Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004, Fall 2011

TEST 4

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, B2, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[3 marks]** Which one of the following values represents the true area under the curve $y = f(x)$ between the points $x = 0$ and $x = 9$ where $f(x) = \sqrt{2x}$.

a (a) $18\sqrt{2}$ (b) $9\sqrt{2}$ (c) 27 (d) $19/\sqrt{2}$

2. **[3 marks]** Evaluate $I = \int_0^1 (3\sqrt{x} - 3x^2)\, dx$.

a (a) $I = 1$, (b) $I = 2$, (c) $I = 3/\sqrt{3}$, (d) $I = 3/2$

3. **[3 marks]** What is the most general antiderivative of the function $f(x) = 3xe^{x^2}$?

c (a) $3e^{x^2} + C$, (b) $6x^2e^{x^2} + C$, (c) $\frac{3}{2}e^{x^2} + C$, (d) xe^{x^3}

4. **[3 marks]** Find the most general antiderivative of the function $\frac{\ln x}{x}$.

c (a) $2(\ln x) + C$ (b) $\ln 1 + C$ (c) $\frac{(\ln x)^2}{2} + C$ (d) $\frac{(\ln x)}{x^2} + C$

5. **[3 marks]** Answer TRUE or FALSE:

a The integral defined by $\int_0^1 x^2e^{-x^2}\, dx$ represents the true area under the graph of the function $x^2e^{-x^2}$ between the lines $x = 0$ and $x = 1$ and above the $x-$axis.

(a) TRUE, (b) FALSE

PART II: Show all work here.
No additional pages will be accepted

6. **[5+4 marks]** Evaluate the following integrals using any method:

a) $\int_0^{\pi/2} \sin x \cos x\, dx$

let $u = \sin x$ ← (2)
$du = \cos x\, dx$ ← (1)

$\Rightarrow F(x)$ an a.d. is given by
$F(x) = \int u \cdot du = \frac{u^2}{2}$ ← (1)
$= \frac{\sin^2 x}{2}$

(1)

$\therefore \int_0^{\pi/2} \cdots = F(\frac{\pi}{2}) - F(0) = \frac{1}{2} - 0 = \frac{1}{2}$

(OR)

$u = \sin x$ ← (2)
$du = \cos x\, dx$ ← (1)
$x = 0,\ u = 0$
$x = \frac{\pi}{2},\ u = 1$ } (1)
$\Rightarrow \int_0^{\pi/2} \cdots = \int_0^1 u\, du$
$= \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}$ ← (1)

b) $\int \frac{x}{x^2+1}\,dx. \quad = I$

Let $u = x^2+1$ 1

$du = 2x\,dx$ 1

$x\,dx = \frac{du}{2}$

$\Rightarrow \quad I = \frac{1}{2}\int \frac{du}{u}$ 1

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

or

$= \frac{1}{2}\ln(x^2+1) + C$ 1

7. **[3+3 marks]** Evaluate the following integrals using any method:

a) $\int \frac{3x}{\sqrt{x^2-1}}\,dx. \quad = I$

b) $\int \sqrt{x}\,(x^{3/2}-1)\,dx.$

a) $u = x^2-1$ 1

$du = 2x\,dx$

$x\,dx = \frac{du}{2}$

$I = \frac{3}{2}\int \frac{2x\,dx}{\sqrt{x^2-1}}$

$= \frac{3}{2}\int \frac{du}{\sqrt{u}} = \frac{3}{2}\int u^{-1/2}\,du$ 1

$= \frac{3}{2}\cdot\frac{u^{1/2}}{\frac{1}{2}} = 3\sqrt{u} + C$

$= \underline{3\sqrt{x^2-1} + C}$

1

b) $I = \int \sqrt{x}\,(x^{3/2}-1)\,dx = \int (x^2 - \sqrt{x})\,dx$ 1

$= \underbrace{\frac{x^3}{3} - \frac{2^{-1}}{\sqrt{x}}}_{1} + \underset{1}{C} = \underline{\frac{x^3}{3} - \frac{1}{2\sqrt{x}} + C}$