ECONOMICS 325 Mid-term Exam Winter 2013/14

Oct 10, 2013 2:00 to 3:15 PM Instructor: D. WHISTLER

Section 2 Exam A Total points = **35**

Question 1 The random variables X and Y are the share prices of two companies with Var(X) = 10 and Var(Y) = 40. A portfolio is the random variable W = 10X + 2Y.

(a) **[5 points]** If X and Y are independent find Var(W).

(b) **[5 points]** With Var(X) = 10 and Var(Y) = 40, state an assumption that will lower Var(W) compared to (a).

Question 2 From a survey of UBC students consider the random variables:

X = 1 frequent library user; X = 0 not a frequent library user, and

Y = 1 live more than 10 km from UBC; Y = 0 live less than 10 km from UBC

Probabilities are: $P(X = 1) = 0.6$, $P(Y = 1) = 0.2$ and $P(X = 1 \mid Y = 1) = 0.5$

Answer the questions below. Clearly explain all answers.

(a) **[5 points]** For a randomly selected student who is a frequent library user find the probability that the student lives more than 10 km from UBC.

(b) **[5 points]** Find the covariance between X and Y.

(c) **[5 points]** Find the conditional probability function for X given Y=0.

(d) **[5 points]** Find the conditional mean and variance of X given Y=0.

(e) **[5 points]** Randomly select 3 students who live less than 10 km from UBC. Find the probability that all of them are frequent library users.

ANSWER KEY **Section 2** Exam A

Question 1 (a) Var(W) = (10)(10)Var(X) + (2)(2)Var(Y) = (100)(10) + (4)(40)

(b) ***negative*** Cov(X,Y)

Question 2 (a) Use Bayes Theorem.

$$P(Y=1 \mid X=1) = \frac{P(X=1 \mid Y=1)P(Y=1)}{P(X=1)} = \frac{(0.5)(0.2)}{0.6}$$

(b) Cov(X,Y) = E(XY) – E(X)E(Y) where E(X) = 0.6 E(Y) = 0.2 and

E(XY) = P(X=1 and Y=1) = $P(X=1 \mid Y=1)P(Y=1) = (0.5)(0.2) = 0.1$

(c) $P(X=1 \mid Y=0) = p$ and $P(X=0 \mid Y=0) = 1-p$

To complete the answer find p.

The joint probabilities can be figured out as:

Y \ X	0	1
0	0.3	0.5
1	0.1	0.1

$$p = P(X=1 \mid Y=0) = \frac{P(X=1 \text{ and } Y=0)}{P(Y=0)} = \frac{0.5}{0.8}$$

(d) The conditional mean is p and the conditional variance is: $p - p^2 = p(1-p)$

(e) Set $P(X=1 \mid Y=0) = p$ where p is the conditional probability calculated in (c).

The answer is: p^3