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LIST OF SESSION VIDEOS

- Chapter 1: What is Statistics
- Chapter 2&3: Graphical Descriptive Techniques I&II
- Chapter 4: Numeric Descriptive Techniques
- Chapter 5: Data Collection and Sampling
- Chapter 6: Probability
- Chapter 7: Probability Distributions
- Chapter 8: Continuous Probability Distributions
- Chapter 9: Sampling Distributions
- Chapter 10: Estimation
- Chapter 11: Hypothesis Testing
- Chapter 12: Inferences - One Population
- Chapter 13: Inferences - Two Populations
- Chapter 14: ANOVA
- Chapter 15: Chi Squared Test
- Ch 16 & Ch 17: Regression (single & multiple)

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Agenda

- Overview BDM Stuff
- Linear Programming
  - Compute, draw them, look at excel
  - Sensitivity analysis!!
  - Common Business Applications
  - Network Problems

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Overview BDM Stuff

- **Management Science process** =
  - Observation -> problem definition -> model construction -> model solution -> implementation.
- **Models** are used because:
  - Less expensive
  - Less time consuming and risky
  - More feasible

  They are abstraction of a real object can be (iconic (replica), analog, or mathematical)

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Linear Programming

- **Linear Programming** is a model of linear relationships representing a decision given an objective and resource constraints.
- Constraint types are ‘=<’ ‘=’ ‘>=’
- They can be any fraction, as long as linear!
- If RHS of constraints are integers then solution will be integer!

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Linear Programming

- **Properties of LP’s**
  - Assumptions
  - Parameters
  - Decision Variables
  - Constraints
  - Objective Function

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Linear Programming

- **Properties of LP's**
  - Assumptions
    - When you read a question, and there is ambiguity, you NEED to state your assumptions that you are using to make your model.

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Linear Programming

- **Properties of LP's**
  - Parameters
    - KNOWN constant values from the Question. They are your coefficients for your variables in your objective function and constraints

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Linear Programming

- **Properties of LP's**
  - Constraints
    - These are resource/capacity issues that affect how many of your product you can produce. Usually labour hours/materials/space available
    - Constraint types are '=<' '=' '>='
  - STANDARD FORM (Constraints are ALL '=')
    - Need to add a slack/surplus variable
    - Constraint: 10x + 20y <= 100
      - Standard Constraint: 10x + 20y + u = 100 (slack)
    - Constraint 3x + 4y >= 30
      - Standard Constraint: 3x + 4y - v = 30 (surplus)

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Linear Programming

- **Properties of LP's**
  - Constraints
    - THEY MUST BE LINEAR!!

Eg: production ratio of product 1 and 2 can't be more then 50% of all products (1,2 and 3).

(X1 + x2) / (X1 + X2 + X3) <= .50

X1+X2 <= .5(X1) + .5(X2) + .5(X3)

.5(X1) + .5(X2) -.5(X3) <= 0

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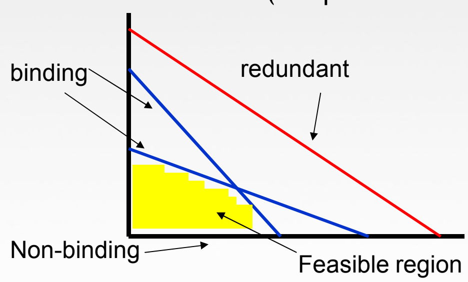
Linear Programming

- **Properties of LP's**
  - Constraints can be:
    - Binding (define part of the optimal solution)
    - Non-Binding (define part of the feasible region, but not the optimal solution)
    - Redundant (not part of either)



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Linear Programming

- **Properties of LP's**
  - Objective Function
    - What are you trying to do??
      - Eg: MAXIMIZE profit!
      - So the objective function needs to be about PROFIT for each possible product option.
        - » NOTE: If they give you sale level and expenses per unit, if they ask for max profit, MAKE SURE you take revenues and subtract expenses and put that number in your objective function.

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Linear Programming

- **Properties of LP's**
  - Decision Variables
    - This is what you are trying to find out – how much x and y you should do to maximize/minimize profit/costs.
    - Put yourself in the shoes of the manager, in the end, what is your problem you are looking to get solved?

Linear Programming

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season. What should you produce to maximize profit?

Linear Programming

(Objective Function Parameters) (Parameters)

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season. What should you produce to maximize profit?

RHS of Constraints

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Linear Programming

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season. What should you produce to maximize profit?

Problem Objective:
- TO MAXIMIZE PROFIT

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Linear Programming

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season.

Decision Variables:
Let x be the number of BLACK shoes
Let y be the number of BROWN shoes

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Linear Programming

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season.

Objective Function:
MAX: $z = 5x + 8y$

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Linear Programming

Ex: You are making 2 types of shoes in your off-season – black shoes and brown shoes. Black shoes bring in $5 profit per shoe, yet take 10 hours of labour and 10 units of cloth, whereas Brown shoes give $8 profit, but take 12 hours of labour and 4 units of cloth. You only have 180 hours of labour and can buy no more then 100 units of cloth. Your warehouse only can store 20 pairs of shoes in the off-season.

Constraints:

Labour Constraint: $10x + 12y \le 180$

Cloth Constraint: $10x + 4y \le 100$

Warehouse Constraint: $x + y \le 20$

NON-NEGATIVE: $x, y \ge 0$

Linear Programming

- **Ways to Solve a LP:**
  - Graphically (will need to do on exam)
  - Excel Solver (will be given and need to analyze)

Linear Programming

- **GRAPHICALLY**
  1. Plot your constraints
  2. Determine your feasible region (area of possible solutions)
  3. Optimal Solution WILL be on an intersection/end point, so calculate the solution for each point. Take the largest/smallest one.

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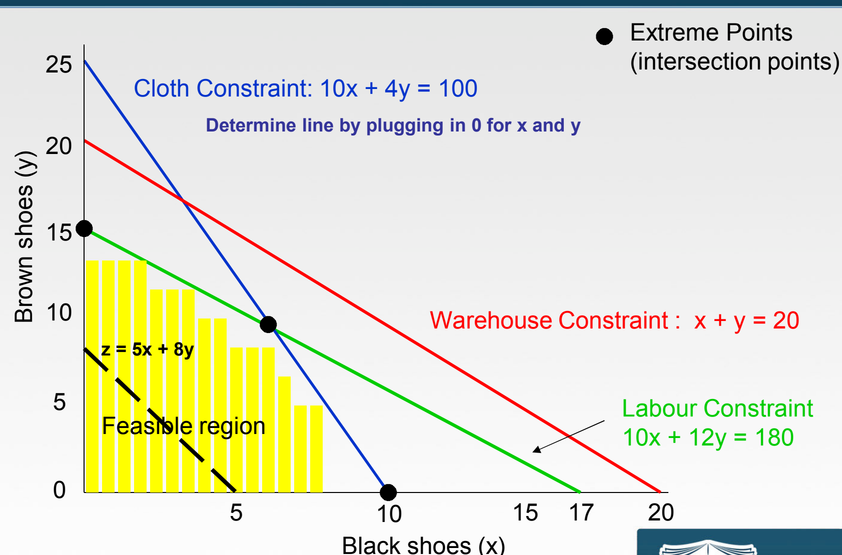
Linear Programming



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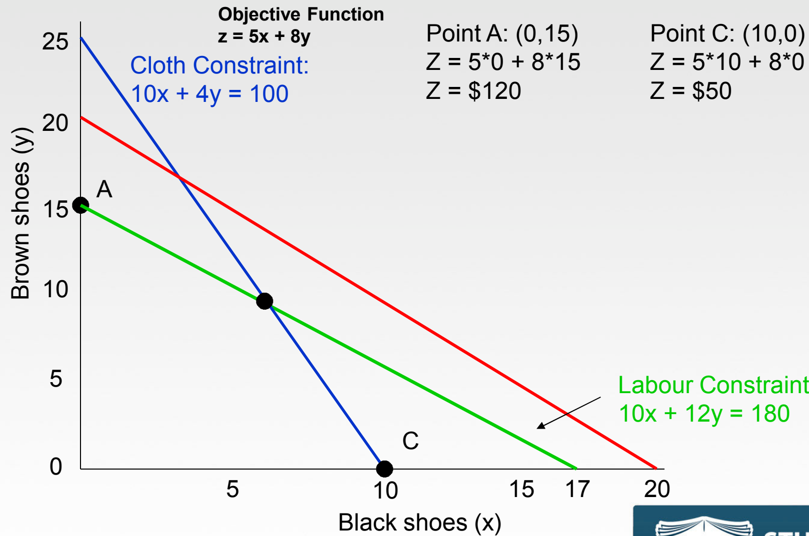
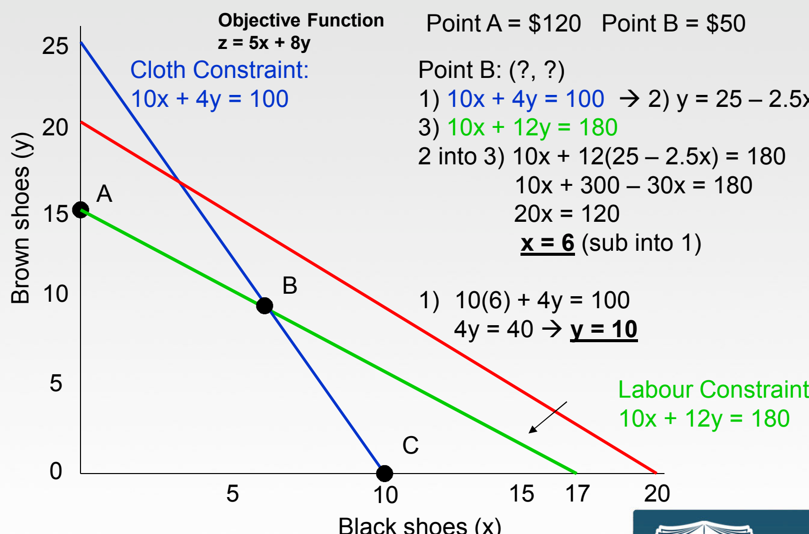
Linear Programming



Point A: (0,15)
Z = 5*0 + 8*15
Z = $120

Point C: (10,0)
Z = 5*10 + 8*0
Z = $50

Linear Programming



Point A = $120   Point B = $50

Point B: (?, ?)

1) 10x + 4y = 100 → 2) y = 25 – 2.5x

3) 10x + 12y = 180

2 into 3) 10x + 12(25 – 2.5x) = 180

10x + 300 – 30x = 180

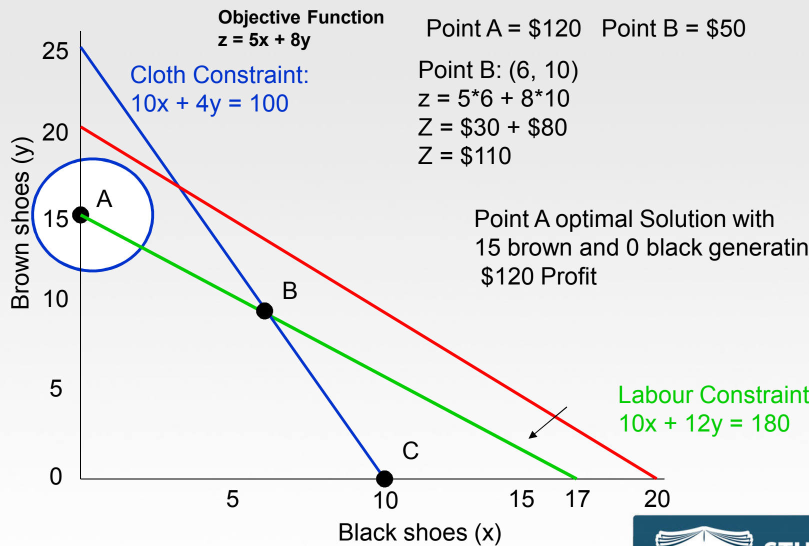
20x = 120

x = 6 (sub into 1)

1) 10(6) + 4y = 100

4y = 40 → **y = 10**

Linear Programming



Point A = $120 Point B = $50

Point B: (6, 10)
z = 5*6 + 8*10
Z = $30 + $80
Z = $110

Point A optimal Solution with 15 brown and 0 black generating $120 Profit

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LP: Excel Solver

	BL	BR				
DecVar	0	15				
					Constraint	Slack
ObFnCoef	5	8	120		Amount	(Surplus)
Labour Constraint	10	12	180	≤	180	0
Cloth Constraint	10	4	60	≤	100	40
Warehouse Constraint	1	1	15	≤	20	5

Amount that is unused

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LP: Sensitivity Analysis

- Objective Function Changes:
  - **Range of Optimality (allowable inc/dec):** the amount the objective coefficients can change without changing the optimal solution. Graphically shown by changing the slope of the obj function until a new optimal solution is found.
    - Find slope of obj func, slope of the constraints, and compare the fractions with the middle coefficient in question as the variable to see how much it can go up/down
  - **Reduced Cost:** the amount the variables obj function coefficient need to be reduced/increased in order for it to become part of the solution. Value is != 0 only if the value of coefficient is 0 (not used) aka non-basic variable

DUAL COST: is amount that coefficient needs to 'improve' to be part of optimal solution → so will ALWAYS be positive.

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LP: Sensitivity Analysis

REDUCED COST

Must increase profit per BLACK shoe by more then $1.66 in order to have BLACK shoes being produced which also maximized profit. NOTE – in a maximization problem, you take the ABSOLUTE value of this.. NOT negative.

Adjustable Cells — Non-basic variable (not in opt sol)

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
C3	Brown Shoes	15	0	8	1E+30	2

Basic variable (in opt sol)

Since BROWN shoes are already part of the solution, it does not need to increase profit per shoe to be part of optimal solution

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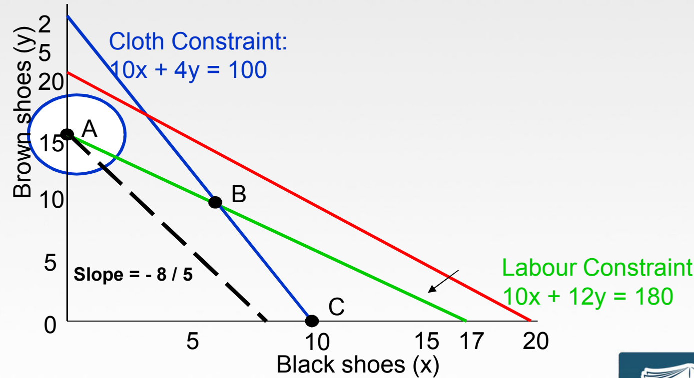
LP: Sensitivity Analysis

RANGE OF OPTIMALITY (Allowable Inc/Dec)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
C3	Brown Shoes	15	0	8	1E+30	2



LP: Sensitivity Analysis

RANGE OF OPTIMALITY (Allowable Inc/Dec)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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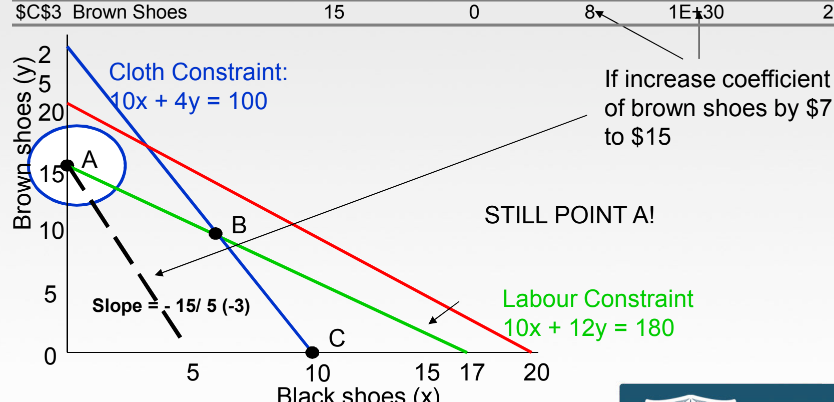

LP: Sensitivity Analysis

RANGE OF OPTIMALITY (Allowable Inc/Dec)

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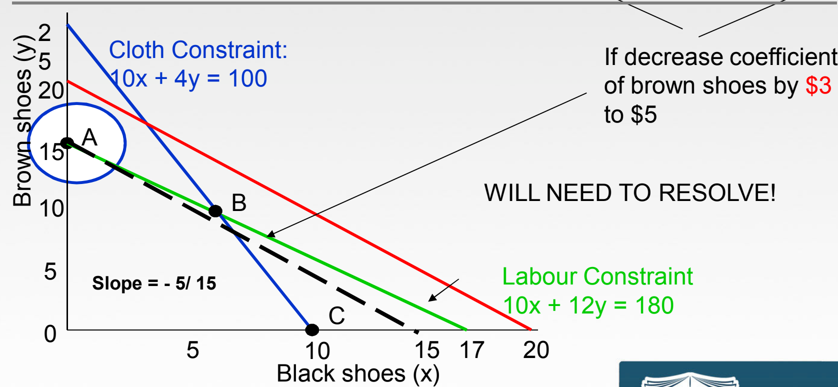

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LP: Sensitivity Analysis

SENSITIVITY RANGE

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
C3	Brown Shoes	15	0	8	1E+30	2

ALBRAICALLY: FOR THE OBJ COFFICIENTS, how much can they go up / down?

Recall: the slope of a line in standard form: a1x1 + a2x2 = b is: - a1/a2.

Our Example: z = 5x + 8y

Slope = -5/8.

So.. How far can it go up?

What is the slope of binding constraints (only 1 here..):

- **Labour Constraint: 10x + 12y = 180**
- **Slope = -10/12**

So we have:

- **Lower constraint slope < function slope < higher constraint slope**
  - **-10/12 < -5/8 < (NA – only 1 binding..)**

Q. How much can the Brown shoes go up/down? (replace the 8 with y)

- **-10/12 < -5 / y (multiply both by -12y)**
- **10y < 60 → y < 6**

y = 8 in the solution, so it can go down 2 (to 6) and it can go up as much as it wants.. (no upper binding constraint)

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LP: Sensitivity Analysis

RANGE OF OPTIMALITY (Allowable Inc/Dec)

- Change within the allowable limit
  - Nonbasic
    - Current optimal solution remains optimal
    - OFV remains the same
  - Basic
    - Current optimal solution remains optimal
    - New OFV determined by plug in the optimal solution in the new objective function
- Change outside the allowable limit
  - **Must re-solve.**

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LP: Sensitivity Analysis

- **RHS CONSTRAINT CHANGES:**
  - **Shadow Price:** the amount the objective function will change per unit increase in the right hand side value of a constraint.
    - Adding 1 to RHS of a constraint will increase/decrease the value of obj function by the shadow price. (if non-binding – shadow price is 0).
    - To find manually, add 1 to rhs and solve for the 2 constraints.
    - **Will be negative for >= and positive for <= (*cheat sheet)**
  - **Range of Feasibility (allowable inc/dec for RHS coeff):** The values of the RHS of the constraint such that the original lines that determine the orig opt sol continue to determine it.
  - Look to the graph to see where the extreme points are that still use the binding constraints, figure out that x/y, sub into constraint to see the range.

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LP: Sensitivity Analysis

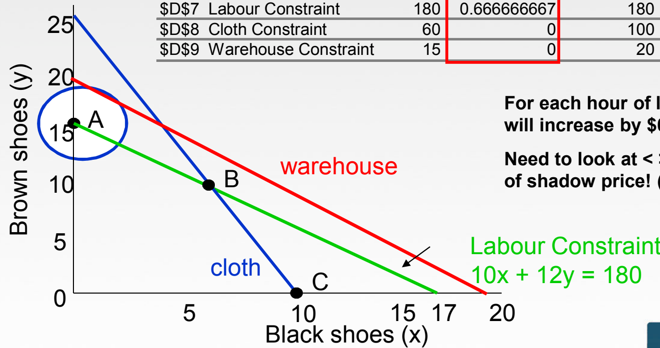
SHADOW PRICE

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
C3	Brown Shoes	15	0	8	1E+30	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D7	Labour Constraint	180	0.666666667	180	60	180
D8	Cloth Constraint	60	0	100	1E+30	40
D9	Warehouse Constraint	15	0	20	1E+30	5



For each hour of labour more, profit will increase by $0.67.

Need to look at < > to know the sign of shadow price! (cheat sheet!)

LP: Sensitivity Analysis

SENSITIVITY RANGE

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
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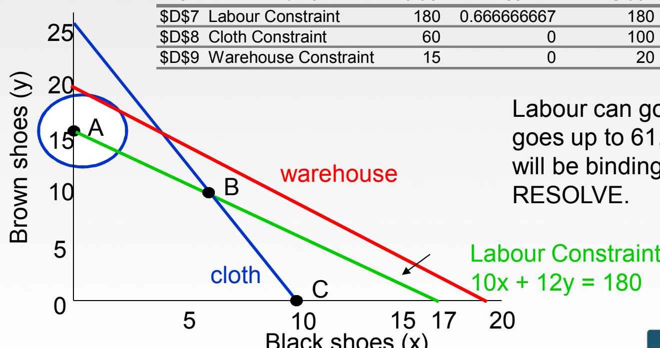

Labour can go up 60 hours, if it goes up to 61, then different lines will be binding, and will need to RESOLVE.

LP: Sensitivity Analysis

SENSITIVITY RANGE

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Black Shoes	0	-1.666666667	5	1.666666667	1E+30
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D8	Cloth Constraint	60	0	100	1E+30	40
D9	Warehouse Constraint	15	0	20	1E+30	5

Solve for Allowable Inc/Dec:
- You will not have to solve for allowable inc/dec for a BINDING constraint (correct me if wrong for this year..)
-You WILL HAVE to solve for the allowable inc/dec for a NON-BINDING constraint:

Non-binding constraint of ≤ type
- Allowable Increase = infinity
- Allowable decrease = RHS – final value

Non-binding constraint of ≥ type
- Allowable Increase = final value - RHS
- Allowable decrease = infinity

THESE ARE ON CHEAT SHEET! ☺

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LP: Sensitivity Analysis

Adding/Deleting Constraint:
- If constraint will be binding, then must resolve.
- If non-binding, then opt sol and OFV are unchanged.

Add variable:
- must resolve

Change LHS coefficient of constraint:
- must resolve

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LP: Sensitivity Analysis

Multiple Changes, 100% rule! (in Obj Funct **or** constraint RHS)

• if you make multiple changes in the RHS constraints, add up the % inc or dec (relating to allowable inc/dec) and if they sum to over 100% -RESOLVE!
• same thing if multiple changes in the obj function variables. (KEY MC/Fill in blanks)
• So if you changed the labour constraint by 50% (of the allowable decrease), and the cloth by 60%, that would be 110% = **re-solve. If not, then the shadow price / obj function doesn't change.**

NOTE: For Non-binding constraint and non-basic variables

- Do **not need** to check 100% rule . As long as each change is within allowable limits, both optimal solution and OFV are unchanged.

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100% rule example

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Advertising Units Television	4	0	1	0.5	1
D14	Advertising Units Print Media	3	0	1	1E+30	0.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E8	Stain Remover Sales	0.03	33.33333333	0.03	0.06	0.008571429
E9	Liquid Detergent Sales	0.18	33.33333333	0.18	0.12	0.12
E10	Powder Detergent Sales	0.08	0	0.04	0.04	1E+30

OB Function: The cost for a tv and print ad is both $1. (for example).
If they asked you to increase the price of Television by .25 cents and the Print Media by 5. What would be the final quantity and price? Can you solve?

Allowable increase for TV is .5, allocation increase for Print Media is infinity.. So the only change that matters is the .25 of .5 which is 50% .. You can!

SO final value would stay as 4 and 3 units of each ($7). The value would be (4*1.25) + 3*5 = $20.

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100% rule example

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C14	Advertising Units Television	4	0	1	0.5	1
D14	Advertising Units Print Media	3	0	1	1E+30	0.333333333

Constraints

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E8	Stain Remover Sales	0.03	33.33333333	0.03	0.06	0.008571429
E9	Liquid Detergent Sales	0.18	33.33333333	0.18	0.12	0.12
E10	Powder Detergent Sales	0.08	0	0.04	0.04	1E+30

RHS: What if they asked you to increase the constraint of the Stain remover by .04 and Liquid detergent by .10?

- Would have to resolve as .04/.06 = 66% and .10/.12 is 83% .. Add them up and it's over 100%.

What if increase Liquid by .10 and Powder by .2 ?

- If you add up the % change, it'd be over 100%... BUT… Powder isn't a binding constraint (final value not equal constraint, sha price = 0). So only consider the liquid, and it's < 100%. So you can!
- To compute the OFV = old OFV ($7) + .10*33.33 = $10.33

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LP Prob: Product Mix

- Four-product T-shirt/sweatshirt manufacturing company.
- Must complete production within 72 hours
- Truck capacity = 1,200 standard sized boxes.
- Standard size box holds 12 T-shirts.
- sweatshirt box (holds 12) is three times size of standard box.
- $25,000 available for a production run.
- 500 dozen blank T-shirts and sweatshirts in stock.
- How many dozens (boxes) of each type of shirt to produce?

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The data

	Processing Time (hr) Per dozen	Cost ($) per dozen	Profit ($) per dozen
Sweatshirt - F	0.10	36	90
Sweatshirt – B/F	0.25	48	125
T-shirt - F	0.08	25	45
T-shirt - B/F	0.21	35	65

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Model Construction

Decision Variables:

x_1 = sweatshirts, front printing
x_2 = sweatshirts, back and front printing
x_3 = T-shirts, front printing
x_4 = T-shirts, back and front printing

Objective Function:

Maximize $Z = \$90x_1 + \$125x_2 + \$45x_3 + \$65x_4$

Model Constraints:

$0.10x_1 + 0.25x_2 + 0.08x_3 + 0.21x_4 \leq 72$ hr
$3x_1 + 3x_2 + x_3 + x_4 \leq 1{,}200$ boxes
$\$36x_1 + \$48x_2 + \$25x_3 + \$35x_4 \leq \$25{,}000$
$x_1 + x_2 \leq 500$ dozen sweatshirts
$x_3 + x_4 \leq 500$ dozen T-shirts

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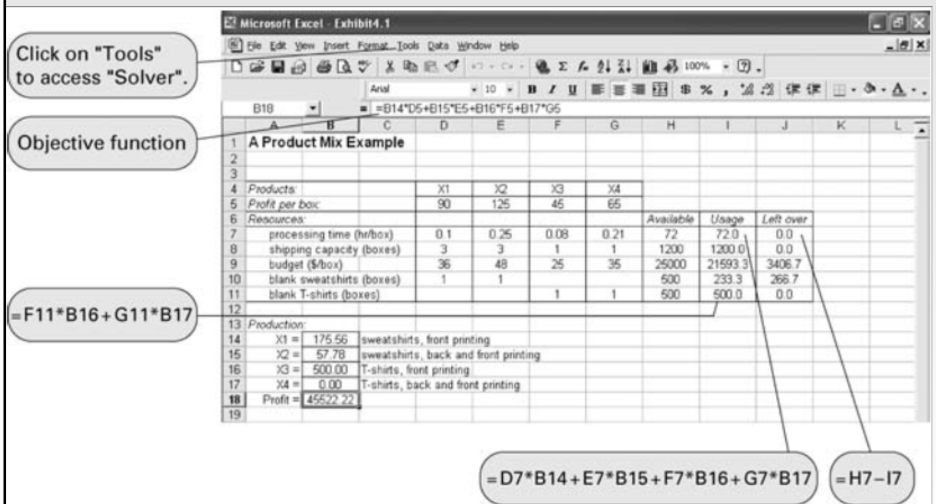
Excel Results



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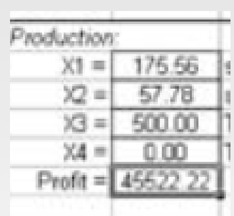
Excel Results

- Profit says 45,522.22
- but that's assuming decimal boxes/shirts. Can't have!
- Would think this question would state that only full boxes are required, in which you'd have another constraint,
- **X1,x2,x3,x3 are integers.**



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LP Prob: Advertising

	Exposure (people/ad or commercial)	Cost
Television Commercial	20,000	$15,000
Radio Commercial	12,000	6,000
Newspaper Ad	9,000	4,000

- Budget limit $100,000
- Television time for 4 commercials
- Radio time for 10 commercials
- Newspaper space for 7 ads
- Resources for no more than 15 commercials and/or ads
- **Maximize exposure!**

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Advertising Ex: Model

Maximize $Z = 20{,}000x_1 + 12{,}000x_2 + 9{,}000x_3$

subject to:

$$15{,}000x_1 + 6{,}000x_2 + 4{,}000x_3 \leq 100{,}000$$
$$x_1 \leq 4$$
$$x_2 \leq 10$$
$$x_3 \leq 7$$
$$x_1 + x_2 + x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

where

x_1 = Exposure from Television Commercial (people)
x_2 = Exposure from Radio Commercial (people)
x_3 = Exposure from Newspaper Ad (people)

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LP Prob: Blend Example

Determine the optimal mix of the three components in each grade of motor oil that will maximize profit. Company wants to produce at least 3,000 barrels of each grade of motor oil.

Component	Maximum Barrels Available/day	Cost/barrel
1	4,500	$12
2	2,700	10
3	3,500	14

Grade	Component Specifications	Selling Price ($/bbl)
Super	At least 50% of 1 Not more than 30% of 2	$23
Premium	At least 40% of 1 Not more than 25% of 3	20
Extra	At least 60% of 1 At least 10% of 2	18

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LP Prob: Blend!

- **Decision variables:** The quantity of each of the three components used in each grade of gasoline (9 decision variables);
- x_{ij} = barrels of component i used in motor oil grade j per day, where i = 1, 2, 3 and j = s (super), p (premium), and e (extra).

Component	Maximum Barrels Available/day	Cost/barrel
1	4,500	$12
2	2,700	10
3	3,500	14

Grade	Component Specifications	Selling Price ($/bbl)
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Blend Model

Maximize $Z = 11x_{1s} + 13x_{2s} + 9x_{3s} + 8x_{1p} + 10x_{2p} + 6x_{3p} + 6x_{1e} + 8x_{2e} + 4x_{3e}$

subject to:

$$x_{1s} + x_{1p} + x_{1e} \leq 4{,}500$$
$$x_{2s} + x_{2p} + x_{2e} \leq 2{,}700$$
$$x_{3s} + x_{3p} + x_{3e} \leq 3{,}500$$
$$0.50x_{1s} - 0.50x_{2s} - 0.50x_{3s} \geq 0$$
$$0.70x_{2s} - 0.30x_{1s} - 0.30x_{3s} \leq 0$$
$$0.60x_{1p} - 0.40x_{2p} - 0.40x_{3p} \geq 0$$
$$0.75x_{3p} - 0.25x_{1p} - 0.25x_{2p} \leq 0$$
$$0.40x_{1e} - 0.60x_{2e-} - 0.60x_{3e} \geq 0$$
$$0.90x_{2e} - 0.10x_{1e} - 0.10x_{3e} \geq 0$$
$$x_{1s} + x_{2s} + x_{3s} \geq 3{,}000$$
$$x_{1p} + x_{2p} + x_{3p} \geq 3{,}000$$
$$x_{1e} + x_{2e} + x_{3e} \geq 3{,}000$$
$$x_{ij} \geq 0$$

Component	Maximum Barrels Available/day	Cost/barrel
1	4,500	$12
2	2,700	10
3	3,500	14

Grade	Component Specifications	Selling Price ($/bbl)
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- **KEY is the Opt Function: '$11' is the profit of using a barrel of component 1 in (s)uper. $23 (price) - $12 (cost) = $11 (PROFIT)**

Blend Model Constraints

- Time to explain → 3 STEPS:

1. **The sign**
   - 'at least' means >= 0
   - 'no more than' mean <= 0
2. **The first item:**
   - The first item is your constraint item. The value is 1 -'x' (so less than 25% would be 1-.25 = .75)
3. **The others:**
   - The others will be the remainder of 100%, both negative.
4. **Put to together!**

$$0.50x_{1s} - 0.50x_{2s} - 0.50x_{3s} \geq 0$$
$$0.70x_{2s} - 0.30x_{1s} - 0.30x_{3s} \leq 0$$
$$0.60x_{1p} - 0.40x_{2p} - 0.40x_{3p} \geq 0$$
$$0.75x_{3p} - 0.25x_{1p} - 0.25x_{2p} \leq 0$$
$$0.40x_{1e} - 0.60x_{2e-} - 0.60x_{3e} \geq 0$$
$$0.90x_{2e} - 0.10x_{1e} - 0.10x_{3e} \geq 0$$

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Blend Model Constraints

- 1st one:
  - At least 50% of 1
  1. The sign: >= 0
  2. The first item: 1-.50 = .50x1
  3. The last ones: - .50x2 - .50x3
  4. Total: .50x1 - .50x2 - .50x3 >= 0

4th one:
- Not more than 25% of 3:
  1. The sign: <= 0
  2. The first item: 1-.25 = .75x3
  3. The last ones: -.25x1 - .25x2
  4. Total:.75x3 - .25x1 - .25x2 <= 0

$$0.50x_{1s} - 0.50x_{2s} - 0.50x_{3s} \geq 0$$
$$0.70x_{2s} - 0.30x_{1s} - 0.30x_{3s} \leq 0$$
$$0.60x_{1p} - 0.40x_{2p} - 0.40x_{3p} \geq 0$$
$$0.75x_{3p} - 0.25x_{1p} - 0.25x_{2p} \leq 0$$
$$0.40x_{1e} - 0.60x_{2e} - 0.60x_{3e} \geq 0$$
$$0.90x_{2e} - 0.10x_{1e} - 0.10x_{3e} \geq 0$$

Component	Maximum Barrels Available/day	Cost/barre
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Multi-Period Example

Production Capacity: 160 computers per week
50 more computers with overtime

Assembly Costs: $190 per computer regular time; $260 per computer overtime

Inventory Cost: $10/comp. per week

Order schedule:

Week	Computer Orders
1	105
2	170
3	230
4	180
5	150
6	250

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Decision Variables:

r_j = regular production of computers per week j (j = 1 - 6)

o_j = overtime production of computers per week j (j = 1 - 6)

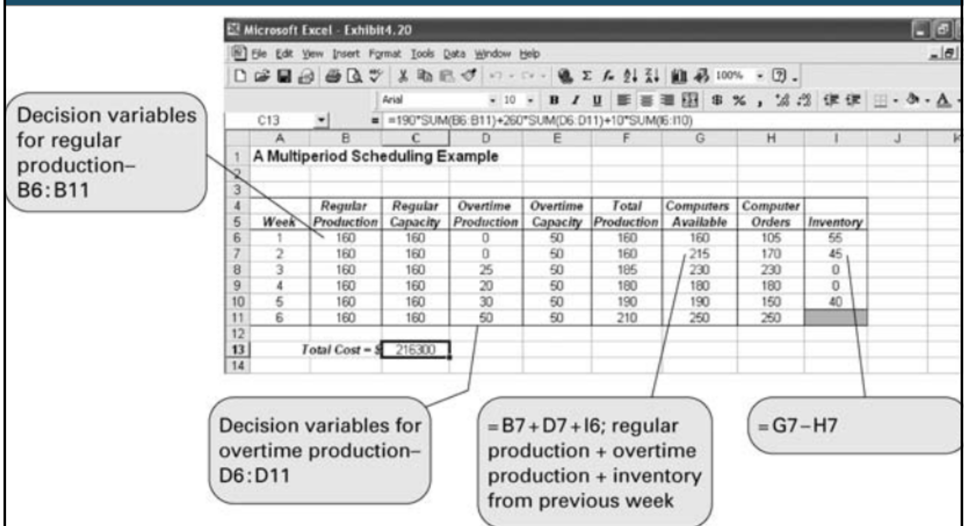
i_j = extra computers carried over as inventory in week j (j = 1 - 5)

Model summary:

Minimize $Z = \$190(r_1 + r_2 + r_3 + r_4 + r_5 + r_6) + \$260(o_1 + o_2 + o_3 + o_4 + o_5 + o_6) + 10(i_1 + i_2 + i_3 + i_4 + i_5)$

subject to:

$r_j \leq 160$ (j = 1, 2, 3, 4, 5, 6)
$o_j \leq 150$ (j = 1, 2, 3, 4, 5, 6)
$r_1 + o_1 - i_1 \geq 105$
$r_2 + o_2 + i_1 - i_2 \geq 170$
$r_3 + o_3 + i_2 - i_3 \geq 230$
$r_4 + o_4 + i_3 - i_4 \geq 180$
$r_5 + o_5 + i_4 - i_5 \geq 150$
$r_6 + o_6 + i_5 \geq 250$
$r_j, o_j, i_j \geq 0$



A Multiperiod Scheduling Example

Week	Regular Production	Regular Capacity	Overtime Production	Overtime Capacity	Total Production	Computers Available	Computer Orders	Inventory
1	160	160	0	50	160	160	105	55
2	160	160	0	50	160	215	170	45
3	160	160	25	50	185	230	230	0
4	160	160	20	50	180	180	180	0
5	160	160	30	50	190	190	150	40
6	160	160	50	50	210	250	250	

Total Cost = $ 216300

LP: Applications

- Network Model: one which can be represented by a set of nodes, set of arcs and function associated with the arcs and/or nodes. Used in transportation, assignment, and transshipment problems.

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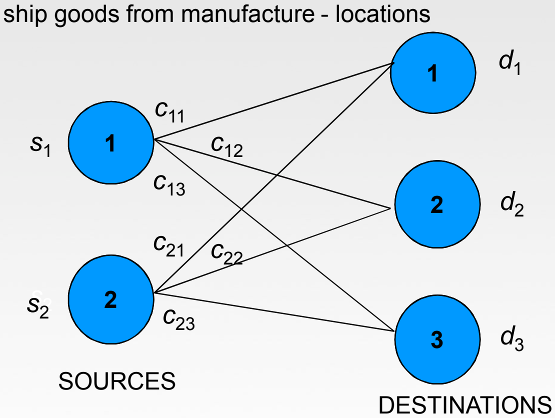
Network Model

s_i = Place that makes the good (supply)
d_i = demand to locations/stores/cities
c_{sd} = cost to ship goods from manufacture - locations



Can also have TRANSSHIPMENT points between (middle-men)

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Network Model

- Formulated in terms of amounts shipped from origins to destinations, ***xij***

$$\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Objective Function – to MINIMIZE Costs!!

X_{ij} is the amount shipped from source i to destination j

st

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad i = 1, 2, \ldots, m \quad \text{Supply}$$

Can't ship more then you can produce

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, 2, \ldots, n \quad \text{Demand}$$

Must satisfy demand!

$$x_{ij} \geq 0 \quad \text{for all i and j}$$

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Network Model

- Special Cases - modifications to LP formulation:
  - Minimum shipping guarantees from ***i*** to ***j***:

    $x_{ij} \geq L_{ij}$ (L = Limit)
  - Maximum route capacity from ***i*** to ***j***:

    $x_{ij} \leq L_{ij}$ (L = Limit)
  - Unacceptable routes: delete the variable or set equal to zero

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Supply ≠ Demand

- If supply exceeds demand, excess supply will simply be slack, unused capacity
- If demand exceeds supply, the solution will not be feasible
  - Solution: add a dummy origin
    - Assign zero cost for each destination route

General Formula For network model:
beginning inventory - demand + production = ending inventory

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Example: BBC

- Building Brick Company (BBC) has orders for 80 tons of bricks at three suburban locations as follows: Northwood – 25 tons, Eastwood – 45 tons and, Westwood – 10 tons.
- BBC has two plants, each of which can produce 50 tons per week
- How should end of week shipments be made to fill above orders given following delivery **cost/ton**:

	Northwood	Eastwood	Westwood
Plant 1	24	30	40
Plant 2	30	40	42

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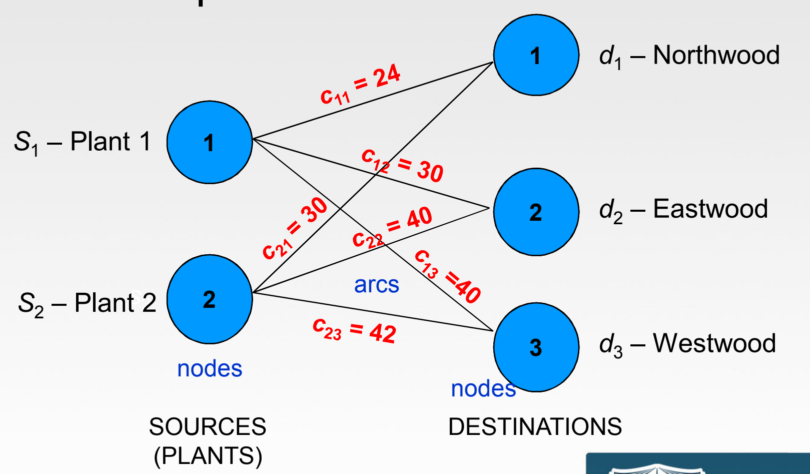
BBC Example

- Network Representation



LP Formulation: Transportation

Decision Variables

x_{ij} = amount shipped from Plant *i* to customer *j*
where *i* = 1 (Plant 1), 2 (Plant 2)
j = 1 (Northwood), 2 (Eastwood), 3 (Westwood)

Objective Function

Minimize Overall Shipping Costs:
Min $24x_{11} + 30x_{12} + 40x_{13} + 30x_{21} + 40x_{22} + 42x_{23}$

LP Formulation: Transportation

Constraints

Plant #1	$x_{11} + x_{12} + x_{13} \leq 50$	Supply Constraints
Plant #2	$x_{21} + x_{22} + x_{23} \leq 50$	
Northwood:	$x_{11} + x_{21} = 25$	Demand Constraints
Eastwood:	$x_{12} + x_{22} = 45$	
Westwood:	$x_{13} + x_{23} = 10$	

EACH NODE WILL HAVE A CONSTRAINT (even transshipment nodes!)

Non-negativities: $x_{ij} \geq 0$, for all i and j.

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Excel Formulation

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}			
DecVar	0	0	0	0	0	0			
									Constraint
ObFnCoef	24	30	40	30	40	42	0		Amount
Plant 1	1	1	1				0	≤	50
Plant 2				1	1	1	0	≤	50
Northwood	1			1			0	=	25
Eastwood		1			1		0	=	45
Westwood			1			1	0	=	10

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Excel Solution

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}				
DecVar	5	45	0	20	0	10				
									Constraint	Constraint
ObFnCoef	24	30	40	30	40	42	2490		Amount	Slack
Plant 1	1	1	1				50	≤	50	0
Plant 2				1	1	1	30	≤	50	20
Northwood	1			1			25	=	25	
Eastwood		1			1		45	=	45	
Westwood			1			1	10	=	10	

- Plant 1 is fully utilized, shipping 45 tons to Eastwood and 5 tons to Northwood. Plant 2 supplies the remaining 20 tons required by Northwood and the 10 tons for Westwood and has 20 tons of spare capacity
- Total delivery cost is $2,490

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Special Cases for Network models!

1. Supply exceeds demand?
   - Change this constraint: $\Sigma x_{ij} \le s_i$ *(not equal to)*
2. Demand exceeds supply? $\Sigma d_j > \Sigma s_i$
   - Add a dummy origin 0
   - Cost c_{0j}=0 (no cost from origin 0 to destination j)
   - Supply $s_0 = \sum_{j=1}^{n} d_j - \sum_{i=1}^{n} s_i$
3. Maximize Profits?
   - Demand exceeds supply and certain customers are more valuable
   - r_j revenue per unit received at destination j
   - **Alternative objective function**
   - **Max** $\Sigma\ \Sigma\ r_j x_{ij}$ ***(revs)*** **-** $\Sigma\ \Sigma\ c_{ij} x_{ij}$ **(costs)**

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Special Cases for Network models!

Side constraints / Business constraints:

4. No one should be awarded less than 10% of the business between.
   - **Minimum shipping guarantees from *i* to *j*:**
     $x_{ij} \geq M_{ij}$ **(M = Minimum)**
5. At most 20% of the supply should be shipped over the US border.
   - **Maximum route capacity from *i* to *j*:**
     $x_{ij} \leq L_{ij}$ **(L = Limit)**
6. There should be no shipments over the US border.
   - Unacceptable route (i,j)
     - Huge cost to c_{ij}
     - Delete decision variable x_{ij}

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General Rules: Application Prob's

- Obj Function in proper format (profit vs revenue vs cost)
- Realize what IT is you are trying to figure out.
  - Is it number of cartons to sell vs number of ounces to use (macro vs micro)
  - Is it number of people working on a particular shift or is it number of people work on a particular time shift? (understanding problem)
- Make sure all variables in the question are addressed for constraints (but may have redundant info..)

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Exam Q: LP Sensitivity

The Almost Natural Baked Goods Company makes four products: mini-cakes, muffins, mini-pies, and fruit-crunches which are cellophane wrapped and shipped to convenience stores all over Ontario. They have two key processes: mix & fill, and baking. The two main ingredients for all four products are Fruit Filling Compound and Premix Pastry Compound. An industrial engineer with ANBG Co. has assigned decision variables to the products as follows:

- x_1 = # of grosses of mini-cakes (recall: 1 Gross = 144 Units = a Dozen Dozen),
- x_2 = # of grosses of muffins,
- x_3 = # of grosses of mini-pies, and
- x_4 = # of grosses of fruit-crunches.

The company's objective is to maximize their profit per this function:

$\text{MAX } 50\cdot x_1 + 58\cdot x_2 + 46\cdot x_3 + 62x_4$ (all coefficients are in \$/gross)

The company is constrained by the factory time available for its key processes and the amount of available ingredients:

$4x_1 + 3.5x_2 + 4.6 + 3.9x_4 \leq 600$ hours (mix & fill process)
$2.1x_1 + 2.6x_2 + 3.5x_3 + 1.9x_4 \leq 500$ hours (baking process)
$15x_2 + 23x_2 + 18x_3 + 25x_4 \leq 3600$ lbs (Fruit Filling Compound)
$8x_1 + 12.6x_2 + 9.7x_3 + 10.5x_4 \leq 1700$ lbs (Premix Pastry Compound)

Historically, sales have been such that the mini-cakes & muffins have been great sellers, but ANBG Co. doesn't want to cannibalize sales of the other products (and potentially lose valuable shelf space) so they've set a cap: the ratio of these two products to the entire product line must be ≤ 60%, i.e.:

$(x_1 + x_2) \div (x_1 + x_2 + x_3 + x_4) \leq .60$

Target Cell (Max)

Cell	Name	Original Value	Final Value
G5	MAX Profit	216.00	9304.49

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	MiniCakes (X1)	25.73	[1]	50	11.53	11.64
C3	Muffins (X2)	49.07	0.00	58	13.23	1.64
D3	MiniPies (X3)	0.00	[2]	46	12.62	[3]
E3	Fruit-Crunches (X4)	[4]	0.00	62	1.72	6.38

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G7	Mix & Fill Process LHS	[5]	7.16	600	145.18	44.03
G8	Baking Process LHS	340.11	[6]	500	[7]	[8]
G9	Fruit Filling Compound LHS	3600	1.14	3600	261.18	111.45
G10	Premix Pastry Compound LHS	1700	0.54	1700	61.48	148.67
G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

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① ___ **0** _____ ② ___ **-12.62** ___ ③ ___**infinity**___ ④ ___ **83.42** ____ ⑤ ___ **600** __

⑥ ____ **0** ____ ⑦ ___**infinity**_ ⑧ ___ **159.89** __ ⑨ ___ **-20.13** ___ ⑩ ____**0**____

1 = 0. Since Minicakes has a final value, then reduced cost is 0 (since the coeff does not need to go up for minicakes to be part of the optimal solution – it already is)

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Target Cell (Max)

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2 = -12.62. This is the allowable increase. Reduced cost is the amount that the coefficient needs to rise in this case, and it's telling us that it is allowed to increase by 12.62 dollars before being part of the optimal solution

reduced cost **= - allowable increase (for max) or decrease (for min)**

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3 = infinity. Since it is already too cheap to be in the optimal solution, it's allowable decrease is infinity because it can be reduced to 0 and still won't affect the optimal solution.

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4 = 83.42. You know that 25.73 minicakes and 49.07 muffins are in the solution, and you know the final value is 9304.49, so find out what is left after subtracting minicakes/muffin profit and divide by the coeff of $62 for Fruit-Cruchers

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G10	Premix Pastry Compound LHS	1700	0.54	1700	61.48	148.67
G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

5 = 600. This is 600 since the mix/fill process has a shadow price, aka the constraint is fully utilized and the RHS of the constraint is 600, so that is the final value. If the shadow price was 0, then we would know that the final value would be < 600.

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Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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C3	Muffins (X2)	49.07	0.00	58	13.23	1.64
D3	MiniPies (X3)	0.00	[2]	46	12.62	[3]
E3	Fruit-Crunches (X4)	[4]	0.00	62	1.72	6.38

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G7	Mix & Fill Process LHS	[5]	7.16	600	145.18	44.03
G8	Baking Process LHS	340.11	[6]	500	[7]	[8]
G9	Fruit Filling Compound LHS	3600	1.14	3600	261.18	111.45
G10	Premix Pastry Compound LHS	1700	0.54	1700	61.48	148.67
G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

6 = 0. The final value is less then the constraint Right hand side value, so this does not use all of the baking process time, so there is no shadow price since if you increase the RHS by 1, it would have NO effect on the optimal solution.

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Target Cell (Max)

Cell	Name	Original Value	Final Value
G5	MAX Profit	216.00	9304.49

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	MiniCakes (X1)	25.73	[1]	50	11.53	11.64
C3	Muffins (X2)	49.07	0.00	58	13.23	1.64
D3	MiniPies (X3)	0.00	[2]	46	12.62	[3]
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G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

7 = infinity. Since the baking constraint is not binding and is redundant, you can increase the 500 value to as much as you want, and still not affect the optimal solution

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Target Cell (Max)

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G5	MAX Profit	216.00	9304.49

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	MiniCakes (X1)	25.73	[1]	50	11.53	11.64
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G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

8 = 159.89. This is the amount that you would have to reduce the RHS of the constraint to make this constraint affect the solution and is the difference between the final value (340.11) and the RHS (500) = 159.89

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Target Cell (Max)

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Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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G10	Premix Pastry Compound LHS	1700	0.54	1700	61.48	148.67
G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

9 = -20.13. To determine the value, your (x1+x2) / (x1+x2+x3+x4) <= .60 needs to be linearized, by multiplying the x1+x2+x3+x4 by .60 and bringing over to the other side – so you get .4*x1 + .4*x2 - .6*x3 -.6*x4 <= 0. Plugging in numbers for the final values for x1 -> x4 will get you -20.13.

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Target Cell (Max)

Cell	Name	Original Value	Final Value
G5	MAX Profit	216.00	9304.49

Adjustable Cells

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G10	Premix Pastry Compound LHS	1700	0.54	1700	61.48	148.67
G11	Ratio Constraint LHS	[9]	0	[10]	1E+30	20.13

10. 0. This is 0 since when you linearize the ratio constraint, the RHS has to be 0.

STUDENTS OFFERING SUPPORT

Q6 An awards committee needs to be formed to review potential award recipients. The only stipulation is that a reviewer cannot be assigned to an applicant if the applicant is a co-worker. The table below shows 9 reviewers and 4 applicants and the time it would take each reviewer to review the application of each candidate in minutes. If an entry in the table contains an "X", then that specific reviewer is ineligible to review an applicant's material. For example, Reviewer 1 cannot review materials submitted by candidate B. It is possible that some reviewers may not receive an assignment. Formulate this as an assignment problem to minimize total time taken to review applications. Two reviewers are assigned to review each applicant's material.

	Applicant			
Reviewer	**A**	**B**	**C**	**D**
1	10	X	15	12
2	X	12	10	13
3	15	13	X	16
4	8	12	14	X
5	X	10	11	12
6	8	X	12	15
7	15	X	9	12

Decision Variables

all the variables below (without the numbers).

Objective Function

MIN $10X_{1A} + 15x_{1C} + 12x_{1D} + 12x_{2B} + 10x_{2C} + 13x_{2D}$**.... keep going..**

Constraints

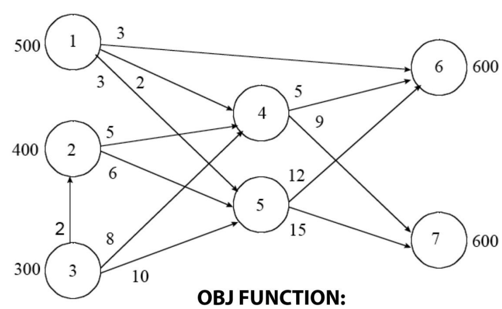
$x_{1A} + x_{3A} + x_{4A} + x_{6A} + x_{7A} = 2$ **(A needs to be reviewed by 2)**

$x_{2B} + x_{3B} + x_{4B} + x_{5B} = 2$ **(B needs to be reviewed by 2)**

... same for C / D

list all variables for non-negativity.

is a maximization problem or a minimization problem. Only write out the constraint that you are asked for. You are NOT asked for all components in every part. The network given below is for a transshipment problem where the objective is to minimize total cost of transportation.



OBJ FUNCTION:

MIN: $3x_{16} + 2x_{14} + 3x_{15} + 5x_{24} + 6x_{25} + 2x_{32} + 8x_{34} + 10x_{35} + 5x_{46} + 9x_{47} + 12x_{56} + 15x_{56}$

Decision Variables

All the x.. listed above

Constraint for Node 2:

$x_{24} + x_{25} <= 400 + x_{32}$ **(need to add the amount that comes from 3)**

$x_{24} + x_{25} - x_{32} <= 400$

Constraint for Node 3:

$x_{32} + x_{34} + x_{35} <= 300$

[1]George Dantzig, the "father of linear programming."

Finished: Next steps!

- Make sure you go through examples of all the types and see how the constraints are created.
- Make sure you know how to play the number game on the excel printout (key!)
- Good luck on the exam!

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Want to experience two weeks in Latin America volunteering to see the impact your donations have?

Go on an Outreach trip!

Laurier SOS has 2 trips this summer in May and August. LaurierSOS.com or info@lauriersos.com