STAT 2606 Assignment # 2 (Chapters 3&4) Fall 2013

Due in class:

Sections A & B, October 22

Sction C, October 21

Section D, October 23

Last Name ——————————————- , First Name ——————-

Student # ————————— ——————-Lab group:—————

Total of marks=100.

Part I. Lab questions. Use only the blanks left to answer lab questions.

1. (Probability as relative frequency)

   You need to use the Filliping a Fair coin applet to answer this question. This applet is available at click here.

   (a) [1] Flip the coin 10 times what is $\frac{\# \ of \ Heads \ observed}{total \ \# \ of \ flips}$?————— Sol: I had 4/10=0.4— ————-

   (b) [1] (press Reset) Flip the coin 100 times. What is $\frac{\# \ of \ Heads \ observed}{total \ \# \ of \ flips}$? ————— Sol: I had 53/100=0.53—————

   (c) [1] (Press Reset) By pressing Flip for $n = 10000$ once, flip the coin 10000 times. What is $\frac{\# \ of \ Heads \ observed}{total \ \# \ of \ flips}$? ————— Sol: I had 5078/10000=0.5078 —————

   (d) [1] Compare the relative frequencies in part (a), part (b) and part (c). Which one is closer to 0.5? ——————- Sol: part (c) —————

   (e) [1] Suppose that you could flip it infinitely many times, then what the value of $\frac{\# \ of \ Heads \ observed}{total \ \# \ of \ flips}$ would be? —————Sol: 1/2 —————

2. (Conditional probability and Independence) the link click here

   gives an experiment of tossing two fair dice (one green and one red).

   Let C be the event that the green die shows a number less than or equal to 2.

   Let D be the event that the red die shows a number less than or equal to 3.

(a) [2] (Press Reset) Obtain $P(C)$= ————Sol: 12/36——— –

(b) [2] (Press Reset) Obtain $P(D)$= ————-Sol: 18/36 ————

(c) [2] (Press Reset) By clicking on the simple events in which the green die has a number at most 2 and the red one has a number at most 3, obtain $P(C \cap D)$= ————-Sol: 6/36 ————

(d) [2] (Use parts (b) and (c)) If you know that the red die showed a number less than or equal to 3, then obtain the probability that the green shows a number less than or equal to 2, by clicking on the relevant simple events in the applet?
I.e., what is $P(C|D)$ = ————Sol: 6/18 ———-

(e) [2] What is your conclusion about the relation between the events C and D? ———— –Sol: Independent as $P(C|D) = P(C)$ ———

3. Suppose that X has a binomial distribution with $n = 21$ and $p = 0.6$. Use minitab to simulate 20 values of X.
random 20 c1;
binomial 21 0.6.

   (a) [3] How many of your values are less than 21?_________Sol: 20 (the answer is a random number so it may vary)

   (b) [3] How many of your values are between 16 and 29 inclusive?_____________Sol: 1 (the answer is a random number so it may vary)

   (c) [3] If Y has a binomial distribution with $n = 15$ and $p = 0.4$, use the *cdf* command, (it gives you the value of $P(Y \leq k)$), which works by typing
cdf;
binomial 15 0.4.
to calculate:

   (d) [3] $P(Y < 10)$ =____________Sol: 0.96617

   (e) [3] If you simulate 100000 values of Y, what would be the expected number of values (among the 100000 values) that are less than 10?____________.Sol: 96617

4. Suppose that X has a Poisson distribution with mean $\mu = 21$. Use the *cdf* command
cdf;
poisson 21.

to answer the following questions:

(a) [2] $P(X < 10) =$ ______________, Sol:0.002766

(b) [3] $P(9 \leq X \leq 10) =$____________. Sol: 0.006251-0.001106=0.005145

(c) [5] The expected number of values (among the 100000 values) that are less than 10 is ____________.Sol: Approximately 277

5. Suppose that Y has a hypergeometric distribution with parameters $N = 20$, $M = 9$, and $n = 6$. Use the command
cdf 3 ;
hypergeometric 20 9 6.
to find

(a) [2] $P(Y \leq 3) =$_________sol: 0.78328

(b) [3] $P(Y > 3) =$_________.sol: 1-0.78328 =0.21672

Part II Comprehension questions

1. A survey has revealed that 75% of all college students study. It is also known that 85% of all students who study will graduate, while only 35% of those students who do not study will graduate.

(a) [7] If a student is randomly selected, what is the probability that he or she will graduate?

(b) [8] A randomly selected student is observed to graduate. What is the probability that this student studied?

Solution: (a) Let $G = \{student\ graduates\}$;
$H = \{student\ doesn?t\ graduate\}$;
$S = \{student\ studies\}$;
$T = \{student\ doesn?t\ study\}$
$P(G) = P(G|S)P(S) + P(G|T)P(T) = 0.85 \times 0.75 + 0.35 \times (1 - 0.75) = 0.725$
(b) Using Bayes' rule: $P(S|G) = \dfrac{P(G|S)P(S)}{P(G|S)P(S) + P(G|T)P(T)} = \dfrac{0.85 \times 0.75}{0.725} = 0.879.$

2. For events A and B we have

$$P(A) = 0.3,\ P(B) = 0.8,\ P(A \cup B) = 0.9$$

(a) [8] Find $P(A|B)$, $P(\bar{A} \cap B)$.

(b) [2] Are A and B independent? Why?

Solution: (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Longrightarrow P(A \cap B) = 0.3 + 0.8 - 0.9 = 0.2$
$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.8} = 0.25$
$P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.8 - 0.2 = 0.6$
(b) No they are not independent, since
$P(A \cap B) \neq P(A)P(B), i.e.,\ 0.2 \neq (0.3)(0.8)$

3. A random variable X can assume five values: $0, 1, 2, 3, 4$. A portion of the probability distribution is shown here:

x	0	1	2	3	4
p(x)	0.2	0.1	0.1	?	0.3

(a) [2] Find p(3).

(b) [2] What is the probability that X is strictly less than 3 and strictly greater than 1;

(c) [4] Calculate the mean of x (μ_x), variance of x (σ_x^2), and standard deviation of σ_x.

(d) [3] What is the probability that x is in the interval $[\mu_x - \sigma_x, \mu_x + \sigma_x]$

(In parts (a)-(d) show steps of your calculations)
SOLUTION

$$(a)\ p(3) = 1 - (0.2 + 0.1 + 0.1 + 0.3) = 0.3.$$

$$(b)\ P(1 < X < 3) = 0.1$$

$$(c)\ \mu_x = 0 \times 0.2 + 1 \times 0.1 + 2 \times 0.1 + 3 \times 0.3 + 4 \times 0.3 = 2.4$$

$$\sigma_x^2 = (0-2.4)^2 \times 0.2 + (1-2.4)^2 \times 0.1 + (2-2.4)^2 \times 0.1 + (3-2.4)^2 \times 0.3 + (4-2.4)^2 \times 0.3 = 2.24$$

$$\sigma_x = \sqrt{\sigma_x^2} = 1.4967$$

$$(d) \text{ first note that} [\mu_x - \sigma_x, \mu_x + \sigma_x] = [0.903, 3.8967]$$

Hence, $P(x \in [\mu_x - \sigma_x, \mu_x + \sigma_x]) = P(1 \leq x \leq 3) = 0.1 + 0.1 + 0.3 = 0.5.$

4. A student is preparing for an upcoming exam. The professor for the course has given the class 30 questions to study from and plans to select 10 of the questions for use on the actual exam. Suppose that the student knows how to solve 25 of the 30 questions.

   (a) [2] What is the probability that the student will get perfect on the test?

   (b) [4] What is the probability that the student will get at least 9 questions correct on the test?

   Solution: (a) $\frac{\binom{25}{10}}{\binom{30}{10}}$.

   Solution: (b) $\frac{\binom{25}{9}\binom{5}{1}}{\binom{30}{10}} + \frac{\binom{25}{10}}{\binom{30}{10}}$

5. The number of flaws on a VHS magnetic tape produced continuously at a factory follows a Poisson distribution with an average of 0.01 flaws per meter. A standard VHS cassette tape contains 200 meters of magnetic tape.

   (a) [2] What is the probability that there are at least two flaws in a single VHS cassette tape?

   (b) [2] What is the probability that there are no flaws in a single VHS cassette tape; that is, a tape is flawless?

   (c) [6] In a random sample of 20 cassettes, what is the probability that at least one of them are flawless? (You are not required to simplify the final answer)

   Solution:
   (a) Let x be the number of flaws in a single VHS. Then x has Poisson distribution with $\mu = 200(.01) = 2$.
   So, $P(x \geq 2) = 1 - P(x \leq 1) = 1 - (.1353 + 0.2707) = 0.594$.
   (b) $P(X = 0) = \exp^{(-2)} = 0.1315$.
   (c) Let Y be the number (out of 20 cassettes) of flawless cassettes. Then Y has binomial distribution with $n = 20$ and $p = P(X = 0) = \exp^{(} -2) = 0.1315$. So $P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - 0.1315)^{20}$.