ECONOMICS 325 Mid-term Exam Winter 2013/14

Oct 10, 2013 12:30 to 1:45 PM Instructor: D. WHISTLER

Section 1 Exam B Total points = **35**

Question 1 **[5 points]** A survey of companies has data on profits in millions of dollars. The sample variance is $\mathbf{s^2}$. The data is rescaled to express profits in thousands of dollars. State the sample variance of the rescaled data. Carefully show your answer.

Question 2 In a large city the number of cars owned by an occupant (or occupants) of an apartment/condo unit varies as given in the table.

Number of cars	0	1	2
Proportion	0.2	0.5	0.3

Let the random variable **X** be the number of cars owned by an an apartment/condo unit. Car owners must pay a monthly parking fee of $30 per car. Let the random variable **Y** be the monthly parking rental payment collected from an apartment/condo unit.

Answer the questions below. Clearly show numeric formula for all answers.

(a) **[5 points]** For four apartment/condo units selected at random find the probability that two of them will own at least one car.

(b) **[5 points]** Find **E(X)** and use this to find **E(Y)**.

(c) **[5 points]** Find $\mathbf{E(X^2)}$ and use this to find **Var(X)** and **Var(Y)**.

(d) **[5 points]** State the probability function for **Y** and use this to find **E(Y)**.

(e) **[5 points]** The random variable **W** is 1 if the apartment/condo unit has a view and 0 otherwise. P(**W**=1) = 0.2.

Of apartment/condo units with no cars, 5% have a view.

For apartment/condo units with a view, find the probability that the unit has no cars.

(f) **[5 points]** For the random variable **W** in the previous question: Are the random variables **W** and **X** independent? Carefully show your answer.

ANSWER KEY **Section 1** Exam B

Question 1 Observations are ***multiplied*** by 1000. The variance is $(1000)^2 \cdot s^2$.

Question 2 (a) Use the binomial distribution.

The probability that a unit owns at least one car is p=0.8. Now find: $\frac{4!}{2!\,2!} p^2 (1-p)^2$

(b) E(X) = (0)(0.2) + (1)(0.5) + (2)(0.3) = 1.1

E(Y) = 30 E(X) = 30(1.1)

(c) $E(X^2) = 0.5 + (4)(0.3) = 1.7$

$Var(X) = E(X^2) - \mu_X^2 = 1.7 - (1.1)^2$

Var(Y) = (30)(30) Var(X)

(d) P(Y=0) = 0.2, P(Y=30) = 0.5 and P(Y=60) = 0.3

E(Y) = (0)(0.2) + (30)(0.5) + (60)(0.3) (a final numeric answer is not needed).

(e) Use Bayes Theorem.

$$P(X = 0 \mid W = 1) = \frac{P(W = 1 \mid X = 0) P(X = 0)}{P(W = 1)} = \frac{(0.05)(0.2)}{0.2} = 0.05$$

(f) $P(X = 0 \mid W = 1) \neq P(X = 0) = 0.2$

Therefore, W and X are ***not*** independent.