Multiple-Choice Questions
Please choose only one answer and insert in PENCIL in your Scantron sheet.

1. **[3 marks]** Evaluate $\lim_{x\to 0} \dfrac{\sin 8x}{16x}$.

   (a) -1
   (b) $1/2$
   (c) $3/2$
   (d) 0

2. **[4 marks]** Let $f(x) = \text{Arctan}\sqrt{x^2+1}$. Evaluate $f'(1)$. (Note that $\text{Arctan}\, x$ and $tan^{-1}x$ represent the same function).

   (a) $f'(1) = \dfrac{1}{\sqrt{3}}$
   (b) $f'(1) = \dfrac{1}{2}$
   (c) $f'(1) = \dfrac{1}{3\sqrt{2}}$
   (d) $f'(1) = 1$

3. **[3 marks]** Let $f(x) = 2|x+1|+1$. Calculate

$$L = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}.$$

   (a) $L = 0$
   (b) $L = 1$
   (c) $L = -1$
   (d) This limit does not exist

4. **[4 marks]** Find the derivative of the function f defined by $f(x) = x^{3x}$.

   (a) $3x^{3x}(1+\ln x)$
   (b) $3x^{3x-1}$
   (c) $3x^{3x}$
   (d) $x^{3x}(1+2\ln x)$

5. **[3 marks]** A differentiable function f with a differentiable inverse, F, has the property that $f'(1) = 1/2$ and $f(1) = 1/2$. What is the value of the derivative of the inverse of f at $x = 1/2$? That is, calculate $F'(1/2)$.

   (a) 1
   (b) 2
   (c) 0
   (d) 1/2

6. **[3 marks]** Let $f(x) = 5^{\sqrt{x+1}}$. Evaluate $f'(8)$. In other words, find the derivative of f at $x = 8$.

   (a) $\dfrac{125 \ln 5}{6}$
   (b) $\dfrac{9 \ln 3}{2}$
   (c) 125
   (d) $\dfrac{9 \ln 5}{2}$

7. **[3 marks]** Find the derivative of the function f defined by $f(x) = \dfrac{x}{x^2+1}$.

   (a) $f'(x) = \dfrac{1}{2x}$
   (b) $f'(x) = \dfrac{1}{(x^2+1)^2}$
   (c) $f'(x) = \dfrac{1-x^2}{(x^2+1)^2}$
   (d) $f'(x) = \dfrac{1}{x(x^2+1)}$

8. **[3 marks]** Evaluate the limit: $L = \lim\limits_{x\to\infty} \sqrt{x}\,(\sqrt{x+1} - \sqrt{x})$.

   (a) $L = 1/3$
   (b) $L = 1/2$
   (c) $L = 1$
   (d) $L = 0$

9. **[2 marks]** Let y be given implicitly as a differentiable function of x by $ye^{xy-1} = 1$. Calculate the value of the derivative $\dfrac{dy}{dx}$ at the point (x, y) where $x = 1$, $y = 1$:

   (a) 1,
   (b) $-1/3$,
   (c) 0,
   (d) $-1/2$

10. **[3 marks]** Evaluate

$$I = \lim_{x \to 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{2t}\, dt$$

(a) $I = 0$

(b) $I = \dfrac{1}{4}$

(c) This limit does not exist

(d) $I = \dfrac{1}{2}$

11. **[3 marks]** Evaluate $L = \lim\limits_{x \to 0^+} \dfrac{\log_2 \sqrt{x}}{\ln x}$ using any method.

(a) $L = \dfrac{1}{2 \ln 2}$

(b) $L = 1$

(c) $L = 2$

(d) $L = \dfrac{1}{\log_2 2}$

12. **[4 marks]** The function f defined by $f(x) = 2x^3 - 6x^2 - 12x + 1$ is concave up at each point of which of the following intervals?

(a) $-12 < x < 0$

(b) $0 < x < 1$

(c) $1 < x < \infty$

(d) $-\infty < x < 1$

13. **[4 marks]** Determine ALL the horizontal asymptotes of the function f defined by

$$f(x) = \frac{x|x|}{x^2 + 1}.$$

(a) $y = 0$ is the only asymptote

(b) $y = -1$, $y = 1$

(c) $y = 1/2$, $y = 1/3$

(d) $y = 1$, $y = -1/2$

14. **[4 marks]** For what value of a does the function defined by

$$f(x) = x^4 + ax^3 - 6$$

have a point of inflection at $x = 1$?

(a) $a = -2$
(b) $a = 1$
(c) $a = -4$
(d) $a = 0$

15. **[5 marks]** Find all the critical points of the function f defined by $f(x) = x^2\, e^{-x}$.

(a) $x = 0$ only
(b) $x = 0$ and $x = 1$ only
(c) $x = e$ and $x = -1$ only
(d) $x = 0$ and $x = 2$ only

16. **[4 marks]** Determine the largest interval I on which the function f, whose domain is the set of all real numbers, and which is defined by $f(x) = e^{-x^2}$ is increasing on I.

(a) $0 < x < \infty$
(b) This function is never increasing.
(c) $-\infty < x < \sqrt{2}/2$
(d) $-\infty < x < 0$

17. **[5 marks]** Find the most general antiderivative of the function f defined by

$$f(x) = \frac{1 - 2\sin 2x}{x + \cos 2x}.$$

(a) $\ln|1 + 2\cos 2x| + C$, where C is a constant
(b) $\ln|1 - \sin 2x| + C$, where C is a constant
(c) $\ln|x + \cos 2x| + C$, where C is a constant
(d) $\ln|x - \sin 2x| + C$, where C is a constant

18. **[3 marks]** Evaluate $f(x) = \int_0^{\pi/8} \tan 2x \; dx$.

(a) $\frac{1}{2} \ln 2 - 1$

(b) $\frac{1}{2} \ln 8$

(c) $\frac{1}{4} \ln 2$

(d) $\frac{1}{2} \ln |\sec^2 2x|$

19. **[4 marks]** Evaluate $\int_0^1 x \; e^x \; dx$

(a) $1 - \frac{2}{e}$

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{e-1}{2}$

20. **[3 marks]** The improper integral $\int_0^\infty x \, 3^{-x^2} \, dx$ has the value

(a) $-\frac{1}{3 \ln 3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{\ln 3}$

(d) $\frac{1}{2 \ln 3}$

21. **[4 marks]** Evaluate $\int x^2 \ln x \; dx$.

(a) $\frac{1}{3} x^3 (\ln x - \frac{1}{3}) \; + \; C$

(b) $\frac{1}{3} x^3 \ln x - \frac{x}{9} \; + \; C$

(c) $3(\ln x)^2 + \frac{1}{9} \; + \; C$

(d) $2x^2 (\ln x) \; + \frac{x}{9} \; + \; C$

22. **[4 marks]** Evaluate and simplify the indefinite integral: $\displaystyle\int \frac{3^{\sqrt{x}}}{\sqrt{x}}\,dx.$

(a) $\dfrac{3^{\sqrt{x}}}{\ln x} + C$

(b) $\dfrac{3^{\sqrt{x}+1}}{4} + C$

(c) $\dfrac{2\,3^{\sqrt{x}}}{\ln 3} + C$

(d) $\dfrac{4\,3^{\sqrt{x}}}{\ln 2} + C$

23. **[3 marks]** Evaluate

$$\int \frac{3x+1}{x(x-1)}\,dx$$

using the method of partial fractions.

(a) $2\ln|x-1| + 3\ln|x| + C$

(b) $\ln|x-1| - 2\ln|x| + C$

(c) $4\ln|x-1| - \ln|x| + C$

(d) $2\ln|x-1| + 4\ln|x| + C$

24. **[4 marks]** Evaluate the indefinite trigonometric integral

$$\int \sec^6 x\ \tan^3 x\ dx.$$

(a) $\dfrac{\sec^3 x\ \tan^3 x}{3} + C$

(b) $\dfrac{\sec^8 x}{8} - \dfrac{\sec^6 x}{6} + C$

(c) $\dfrac{\sec^8 x}{7} + \dfrac{\sec^6 x}{5} + C$

(d) $\dfrac{\sec^3 x}{5} - \dfrac{\tan^4 x}{4} + C$

25. **[3 marks]** Evaluate the improper integral $I = \int_0^1 x \ln x \, dx$

    (a) $I = -\frac{1}{4}$
    (b) $I = 0$
    (c) $I = \frac{1}{3}$
    (d) $I = -\frac{5}{12}$

26. **[4 marks]** Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \sin x$ between $x = 0$ and $x = \pi$ about the $y-$axis.

    (a) $2\pi \int_0^\pi \sin^2 x \; dx$
    (b) $2\pi \int_0^\pi x^2 \; \sin^2 x \; dx$
    (c) $2\pi \int_0^\pi x \; \sin x \; dx$
    (d) $2\pi \int_0^\pi \sin x \; dx$

27. **[4 marks]** Find the area A of the region bounded by the curves defined by $y = x^2$, $y = x^3$ and the lines $x = 0$ and $x = 1$.

    (a) $A = \frac{3}{2}$
    (b) $A = \frac{3}{4}$
    (c) $A = \frac{1}{12}$
    (d) $A = \frac{1}{3}$

28. **[4 marks]** Evaluate $I = \int_0^1 \sqrt{1 - x^2} \, dx$. (Hint: There is an easy way to do this.)

    (a) $I = \pi$
    (b) $I = \frac{\pi}{4}$
    (c) $I = \frac{\pi}{2}$
    (d) $I = \frac{\pi}{3}$

END
Total: [100 marks]

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE

ROUGH WORK HERE