CONCORDIA UNIVERSITY

Department of Civil Engineering

MID-TERM EXAM

CIVI 341/2, SEC.V
Civil Engineering Systems
DATE : October 18, 1990
Prof. Y. Lu

NAME : _____
I.D. : _____

INSTRUCTIONS :

1. No books or notes are allowed.
2. Do all FOUR QUESTIONS
3. ~~Write your answers on the mid-term examination itself, attaching extra pages if necessary.~~
4. To receive partial credit for wrong answers, you must show your work.

25% 1. Solve the following optimization model :

Minimize $Z = 2x+2x^2 + 6xy$

Subject to $x + y = 1$

$x, y \geq 0$

a) By the method of direct substitution

b) By the method of Lagrangian multipliers

25% 2. The systems method is a general problem-solving technique. Supposed that Concordia University is planning a sports stadium for football, track and field, and baseball in the near future. Please describe BRIEFLY what might be involved in applying each phase of the systems method in order to assist in this planning?

25% 3. You are an engineer working for a construction firm that plans to build an apartment complex. You have a choice of building some combination of one-, two-, and three-bedroom apartments. A one-bedroom apartment requires 1000 board ft of lumber, 10 yd^3 of concrete, and 20 rolls of insulation. A two-bedroom apartment requires 1400 board ft of lumber, 12 yd^3 of concrete, and 24 rolls of insulation. A three-bedroom apartment requires 1600 board ft of lumber, 13 yd^3 of concrete, and 26 rolls of insulation. A one-bedroom apartment rents for $250 per month, a two-bedroom apartment for $300 per month, and a three-bedroom for $325 per month. Local ordinances require at least one three-bedroom apartment for every three one-and-two bedroom apartments. Your firm has set aside 100,000 board ft of lumber, 1200 yd^3 of concrete, and 2500 rolls of insulation for this project. Formulate this as a linear programming problem to maximize rental income. Give objective function and constraints.

25% 4. Solve the following linear programming problem by the graphical method. Please show the feasible region and also give the optimal solution.

Maximize $Z = 6x + 5y$

Subject to

$2x + 2y \leq 24$

$2x - y \geq 4$

$3x - 4y = 6$

$x \geq 2$

$y \geq 1$

94/100

good

CONCORDIA UNIVERSITY

Department of Civil Engineering

CIVI 341/2 Civil Engineering System

Midterm Exam

Date : Thursday, Oct.31, 1991 **Time :** 11:45 a.m -- 1:00 p.m

Instructor : Dr. Y. Lu

Special Instructions:

1. Closed book examination.
2. Write your answer on the exam paper.
3. Please write your name and I.D. at the top of each page.

Question 1 : (25 marks)

The unit selling price of an item is \$150/unit, thus the total revenue is R=150q. The production cost C is a function of output level q, $C=100q^{1.2}$. The maximum output of the firm is 50 unit per year. Formulate a mathematical model to maximize profit subject to the production constraint. Find the optimum level of production. Prove your answer is a maximum.

$P = R - C$

Maximize $P = 150 q_j - 100 q_j^{1.2}$

$0 \le q_j \le 50$ units ✓

$\frac{dP}{dq} = 150 - 120 q_j^{0.2} = 0$

$q_j = 3.05$

$\frac{d^2P}{dq^2} = -24 q_j^{-0.8} = -$ ✓ $\Rightarrow$

optimum [illegible]

$= 76.$ ✓

Question 5 : (25 marks)

Given the following information, develop a model that could be used to find the minimum transportation cost for supplying each project. (No calculation is needed)

Project	Requirement (truckloads/week)	Plant	Production Capacity (truckloads/week)
A	45	W	45
B	50	X	40
C	20	Y	50

Transportation costs ($ ×1000/truckload)

To:	Project A	Project B	Project C
From			
Plant W	5	10	10
Plant X	20	30	20
Plant Y	5	8	12

REQUIRED/WEEK

From A B C

W x_1 x_2 x_3

X x_4 x_5 x_6

Y x_7 x_8 x_9 ✓

(1) $x_1 + x_4 + x_7 = 45$

$x_2 + x_5 + x_8 = 50$

$x_3 + x_6 + x_9 = 20$

FULFILL REQUIRED

(2) $x_1 + x_2 + x_3 \leq 45$

$x_4 + x_5 + x_6 \leq 40$

$x_7 + x_8 + x_9 \leq 50$

DON'T TAKE TOO MUCH FROM ANY OF THE PLANTS

(3) Cost at A = $5x_1 + 20x_4 + 5x_7$

Cost at B = $10x_2 + 30x_5 + 8x_8$

Cost at C = $10x_3 + 20x_6 + 12x_9$

min C = cost A + cost B + cost C

$x_i \geq 0$ where $i = 1, 2, 3 \ldots 9$ ✓

-3

Question 4 : (25 marks)

Solve the following linear programming problem using the simplex method

Maximize $z = -x_1 + 2x_2$

$$-x_1 + x_2 \leq 8$$

$$x_1 - x_2 \leq 8$$

$$x_1 + x_2 \leq 15$$

$$x_2 \leq 9$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
1	1	-2	0	0	0	0	0
0	-1	1	1	0	0	0	8
0	1	1	0	0	1	0	15
0	0	1	0	0	0	1	9
1	-1	0	2	0	0	0	16
0	-1	1	1	0	0	0	8
0	0	0	1	1	0	0	16
0	2	0	-1	0	0	1	7
1	1	0	-1	0	0	1	1
1	0	0	1	0	0	1	17
0	0	1	0	0	0	1	9
0	0	0	1	0	0	-1	16
0	0	0	1	0	0	-1	5
0	1	0	-1	0	0	1	1

$x_1 = 1$
$x_2 = 9$
$z = 17$

OPTIMUM SOLUTION

Question 3 : (25 marks)

Solve the problem by graphical means and clearly show the feasible region.

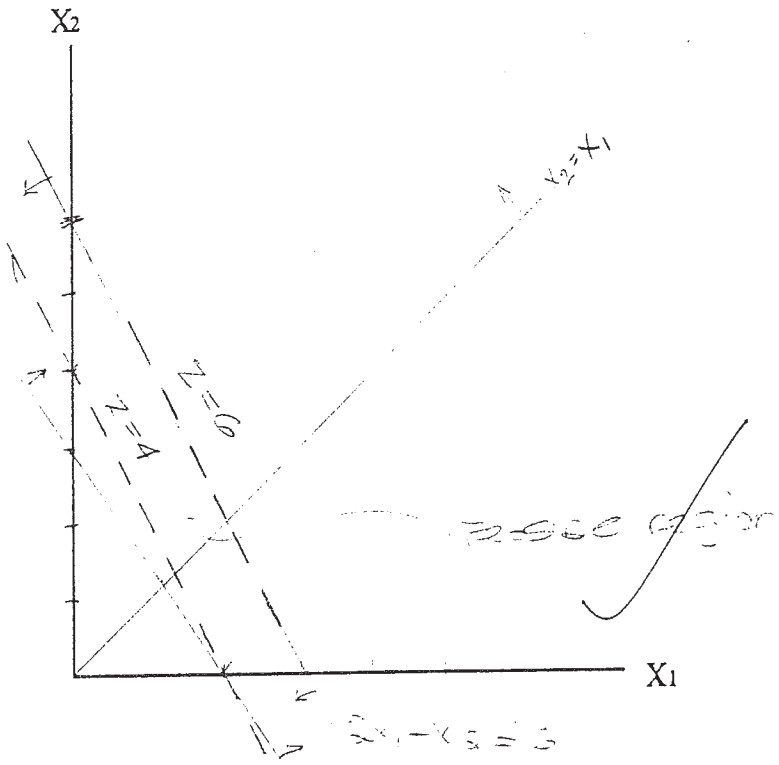
Maximize $z = 2x_1 + x_2$

$$2x_1 + 3x_2 \geq 6$$

$$2x_1 + x_2 \leq 6$$

$$x_2 \geq x_1$$

$$x_1 \geq 0, \quad x_2 \geq 0$$



Maximum $z = 6$ at the line $2x_1 + x_2 = 6$

Question 2 : (~~25~~ 20 marks)

Define the following terms briefly :

1. Problem definition. (~~5~~ 4 marks)
2. System structure. (~~5~~ 4 marks)
3. Optimization. (~~5~~ 4 marks)
4. Objective function. (~~5~~ 4 marks)
5. Hierarchical system. (~~5~~ 4 marks)

1 - Problem definition

Finding out the scope of the problem - what data will be required what the output will be. Generally - requirements for writing proposal ✓

2 - System structure

The way the components of a system [illegible] and interact with each other ✓

3 - Optimization

The [illegible] alternative in solving a problem based on a given set of requirements [illegible] ✓

4. Objective function

The [illegible] maximize or minimize [illegible] ✓

5. Hierarchical system

[illegible] and consider only the [illegible] of a problem ✓

(-3)