Test 1 Solution (version A)

MAT1320C, Fall 2011
Total = 23 marks

1. (2 marks) Solve the equation $\log_2(x^2 - 2x) = 3$.

$x^2 - 2x = 2^3$, $x^2 - 2x - 8 = 0$, $x = 4$, $x = -2$.

2. (2 marks) Find $\frac{d}{dx}\left(2e^x + 2\sqrt{x} + \frac{1}{3x^3}\right)$.

$$\frac{d}{dx}\left(2e^x + 2\sqrt{x} + \frac{1}{3x^3}\right) = 2e^x + 2\left(\frac{1}{2\sqrt{x}}\right) - \frac{3x^{-4}}{3} = 2e^x + \frac{1}{\sqrt{x}} - \frac{1}{x^4}.$$

3. (2 marks) Evaluate $\int_1^4 \left(\frac{1}{t^2} + e^t\right) dt$.

$$\int_1^4 \left(\frac{1}{t^2} + e^t\right) dt = \left[-\frac{1}{t} + e^t\right]_{t=1}^4 = \left(-\frac{1}{4} + e^4\right) - (-1 + e) = e^4 - e + \frac{3}{4}.$$

4. (2 marks) Find $\frac{d}{dx}\int_0^x \ln(t^2 + 1)\,dt$.

$$\frac{d}{dx}\int_0^x \ln(t^2 + 1)\,dt = \ln(x^2 + 1).$$

5. (3 marks) If $\int_2^8 f(x)dx = 3$ and $\int_2^5 f(x)dx = -2$, what is $\int_5^8 (f(x) - 3)dx$?

$$\int_5^8 (f(x) - 3)dx = \int_2^8 f(x)dx - \int_2^5 f(x)dx - 3\int_5^8 dx = 3 - (-2) - 9 = -4.$$

6. (4 marks) A car is accelerating from $t = 3$ to $t = 9$. The velocity of the car at time t is denoted by $v(t)$. Denote the distance that this car traveled from $t = 3$ to $t = 9$ by $s(3, 9)$.

(a) Write a definite integral that can be used to calculate $s(3, 9)$ from the velocity function $v(t)$.

(b) The following table lists the velocity of the car at $t = 3, 5, 7$, and 9:

t (sec)	3	5	7	9
v (m / sec)	2	3	6	10

Find the best lower bound and the best upper bound of $s(3, 9)$ that you can obtain from the data given in the table.

$\boldsymbol{S}_{\text{Left}}(3) = 2 \times (2 + 3 + 6) = 22$ is the best lower bound.

$\boldsymbol{S}_{\text{Right}}(3) = 2 \times (3 + 6 + 10) = 38$ is the best upper bound.

7. (4 marks) Let

$$g(x) = \begin{cases} 2 + x, & 0 \le x \le 2 \\ 5 - (x/2), & 2 \le x \le 4 \\ 3, & 4 \le x \le 6 \end{cases}$$

Evaluate $\int_0^6 g(x)dx$.

$$\int_0^6 g(x)dx = \int_0^2 (2+x)dx + \int_2^4 \left(5 - \frac{x}{2}\right)dx + \int_4^6 xdx = 2\int_0^2 dx + \int_0^2 xdx + 5\int_2^4 dx - \frac{1}{2}\int_2^4 xdx + \int_4^6 3dx$$

$= 4 + 2 + 10 - 3 + 6 = 19.$

8. (4 marks) State the definition of the derivative $f'(x)$ of a function $y = f(x)$, and **use the definition of the derivative** to find the derivative of the function $y = \sqrt{2x+1}$.

The definition of the derivative is $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$.

The derivative of the function $y = \sqrt{2x+1}$ is

$$\lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} = \lim_{h\to 0} \frac{(\sqrt{2(x+h)+1} - \sqrt{2x+1})(\sqrt{2(x+h)+1} + \sqrt{2x+1})}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})}$$

$$= \lim_{h\to 0} \frac{(2(x+h)+1) - (2x+1)}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} = \lim_{h\to 0} \frac{2h}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})} = \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}.$$