ECONOMICS 325 Mid-term Exam Winter 2013/14

Oct 10, 2013 2:00 to 3:15 PM Instructor: D. WHISTLER

Section 2 Exam B Total points = **35**

Question 1 The random variables X and Y are the share prices of two companies with Var(X) = 50 and Var(Y) = 30. A portfolio is the random variable W = 4X + 8Y.

(a) **[5 points]** If X and Y are independent find Var(W).

(b) **[5 points]** With Var(X) = 50 and Var(Y) = 30, state an assumption that will increase Var(W) compared to (a).

Question 2 From a survey of UBC students consider the random variables:

X = 1 live more than 10 km from UBC; X = 0 live less than 10 km from UBC.

Y = 1 have an 8 AM class; Y = 0 do not have an 8 AM class.

Probabilities are: P(X = 1) = 0.4, P(Y = 1) = 0.2 and $P(X = 1 \mid Y = 1) = 0.3$

Answer the questions below. Clearly explain all answers.

(a) **[5 points]** For a randomly selected student who lives more than 10 km from UBC find the probability that the student has an 8 AM class.

(b) **[5 points]** Is the covariance between X and Y negative or positive? Clearly show your answer.

(c) **[5 points]** Find the conditional probability function for Y given X=0.

(d) **[5 points]** Find the conditional mean and variance of Y given X=0.

(e) **[5 points]** Randomly select 4 students who live less than 10 km from UBC. Find the probability that two of them have an 8 AM class.

ANSWER KEY **Section 2** Exam B

Question 1 (a) Var(W) = (4)(4)Var(X) + (8)(8)Var(Y) = (16)(50) + (64)(30)

(b) ***positive*** Cov(X,Y)

Question 2 (a) Use Bayes Theorem.

$$P(Y=1 \mid X=1) = \frac{P(X=1 \mid Y=1)P(Y=1)}{P(X=1)} = \frac{(0.3)(0.2)}{0.4}$$

(b) Cov(X,Y) = E(XY) – E(X)E(Y) where E(X) = 0.4 E(Y) = 0.2 and

E(XY) = P(X=1 and Y=1) = $P(X=1 \mid Y=1)P(Y=1) = (0.3)(0.2) = 0.06$

Therefore, Cov(X,Y) = 0.06 – (0.4)(0.2) ***negative***

(c) $P(Y=1 \mid X=0) = p$ and $P(Y=0 \mid X=0) = 1-p$

To complete the answer find p.

The joint probabilities can be figured out as:

	X	
Y	0	1
0		.4–.06
1	.2–.06	0.06

$$p = P(Y=1 \mid X=0) = \frac{P(X=0 \text{ and } Y=1)}{P(X=0)} = \frac{0.2-0.06}{0.6}$$

(d) The conditional mean is p and the conditional variance is: $p - p^2 = p(1-p)$

(e) Set $P(Y=1 \mid X=0) = p$ where p is the conditional probability calculated in (c).

Now use the binomial distribution to find: $\frac{4!}{(2)(2)} p^2 (1-p)^2$