Math 1119B: Week 4, Lecture 2

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Recap

- The difference between vector equations $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = b$, and matrix equations $Ax = b$.

Recap

- The difference between vector equations $c_1v_1 + c_2v_2 + \cdots + c_nv_n = b$, and matrix equations $Ax = b$.
- The similarities between vector equations and matrix equations.
- Rephrasing vector equations into matrix equations and vice versa.
- Homogeneous linear systems.

Test 1, Question 1

(a) Place the following system of equations into an augmented matrix and then row reduce to *reduced* row-echelon form. Circle the pivots and indicate the pivot columns in the *reduced* matrix. Indicate whether the system is consistent or inconsistent, and list the number of solutions of the system (0, 1 or infinite).

$$\begin{array}{rcrcrcr} 3x_1 & - & 2x_2 & + & 4x_3 & = & 0 \\ 9x_1 & - & 6x_2 & + & 13x_3 & = & 0 \\ -6x_1 & + & 4x_2 & - & 8x_3 & = & 0 \end{array}$$

(b) If the solution is unique, check your solution in one of the equations. If you have infinitely many solutions, indicate which variables are basic, and which are free. Express your solution as a column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \cdots$.

Test 1, True/False

Due to the skill-testing nature, I thought it was a good idea to talk about why the following questions were true/false.

T / F [a] A basic variable in a linear system is a variable that corresponds to a pivot row in the coefficient matrix.

T / F [b] Whenever a system contains free variables, the solution of the system is consistent.

T / F [c] The echelon form of a matrix is unique.

T / F [d] If every column of an augmented matrix has a pivot, the corresponding system is consistent.

T / F [e] You will obtain the same solution set of a system of linear equations whether you use substitution or the row-reduction algorithm.

T / F [f] If A is an $m \times n$ matrix and B is an $n \times p$ matrix, the product $(3A)(-2B)$ has dimensions $m \times p$.

Homogeneous systems of equations

Recall. A homogeneous system of equations can be written as a matrix equation $Ax = 0$.

We have the following properties of homogeneous linear systems:

1. They always have at least one solution $x_1 = x_2 = \cdots = x_n = 0$. This is called the trivial solution. (They are always consistent).
2. We are really interested in if there is a non-trivial solution to the system (i.e., do we have free variables when we solve the system?)

A telling example

Find a solution to $Ax = 0$, where $A = \begin{bmatrix} 3 & -4 & 5 \\ -3 & 4 & -2 \\ 9 & -12 & 6 \end{bmatrix}$.

Ans.:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix}.$$

NOTE

A telling example

Find a solution to $Ax = 0$, where $A = \begin{bmatrix} 3 & -4 & 5 \\ -3 & 4 & -2 \\ 9 & -12 & 6 \end{bmatrix}$.

Ans.:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix}.$$

NOTE(There's always one...): x_3 here always equals 0, but this solution is not the trivial solution. The only solution that we call trivial is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

Leontief

See the supplemental slides for Leontief's Exchange process. I felt as if my explanation was unclear for this, and so I want to go through it in more detail before moving on.

See Wednesday's class.