

Oct 10, 2013 12:30 to 1:45 PM Instructor: D. WHISTLER

Section 1 Exam A

Total points = 35

Question 1 [5 points] A survey of companies has data on sales in thousands of dollars. The sample variance is s^2 . The data is rescaled to express sales in millions of dollars. State the sample variance of the rescaled data. Carefully show your answer.

Question 2 In a large city the number of cars owned by an occupant (or occupants) of an apartment/condo unit varies as given in the table.

Number of cars	0	1	2
Proportion	0.3	0.6	0.1

Let the random variable X be the number of cars owned by an an apartment/condo unit. Car owners must pay a monthly parking fee of \$40 per car. Let the random variable Y be the monthly parking rental payment collected from an apartment/condo unit.

Answer the questions below. Clearly show numeric formula for all answers.

- (a) **[5 points]** Find $E(X)$ and use this to find $E(Y)$.
- (b) **[5 points]** State the probability function for Y and use this to find $E(Y)$.
- (c) **[5 points]** For five apartment/condo units selected at random find the probability that three of them will own at least one car.
- (d) **[5 points]** Find $E(X^2)$ and use this to find $\text{Var}(X)$ and $\text{Var}(Y)$.
- (e) **[5 points]** The random variable W is 1 if an occupant of an apartment/condo unit has a university degree and 0 otherwise. $P(W=0) = 0.7$.
Of apartment/condo units with no cars, 40% had an occupant with a university degree. For apartment/condo units with an occupant with a university degree, find the probability that the unit has no cars.
- (f) **[5 points]** For the random variable W in the previous question: Are the random variables W and X independent? Carefully show your answer.

Question 1 Observations are *divided* by 1000. The variance is $s^2 / (1000)^2$.

Question 2 (a) $E(X) = (0)(0.3) + (1)(0.6) + (2)(0.1) = 0.8$

$$E(Y) = 40 E(X) = 40(0.8)$$

(b) $P(Y=0) = 0.3$, $P(Y=40) = 0.6$ and $P(Y=80) = 0.1$

$$E(Y) = (0)(0.3) + (40)(0.6) + (80)(0.1) \quad (\text{a final numeric answer is not needed}).$$

(c) Use the binomial distribution.

The probability that a unit owns at least one car is $p=0.7$. Now find: $\frac{5!}{3! 2!} p^3 (1-p)^2$

(d) $E(X^2) = 0.6 + (4)(0.1) = 1$

$$\text{Var}(X) = E(X^2) - \mu_X^2 = 1 - (0.8)^2$$

$$\text{Var}(Y) = (40)(40) \text{Var}(X)$$

(e) Use Bayes Theorem.

$$P(X=0 | W=1) = \frac{P(W=1 | X=0)P(X=0)}{P(W=1)} = \frac{(0.4)(0.3)}{0.3} = 0.4$$

(f) $P(X=0 | W=1) \neq P(X=0) = 0.3$

Therefore, W and X are *not* independent.