

Concordia University
Department of Electrical and Computer Engineering
Fundamentals of Control Systems (ELEC 372)
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Final Exam
7 Questions, 10 pages
December 5, 2008

Family name:

Given name:

Student ID:

Time allotted: Three hours.

Instructions:

- The exam is closed-book.
 - No crib sheet is permitted.
 - Only faculty-approved calculators are permitted.
 - Answer each question within the space provided.
 - You may use the last blank sheet if necessary.
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- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.

List of the formulas

$$PO = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$T_s = \frac{4}{\zeta\omega_n}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$T_{r1} = \frac{2.16\zeta + 0.6}{\omega_n} \quad (0.3 \leq \zeta \leq 0.8)$$

$$M_{p\omega} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (0 < \zeta < \frac{\sqrt{2}}{2})$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} \quad (0 < \zeta < \frac{\sqrt{2}}{2})$$

$$\omega_B = (-1.196\zeta + 1.85)\omega_n \quad (0.3 \leq \zeta \leq 0.8)$$

$$\zeta = \frac{\phi_{pm}}{100}$$

$$\omega_B = 1.6\omega_c$$

$$e^{-sT} \approx \frac{1 - sT/2}{1 + sT/2} \quad (\text{First-order Padé approximation})$$

$$e^{-sT} \approx \frac{1 - sT/2 + s^2T^2/12}{1 + sT/2 + s^2T^2/12} \quad (\text{Second-order Padé approximation})$$

$$\sin \phi_m = \frac{\alpha - 1}{\alpha + 1} \quad (\alpha = \frac{p}{z})$$

$$\omega_m = \sqrt{zp}$$

1. (15%) Consider a second-order system with a transfer function of

$$\frac{Y(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Here $R(s) = \mathcal{L}\{r(t)\}$ and $Y(s) = \mathcal{L}\{y(t)\}$ are the input and output. In order to determine the parameters K , ζ and ω_n , a step input of size 2 ($R(s) = 2/s$) is applied to the system. The corresponding response is given in the following figure. Find estimates for K , ζ and ω_n . Briefly explain your answer.

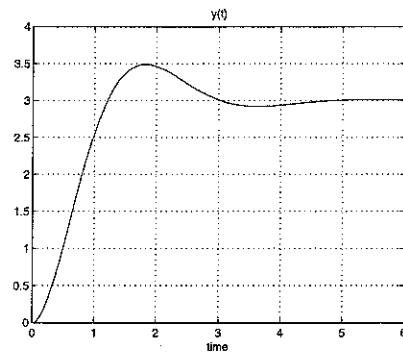


Figure 1: Problem 1: Output $y(t)$.

$$K=1.5, \quad \zeta=0.5, \quad \omega_n=2$$

2. (15%) Consider the following feedback system where

$$K(s) = \frac{K}{s} \quad G(s) = \frac{2}{s^2 + 4s + 1}$$

- Determine the range of K so that the closed-loop system is stable.
- Assume $K = 2$. If $r(t) = 0$ and $d(t) = 3 \sin 2t$ (for $t \geq 0$), find the output $y(t)$ in the *steady state*.

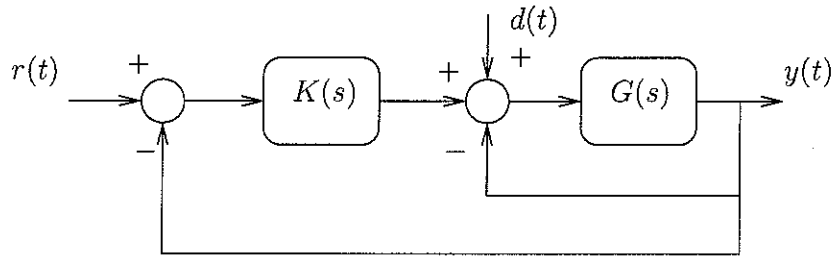


Figure 2: Problem 2.

$$0 < K < 6$$

$$y_{ss}(t) = 0.9864 \sin(2t - 1.74)$$

3. (15%) The open-loop transfer function of the following feedback system is given by

$$G(s) = \frac{s - 1}{(s + 1)(s + 3)(s + 4)}$$

where $K > 0$. Plot the root locus of the closed-loop system for $K > 0$, showing all relevant steps clearly. Calculation of breakaway points (if any) and angles of departure from poles and arrival at zero are not required.

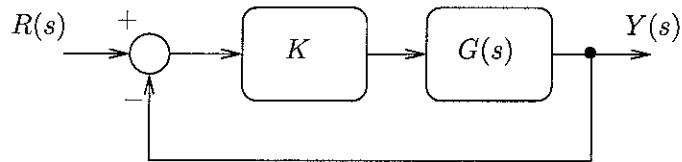
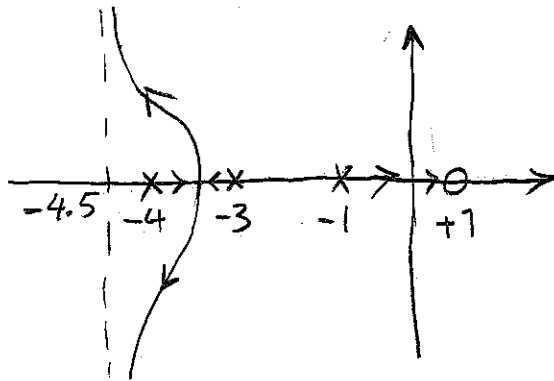


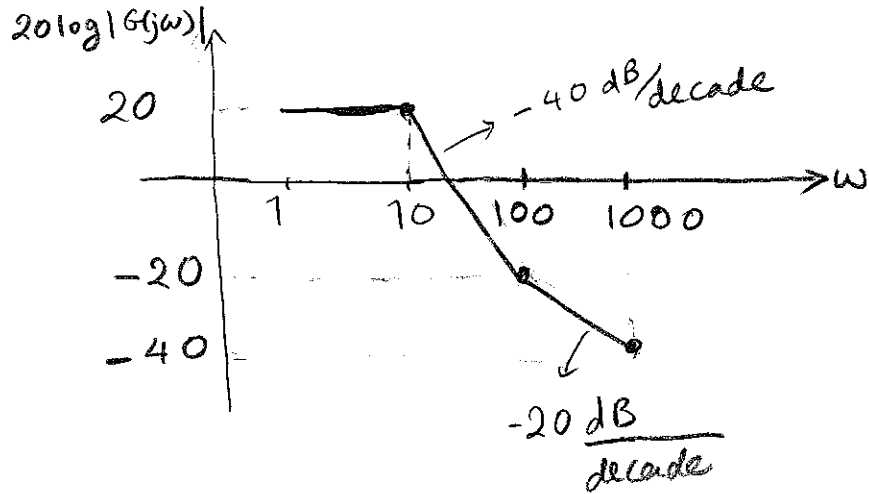
Figure 3: Problem 3



4. (10%) Consider a system with the transfer function

$$G(s) = \frac{10(s + 100)}{(s^2 + 8s + 100)}$$

Draw $20 \log |G(j\omega)|$ versus ω , for $1 \leq \omega \leq 1000$ (Use a logarithmic scale for the ω axis).



5. (15%) Consider the following feedback system. The Bode diagrams for $G(j\omega)$ (magni-

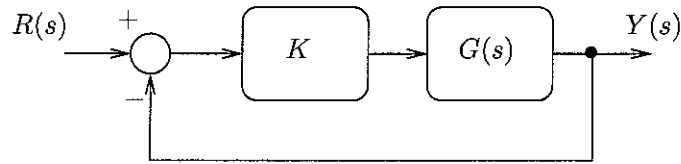
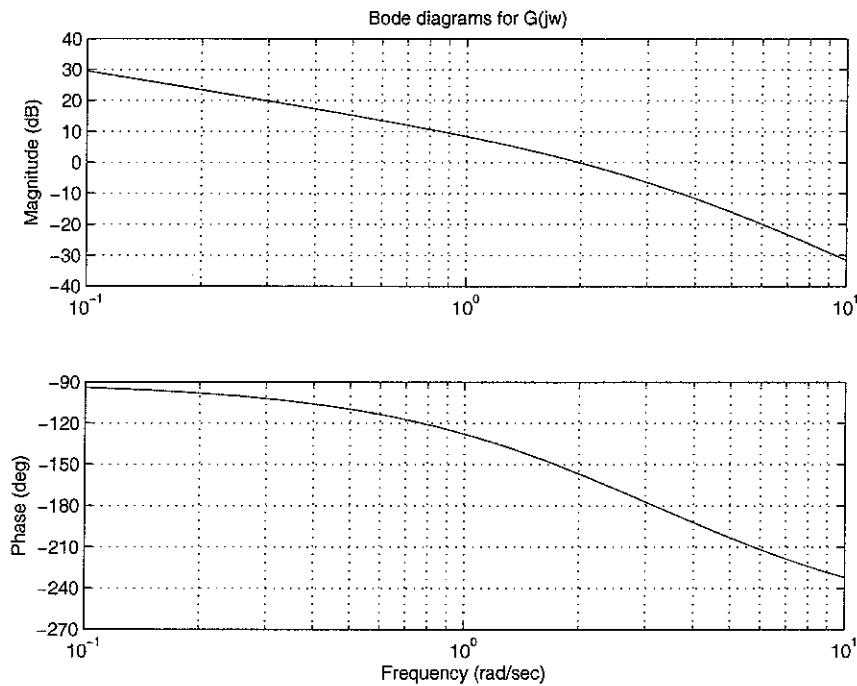


Figure 4: Problem 5

tude (dB) and phase (deg)) are given below.

- Select K so that the phase margin of the system is 70° . Provide clear explanation.
- Suppose K is selected according to part(a). Estimate (i) the bandwidth of the closed-loop system, (ii) the percentage of overshoot and the rise-time of the step response of the closed-loop system.



(a) $K = 0.18$ or -15 dB

(b) $\omega_B = 0.8$ rad/s, P.O. = 5%, $T_r = 2.7$ s

6. (10%) In the following feedback system $G(s)$ is stable and $K > 0$. The polar plot of

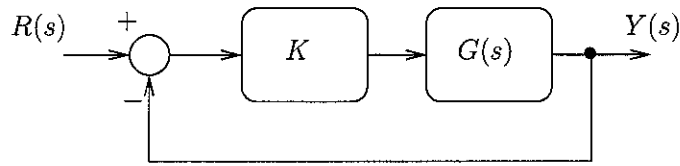
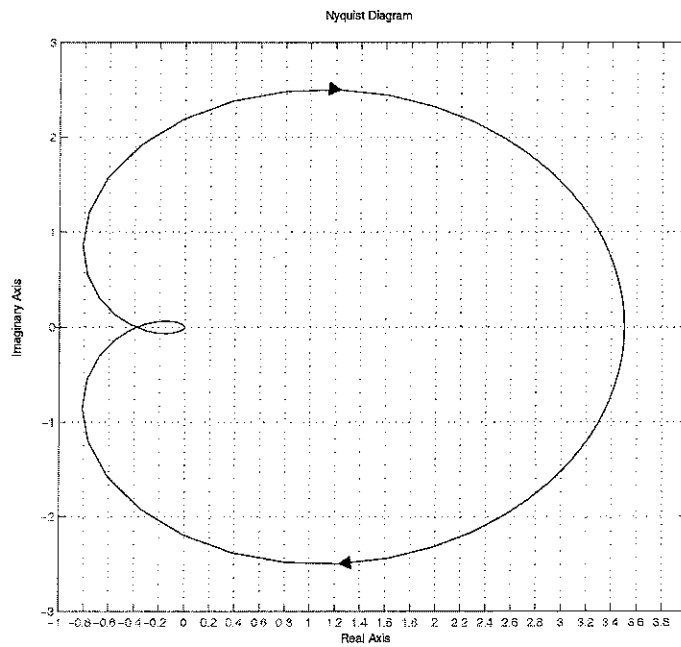


Figure 5: Problem 6

$G(j\omega)$ (for $-\infty < \omega < +\infty$) is given below. Use the Nyquist criterion to determine the range of K for closed-loop stability.



$$0 < K < 2.5$$

7. (20%) The unity feedback control system shown in the following figure has a plant

$$G(s) = \frac{10}{s^2} .$$

Design the compensator ($K(s)$) so that

- the percentage of overshoot of step response is less than 20%
- the settling time of step response is less than 2 sec and
- the acceleration error constant K_a is greater than or equal to 250.

Provide your calculations and a rough sketch of the root locus.

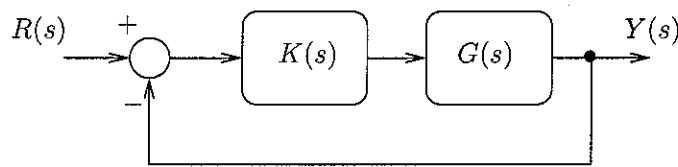


Figure 6: Problem 7

$$K(s) = 16.60 \underbrace{\frac{s+3.6153}{s+41.5948}}_{K_{\text{lead}}(s)} \times \underbrace{\frac{s+0.2}{s+0.0115}}_{K_{\text{lag}}(s)}$$

(Note: The solution is not unique.)

