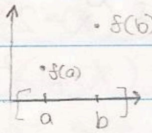
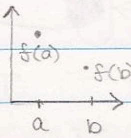
Wed, Sept 11, 2013

Increasing + Decreasing Functions: a function f is called (strictly) increasing on an interval I if $f(a) < f(b)$ whenever $a < b$ on I



- a function f is called (strictly) decreasing on an interval I if $f(a) > f(b)$ whenever $a < b$ in I



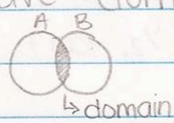
- not called strictly increasing or decreasing if the points are ever equal

Algebra + Functions

- let f and g be any functions, then the sum of $f+g$, the difference $f-g$, and the product fg are functions with:

$$(f+g) = f(x) + g(x)$$
$$(f-g) = f(x) - g(x)$$
$$(fg) = f(x)\,g(x)$$
$$(f/g) = f(x)/g(x) \text{ when } g(x) \neq 0$$

- If f and g have domains A and B respectively, then $f+g$, $f-g$, $f \cdot g$ have domain $A \cap B$



- If f and g have domains A and B respectively, then f/g have domain $A \cap B$ excluding all points where $g(x) = 0$

Composition Functions

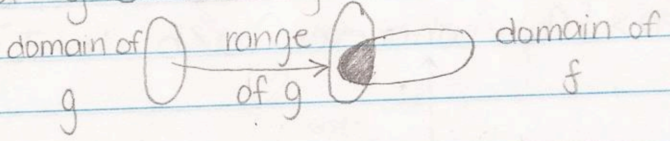
- The composition of f and g is the function
$f \circ g(x) = f[g(x)]$

ie. $f(x) = x^2$ $\qquad f \circ g = f[g(x)]$

$g(x) = x+2$ $\qquad = f(x+2)$

$\qquad = (x+2)^2$

- The domain of $f \circ g$ is the set of all x in the domain of g so that $g(x)$ lies in the domain of f



ie. $g \circ f(x) = g[f(x)]$ $\qquad f \circ g(x) \neq g \circ f(x)$

$= g(x^2)$

$= x^2+2$

ie. $f(x) = x^2-3$ $\longrightarrow$ domain: $(-\infty, \infty)$ range: $[-3, \infty)$

$g(x) = 2-\sqrt{x+1}$ $\longrightarrow$ domain: $[-1, \infty)$ range: $(-\infty, 2)$

↳ domain $f[g(x)]$ is $[-1, \infty)$

↳ domain $g[f(x)]$ is $[\sqrt{2}, \infty)$ or $(-\infty, \sqrt{2})$

↳ $x^2-3 \geq -1$

$x^2 \geq 2$

$x \geq \sqrt{2}$

ie. $f(x) = \sqrt{2-x}$ $\rightarrow$ domain: $(-\infty, 2]$

$g(x) = 1/x$ $\rightarrow$ domain: $(-\infty, 0) \cup (0, \infty)$

↳ domain $f[g(x)]$ is $[1/2, \infty)$

↳ $1/x \leq 2$

$x \geq 1/2$

↳ domain $g[f(x)]$ is $(-\infty, 2) \cup (2, \infty)$

↳ $\sqrt{2-x} \neq 0$

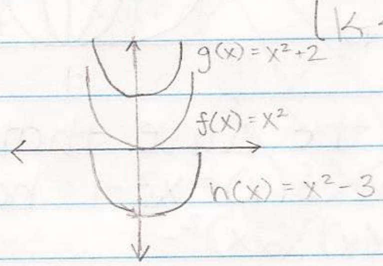
$x \neq 2$

Operations on Graphs

Shifting

A. Given a function/graph $f(x)$

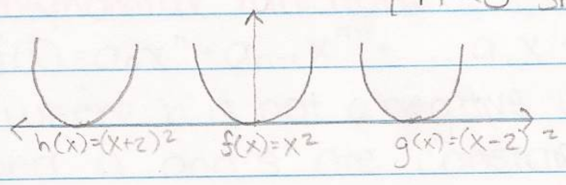
$g(x) = f(x) + k$ $\begin{cases} k > 0 \text{, shifts the graph up k units} \\ k < 0 \text{, shifts the graph down k units} \end{cases}$



ie. Give the equation of the function which shifts down the graph of the function $f(x) = x^2$ by 2 units. $g(x) = x^2 - 2$

B. Given a function/graph $f(x)$

$g(x) = f(x+h)$ $\begin{cases} h > 0 \text{ shifts the graph to the left} \\ h < 0 \text{ shifts the graph to the right} \end{cases}$



ie. Given the equation of the function which shifts the graph of $f(x) = x^2 + x + 2$ 2 units to the right. $g(x) = (x-2)^2 + (x-2) + 2$

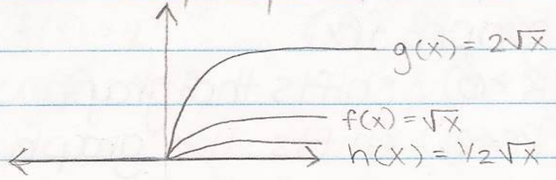
$= x^2 + 3x + 4$

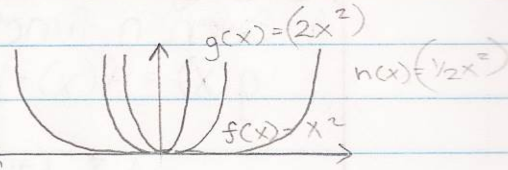
Scaling

$c > 1$

- $g(x) = cf(x)$ stretches the graph of f vertically by a factor of c
- $g(x) = \frac{1}{c} f(x)$ compresses the graph of f vertically by a factor of c.
- $g(x) = f(cx)$ compresses the graph of f horizontally by a factor of c

- $g(x) = f(\frac{1}{c}x)$ stretches the graph of f horizontally by a factor of c





ie. Give the equation of the curve that compresses the graph of $f(x) = x^2 - 1$ horizontally by a factor of 2. $g(x) = (2x)^2 - 1$

$= 4x^2 - 1$

Reflecting

- $g(x) = -f(x)$ reflects the graph of f across the x-axis
- $g(x) = f(-x)$ reflects the graph of f across the y-axis