \dim \mathrm{null}(A) + \dim \mathrm{Col}(A) = t.$$

Math 1119B, Tutorial 3

Monday, October 24, 2011

Do all of the following questions that you have covered in class to-date. If you have not covered, say, null(A) yet, then don't worry about it; simply move on to the next part. I am making this tutorial before I leave for South America. Go back later and do the rest of it while you're studying!

1. Let $A = \begin{bmatrix} 2 & -1 & 11 & 5 & 4 \\ 1 & -2 & -5 & 10 & 12 \\ 7 & -2 & 17 & 10 & 4 \end{bmatrix}$. Determine

1. rank(A),
2. A basis for Row(A),
3. A set of 3 independent vectors in $\mathbb{R}^3$.
4. Write down linear combinations giving each dependent vector in terms of the independent vectors.
5. dim null(A),
6. A basis for null(A)
7. A basis for Row(A) containing at least 2 zeroes per vector.

2. Determine by inspection if the following vectors are linearly independent. Make sure you can explain why!

1. Let $v_1 = \begin{bmatrix} 2 \\ -2 \\ -3 \\ 0 \\ -1/2 \end{bmatrix}, v_2 = \begin{bmatrix} 6 & -6 & -9 & 0 & -3/2 \end{bmatrix}^T$.
2. Let $s_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, s_2 = \begin{bmatrix} -1 \\ -2 \\ -4 \end{bmatrix}, s_3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T, s_4 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.
3. $w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, w_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w_4 = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 1 \end{bmatrix}$

3. Let $A = \begin{bmatrix} 3 & -24 & -15 \\ 2 & 0 & 6 \\ -2 & 4 & -2 \end{bmatrix}$, and $b = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$. Denote the columns of A as a_1, a_2 and a_3, and let $W =$ Span(a_1, a_2, a_3).

1. Is $b \in \{a_1, a_2, a_3\}$?
2. Is $b \in W$?
3. Do not row reduce, but show that $a_1 \in W$.
4. Does $W = \mathbb{R}^3$? why or why not? Can you re-use your work from part 2?

End of test material

Sections covered:

- Systems of linear equations (1.1-1.2),
- matrices (2.1),
- vector equations and matrix equations (1.3-1.4),
- homogeneous systems of equations (1.5),
- Leontief exchange system ($\sim$ 1.6),
- linear independence (1.7),
- spans of vectors and spanning $\mathbb{R}^m$ (1.3-1.4),
- Row/Column spaces and null space (2.7) **check!**,

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End of test content.

But the best way of seeing what is on the test is to look over the lecture notes provided!!!!

What does linear mean?

Let's go to the blackboard.